Matrix Description of Interacting Theories in Six Dimensions

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Abstract

We propose descriptions of interacting (2,0) supersymmetric theories without gravity in six dimensions in the infinite momentum frame. They are based on the large N limit of quantum mechanics or 1+1 dimensional field theories on the moduli space of N instantons in $\mathbb{R}^4$.

1   Introduction

Recent advances in string theory have led to the discovery of many new theories without gravity. These theories are found by taking various limits of the full M-theory in which many of the degrees of freedom decouple. In particular, by restricting attention to processes with energy much lower than the Planck scale, M_p, we decouple gravity. In most cases the resulting theory in this limit is simply a free field theory. In other cases the low energy theory is an interacting local quantum field theory at a nontrivial fixed point of the renormalization group. Finally, as in [1], the limit can yield a string theory,

which is not a standard local quantum field theory (these theories had been anticipated in [2–5]).

The first examples of such nontrivial field theories were found [6] in compactifications of the IIB theory on singular K3 surfaces. These theories are labeled by the A-D-E singularities of the underlying K3. The A type theories also appear when several 5-branes of M-theory in eleven dimensions approach each other [7], and the D type theories appear when several 5-branes of M-theory approach each other at an $\mathbb{R}^5/\mathbb{Z}_2$ singularity.

We now review some of the basic properties of these theories (for a more detailed review see [8]). At generic points in the moduli space of these theories there are k tensor multiplets, each consisting of a two-form field B with a self-dual field strength, five real scalars $\Phi_i, i = 1, \ldots, 5$, and a fermion. The moduli space of vacua is parametrized by the expectation values of the scalars. It is

$$\frac{\mathbb{R}^{5k}}{\mathcal{W}}, \tag{1.1}$$

where $\mathcal{W}$ is a discrete group. The theory at the singularities of this space is an interacting local quantum field theory. Along the moduli space of vacua the theory has BPS string-like excitations. Their tensions vanish at the singularities. Hence, these theories are often referred to as tensionless string theories. However, there is a lot of evidence that they are standard interacting local quantum field theories. In particular, these are (2,0) superconformal field theories with a *Spin*(5) R-symmetry [9]. When these theories are compactified on a circle of radius R, the effective five-dimensional theory is a gauge theory with gauge group G of rank k and $\mathcal{W}$ is its Weyl group. The five-dimensional gauge coupling is $g_5^2 = R$. For the special case of k coincident 5-branes the gauge group is $G = U(k)$.

The five-dimensional gauge theory has, in addition to the gauge bosons, static particle-like BPS configurations, which are instantons in four dimensions. Their energy is N/R where N is the instanton number. This leads to the identification of N with the momentum quantum number along the S^1 [10–12]. As in [13] we can use this fact to study the six-dimensional theory in the lightcone frame. We consider the theory in the infinite momentum frame and compactify the longitudinal direction on a circle of radius R. The momentum N along the circle is the instanton number in the four transverse dimensions. Let us denote the moduli space of instantons by $\mathcal{M}_N(G)$. As in [13], we suggest that the six dimensional (2,0) theory is described by the large N limit of the quantum mechanics on $\mathcal{M}_N(G)$. The finite N theory may provide a discrete light cone description of the theory, as in [14].

Many people [15] have independently suggested that the (2,0) field theory and its stringy extension have a matrix model description. In particular, there was an inspiring paper [4], which is closely related to our work. We

will discuss this further in §4.

In the following we will describe this quantum mechanics and some of its generalizations. In section 2 we describe the quantum mechanical system. In section 3 we will motivate the proposal above by studying the matrix description of several longitudinal 5-branes along the lines of [16]. We will also motivate it by examining the matrix description of IIB compactifications on singular K3 surfaces along the lines of [1] or its low energy limit [17]. In both cases we can scale out gravity to derive the prescription above. Section 4 contains generalizations of this construction to the string theories of [1].

2   The Quantum Mechanical System

Since we are going to describe a space-time theory with 16 supercharges we should study a quantum mechanical system with 8 supersymmetries. The maximal R symmetry is $Spin(8)$ but our system will have only a $Spin(3) \times Spin(5)$ subgroup under which the supercharges transform as $(\mathbf{2}, \mathbf{4})$. One way to construct such theories is to start with a (1,0) supersymmetric field theory in six dimensions and to dimensionally reduce it to quantum mechanics. The first $Spin(3) \cong SU(2)$ factor is an R symmetry, which is present already in six dimensions. The $Spin(5)$ factor appears from the dimensional reduction. (These theories were recently discussed in [18])

