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Corrections to 'Spin foam quantization of SO(4) Plebanski's action'

A. Perez

Center for Gravitational Physics and Geometry
Pennsylvania State University
University Park, PA

perez@gravity.psu.edu

Abstract

In [1] a systematic spin foam quantization of $SO(4)$ Plebanski's theory is proposed. Both the non-degenerate and degenerate sectors of the theory are studied. In the first part of that work, a mistake in the derivation led to the wrong conclusion that only a special sector of the Barrett-Crane (BC) model[2] could be obtained by our prescription. Correction of the mistake leads to the full BC model providing a clear-cut connection of the spin foam model and an action. One does not need to invoke any ad hoc quantization principle and the model follows directly from a bona fide path integral quantization of simplicial Plebanski's action (more precisely from the corresponding gravity sector[3]). The result is stated here, while a detailed corrected version of [1] can be found in [4].

The second part of [1] regarding the quantization of the degenerate sectors of Plebanski's formulation remains unchanged. Indeed, the second part of the paper is fully independent of the first one.

The building block of the BF amplitude is the projector

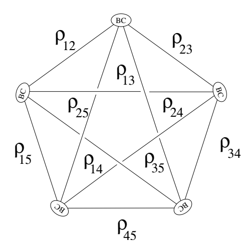
$$P^4_{inv} : \mathcal{H}_{\rho_2} \otimes \mathcal{H}_{\rho_2} \otimes \mathcal{H}_{\rho_3} \otimes \mathcal{H}_{\rho_4} \to \mathrm{Inv}\left[\mathcal{H}_{\rho_2} \otimes \mathcal{H}_{\rho_2} \otimes \mathcal{H}_{\rho_3} \otimes \mathcal{H}_{\rho_4}\right]$$

which is associated with spin foam edges defined on the dual 2-complex of a simplicial decomposition of the space time manifold. P^4_{inv} has clearly an invariant meaning. However, writing the BF amplitudes in a spin foam form requires choosing of a basis in $\mathcal{H}_{\rho_2} \otimes \cdots \otimes \mathcal{H}_{\rho_4}$ at each edge. In [1] (Section 4.2) we overlooked the fact that since the Barrett-Crane intertwiner is in $\mathrm{Inv}\left[\mathcal{H}_{\rho_2} \otimes \cdots \otimes \mathcal{H}_{\rho_4}\right]$ the projector can be simply written as

$$P^4_{inv} = |\Psi_{BC}\rangle \langle \Psi_{BC}| + \quad orthogonal \quad terms, \tag{1}$$

where $|\Psi_{BC}\rangle \in \mathrm{Inv}\left[\mathcal{H}_{\rho_2} \otimes \cdots \otimes \mathcal{H}_{\rho_4}\right]$ is the normalized Barrett-Crane intertwiner. We write the BF partition function in a spin foam form by choosing such decomposition of P^4_{inv} at each edge. It is clear now that the action of the (formal) delta function of the constraints in equation (17) of [1] will project out the *orthogonal terms* in P^4_{inv}. This is because $|\Psi_{BC}\rangle \in \mathrm{Inv}\left[\mathcal{H}_{\rho_2} \otimes \cdots \otimes \mathcal{H}_{\rho_4}\right]$ is the unique solution [5] of Plebanski's constraints defined by equation (20) in [1]. The constrained amplitudes that define our quantization of the gravity sector of Plebanski's theory becomes

$$Z_{const}(\Delta) = \sum_{\mathcal{C}_f:\{f\}\to j_f} \prod_{f\in\Delta^*} \Delta_\rho \prod_{e\in\Delta^*} A_e \prod_{v\in\Delta^*} \tag{2}$$



where $\rho_f = j_f \otimes j^*_f$ and $j_f \in \mathrm{Irrep}[SU(2)]$. A_e is the appropriate edge amplitude (to be determined from the modified path integral measure [6]). The value of A_e has to do with the issue of normalization of the model. Equation (2) is precisely the state sum amplitude of the Barrett-Crane model! Our prescription can be naturally generalized to the Lorentzian sector.

References

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