For describing the theories with $G = U(k)$ we use a $U(N)$ gauge theory with hypermultiplets in the adjoint representation and k fundamentals. This theory has an $SU(k) \times SU(2) \times SU(2) \times Spin(5)$ global symmetry, where the first $SU(2)$ factor acts on the field in the adjoint representation and the second is part of the R symmetry. For the theories with $G = SO(2k)$ we use an $Sp(N)$ gauge theory with hypermultiplets in an antisymmetric tensor and k fundamentals. This theory has an $SO(2k) \times SU(2) \times SU(2) \times Spin(5)$ global symmetry, where the first $SU(2)$ factor acts on the field in the antisymmetric representation. These theories have Higgs branches, in which both types of hypermultiplets obtain VEVs, which are isomorphic to the moduli spaces $\mathcal{M}_N(SU(k))$ and $\mathcal{M}_N(SO(2k))$, respectively [19]. The $Spin(5)$ factor in the global symmetry does not act on the Higgs branch (it acts only on the fermions). The other global symmetries are spontaneously broken to a subgroup at a generic point in the moduli space.

The quantum mechanical systems, which we are interested in, differ from these in two ways. First, along the Higgs branch the gauge theories have effects from the massive gauge bosons, which have been integrated out. These are not present in the minimal quantum mechanical system with target space $\mathcal{M}_N(G)$. Second, the gauge theories have Coulomb branches emanating from the singularities of $\mathcal{M}_N(G)$. These two differences are removed if we consider the limit in which the gauge coupling g_{QM} goes to infinity. This gives

the Higgsed gauge multiplets infinite masses and, therefore, decouples the Coulomb branch from the interior of the Higgs branch.

The $g_{QM} \to \infty$ limit is the IR limit of the quantum mechanics. In this limit all the higher derivative operators in the effective action are negligible. Furthermore, as in [20], we can promote $1/g_{QM}^2$ to a background vector superfield and use the decoupling of vector superfields from the Higgs branch to show that the metric on the Higgs branch is independent of g_{QM}. This nonrenormalization theorem shows that the strong coupling limit of the quantum mechanics is simply the supersymmetric quantum mechanics with target space $\mathcal{M}_N(G)$. It should be stressed that unlike higher dimensional field theories, in quantum mechanics the singularities in $\mathcal{M}_N(G)$ are quite tame. They merely limit the wave functions to be single-valued around them.

We can generalize this construction for other gauge groups G by studying the supersymmetric quantum mechanics with target space $\mathcal{M}_N(G)$. The global symmetry of this theory is $G \times SU(2) \times SU(2) \times Spin(5)$, where as before the $Spin(5)$ symmetry acts only on the fermions.

3   Motivation From M-Theory

We will now motivate this proposal by studying k coincident 5-branes in eleven dimensions. The theory along the 5-branes is the (2,0) theory of $U(k)$. Let us compactify the longitudinal direction along the 5-branes on a circle of radius R. The k longitudinal branes are now D4-branes. The system with N units of momentum around the circle is then described by k D4-branes and N D0-branes. This system was studied in detail in [11]. The D0-branes are described by a quantum mechanical system with a $U(N)$ gauge symmetry with hypermultiplets in the adjoint and k fundamentals. The Coulomb branch corresponds to the motion of the D0-branes away from the D4-branes. In matrix theory, graviton states live on this branch. The Higgs branch corresponds to D0-branes in the D4-branes where they can grow. This fact can be interpreted by a D4-brane observer as instantons in the $U(k)$ gauge theory on the D4-brane [21].

This quantum mechanical system was used in [16] to describe the longitudinal 5-brane in the matrix theory. Since these theories can be viewed as dimensional reduction of a higher dimensional gauge theory, we find it convenient to rescale all the scalars to have dimension one. Then, the gauge coupling of the quantum mechanical system is

$$g_{QM}^2 = M_p^6 R^3. \tag{3.1}$$

The authors of [16] studied the case $k = 1$, where there is no Higgs branch

in the quantum mechanical system[1]. The Coulomb branch was interpreted there as leading to the space-time transverse to the fivebrane. We are interested in larger values of k, where there is also a Higgs branch. Should it be interpreted as leading to a new branch of space-time? In order to answer this question we should recall how space-time appears in the quantum mechanical systems [13, 22]. One studies low energy states in the quantum system, which are localized far from the singularities and interprets them as particles moving in space-time. The noncompact nature of the Higgs branch might lead to an interpretation as a new branch of space time. We would like to suggest, however, a different interpretation. The degrees of freedom along the Higgs branch correspond to states of the interacting (2,0) theory (other states may be concentrated at the singularities). As an interacting conformal field theory, this theory does not have a particle interpretation. Therefore, we should not interpret the Higgs branch as a new branch of space time. As the space-time moduli are changed so that two 5-branes coincide, the Higgs branch opens up. This leads to IR divergences in the S-matrix corresponding to the lack of well defined particle states and S-matrix. It will be interesting to understand explicitly how the quantum mechanics along the Higgs branch captures the dynamics of the (2,0) field theory, and how to extract the operators and correlation functions of this theory.

In order to focus on the degrees of freedom of the (2,0) field theory we should consider the limit $M_p \to \infty$ holding all other energies fixed. In this limit the degrees of freedom on the eleven-dimensional five-brane decouple from the degrees of freedom in the bulk of space-time. In particular, they decouple from gravity. Now, it is clear from 3.1 that in this limit g_{QM} goes to infinity. As we discussed above, this limit removes the Coulomb branch and restricts the quantum mechanics to the Higgs branch[2]. The $Spin(5)$ R-symmetry in the superconformal algebra arises from the $Spin(5)$ R-symmetry in the quantum mechanics discussed in §2. The $SU(2) \times SU(2)$ global symmetry of the quantum system is interpreted as the transverse Lorentz symmetry. It is spontaneously broken on the Higgs branch. This corresponds to the "breaking" of Lorentz invariance by the instanton positions in the transverse spacetime.

We can also derive the same conclusion by examining the compactification of the type IIB theory on a singular K3, which is dual to M-theory

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[1]There is always an $\mathbb{R}^4$ component of the Higgs branch which corresponds to the center of mass of all the instantons inside the 5-brane. This arises in the gauge theory from the singlet component of the adjoint hypermultiplet (for $U(N)$) or of the antisymmetric multiplet (for $Sp(N)$), and it will give rise to the spacetime on the 5-brane in our description.

[2]In the case $G = U(k)$ corresponding to k coincident fivebranes, there is also a decoupled free tensor multiplet, which has asymptotic scattering states. The corresponding states in the quantum mechanics come from the singularities at the intersection of the Higgs and Coulomb branches. These also decouple from the interacting SCFT.

on $T^5/\mathbb{Z}_2$ [23]. A matrix model description of this theory [1] is based on heterotic fivebranes at zero string coupling, wrapped on a T^5 with cycles of lengths $\Sigma_1, \ldots, \Sigma_5$. In a region of the moduli space, at low energies, this is an $Sp(N)$ gauge theory in six dimensions, with 16 fundamental hypermultiplets and one hypermultiplet in the antisymmetric tensor [24]. The gauge coupling of this theory is given by $1/g_6^2 = M_s^2$. At energies of the order of the heterotic string scale M_s, string degrees of freedom become important.

The masses of the fundamentals correspond to some of the moduli of the space-time theory. In the interpretation of the theory as M-theory on $T^5/\mathbb{Z}_2$, they are related to the positions of the 16 5-branes in spacetime [23]. When k of these masses are equal, the K3 has an A_{k-1} singularity and we find the (2,0) conformal theory associated with $U(k)$. The gauge theory near this singularity is the $U(N)$ theory described in the previous section. When k of the masses are zero, the K3 has a D_k singularity leading to the $SO(2k)$ (2,0) theory. These theories are only nontrivial for $k > 1$. Then, there are finite-size instantons in the corresponding Yang-Mills theory, and in the quantum mechanics there is a Higgs branch isomorphic to the moduli space of these instantons. In the context of the full six dimensional string theory of [1] this can be extended to exceptional groups corresponding to $E_{6,7,8}$ singularities of the underlying K3.

In the spacetime theory, we would like to scale out gravity, i.e. send $M_p \to \infty$. We can now consult the formulas relating the spacetime quantities to the matrix theory parameters [1] to determine the corresponding limit in the matrix theory. Taking a square $T^5/\mathbb{Z}_2$ with lengths $L_1, \ldots, L_5$ in spacetime, and letting the radius of the longitudinal direction be R, the map is

$$\Sigma_i = \frac{1}{M_p^3 R L_i} \tag{3.2}$$

$$M_s^2 = M_p^9 R^2 L_1 L_2 L_3 L_4 L_5. \tag{3.3}$$

Now let us take $M_p \to \infty$. Then $M_s \to \infty$, so the string degrees of freedom in the matrix theory decouple. At the same time, $\Sigma_i \to 0$, so the matrix theory reduces to quantum mechanics. The gauge coupling g_{QM} in this quantum mechanics behaves as

$$\frac{1}{g_{QM}^2} = M_s^2 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 = \frac{1}{M_p^6 R^3} \to 0. \tag{3.4}$$

So we have $g_{QM} \to \infty$, which means, as discussed above, that the Coulomb branch decouples from the Higgs branch. We see then that in the limit in which space-time gravity decouples from the theory on the fivebranes, the Coulomb branch in the matrix quantum mechanics decouples from the Higgs branch. Thus, the quantum field theory on the fivebrane is reformulated à la

matrix theory as the quantum mechanics on the moduli space of instantons. Note that deriving the E_n theories in this way is more complicated, since in the M-theory description they require taking some of the L_i to be small, but we still expect the description to go over to the quantum mechanics on the moduli space of instantons as described in the previous section.

4   A Description of the Six-Dimensional String Theories

A simple generalization of the methods used in the previous sections provides a description[3] of the new string theories in six dimensions [1]. To derive these theories from M-theory, we look at k five-branes on a transverse circle of radius $L_1 \to 0$. We take a scaling limit such that $\widetilde{M}_{strin}^2 = L_1 M_p^3$ (which is the tension of membranes wrapped around this circle) remains constant [1]. As in the previous section, we can describe fivebranes in M-theory by going to particular points in the moduli space of M-theory on $T^5/\mathbb{Z}_2$, where k fivebranes come together.

Starting with the matrix theory description of this in terms of $Spin(32)/\mathbb{Z}_2$ heterotic fivebranes on T^5, we can determine the limit corresponding to this string theory by using the relations 3.2 and 3.3, and get a description of this string theory. We find that we need to take the size of the circles $\Sigma_{2,3,4,5}$ to zero, take M_s^2 to infinity, and scale Σ_1 as $1/R\widetilde{M}_{string}^2$. The resulting 1+1 dimensional gauge coupling is given by

$$\frac{1}{g_2^2} = M_s^2 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 = \frac{L_1}{R^2 M_p^3}, \qquad (4.1)$$

so again we find that the description involves the $g_2 \to \infty$ limit of the gauge theory. In this case, however, Σ_1 is kept finite, so that the description is in terms of a $1+1$ dimensional gauge theory. The $g_2 \to \infty$ limit is the IR limit of this theory. In this limit the theory becomes scale invariant. However, the presence of the finite circle Σ_1 creates a scale. Equivalently, we can set it to one and measure all other scales relative to it.

Note that we are taking the 5-brane to be far from the orbifold points, so the gauge theory is simply a $U(N)$ gauge theory with an adjoint hypermultiplet and k fundamental hypermultiplets, as in section 2. When we are far from the singularities the fact that we started from M-theory on $T^5/\mathbb{Z}_2$ and not in flat space should be unimportant, so we expect to get the usual string theory corresponding to a type IIA 5-brane in spacetime.

The theory we have found in the scaling limit basically recovers the theory discussed in [4]. These authors studied the theory on the fivebrane in terms

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[3]Similar ideas have been independently studied by E. Witten [25].

of a matrix string theory [26–28]. The main new element that we add is the limit $g_2 \to \infty$. This limit can be motivated as we did above. Alternatively, we are trying to study the string theories of [1], where the string coupling was taken to zero. In terms of the matrix string theory this is precisely the limit where $g_2 \to \infty$. This limit enabled us to decouple the Higgs branch from the Coulomb branch. This is analogous to the statement in [1] that in this limit the fundamental strings are trapped in the fivebrane (the Coulomb branch corresponds to motion of the strings away from the fivebrane).

There are a few other differences between our proposal and that of [4]. We consider moduli spaces of instantons on $\mathbb{R}^4$, instead of instantons on T^4. More importantly, as we explained above, the nonrenormalization theorem of [20] guarantees that the Higgs branch of our quantum field theory receives no quantum corrections. Therefore there is no room for any marginal operator to correct $\mathcal{M}_N(G)$.

Other novel six-dimensional theories (without gravity) can be defined in a similar fashion. For instance, the theory of a five-brane in M-theory on a transverse T^2 with $M_p \to \infty$ and $L_{1,2}M_p^3$ kept constant would be described by a $2+1$ dimensional field theory compactified on two circles whose length scales as $1/R$.

Unfortunately, we can only describe in this way the six-dimensional theories and not their compactifications, which are useful for Matrix theory descriptions of M-theory compactifications [1, 5]. Since these theories are non-local, their definition in infinite six-dimensional space does not completely describe their compactifications, so that a concrete description of the compactified theories is still an interesting open problem.

Acknowledgments

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