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STRINGY HODGE NUMBERS AND VIRASORO ALGEBRA

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Abstract. In this paper we define for singular varieties X a rational number $c_{\rm st}^{1,n-1}(X)$ which is a stringy version of the product of Chern numbers c_1 and c_{n-1} We show that the number $c_{\rm st}^{1,n-1}(X)$ can be expressed via stringy Hodge numbers of singular X in the same way as c_1c_{n-1} expresses via usual Hodge numbers for smooth manifolds. Our result provides some evidences for the existence of quantum cohomology theory of singular varieties X based on representation of the Virasoro algebra whose central charge is the rational number $e_{\rm st}(X)$ which equals the stringy Euler number of X.

1. Introduction

Let X be an arbitrary smooth n-dimensional projective variety. It was discovered by Libgober and Wood that the product of the Chern classes $c_1(X)c_{n-1}(X)$ depends only on the Hodge numbers of X [11]. This result has been used by Eguchi, Jinzenji and Xiong in their approach to the quantum cohomology of X via a representation of the Virasoro algebra with the central charge $c_n(X)$ [8, 9].

We recall that the E-polynomial of X is defined as

$$E(X;u,v) := \sum_{p,q} (-1)^{p+q} h^{p,q}(X) u^p v^q,$$

where $h^{p,q}(X) = \dim H^q(X, \Omega_X^p)$ are Hodge numbers of X. Using the Hirzebruch-Riemann-Roch theorem, Libgober and Wood [11] have proved the following equality (see also results of Borisov [6] and Salamon [12]):

Theorem 1.1.

$$\frac{d^2}{du^2} E_{\rm st}(X;u,1)|_{u=1} = \frac{3n^2-5n}{12} c_n(X) + \frac{c_1(X)c_{n-1}(X)}{6}.$$

By Poincaré duality for X, one immediately obtains the following equivalent reformulation of the above equality:

Theorem 1.2. *Let X be an arbitrary smooth n-dimensional projective variety. Then $c_1(X)c_{n-1}(X)$ can be expressed via the Hodge numbers of X using the following equality*

$$\sum_{p,q} (-1)^{p+q} h^{p,q}(X) \left(p - \frac{n}{2}\right)^2 = \frac{n}{12} c_n(X) + \frac{1}{6} c_1(X) c_{n-1}(X),$$

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where

$$c_n(X) = \sum_{p,q} (-1)^{p+q} h^{p,q}(X)$$

is the Euler number of X.

In particular, one has:

Corollary 1.3. *Let X be an arbitrary smooth n-dimensional projective variety with $c_1(X) = 0$. Then the Hodge numbers of X satisfy the following equation*

$$\sum_{p,q} (-1)^{p+q} h^{p,q}(X) \left(p - \frac{n}{2}\right)^2 = \frac{n}{12} \sum_{p,q} (-1)^{p+q} h^{p,q}(X).$$

Remark 1.4. We note that if X is a $K3$-surface, then the relation 1.3 is equivalent to the equality $c_2(X) = 24$. For smooth Calabi-Yau 4-folds X the equality 1.3 has been observed by Sethi, Vafa, and Witten [13]. It is equivalent to the equality

$$c_4(X) = 6(8 - h^{1,1}(X) + h^{2,1}(X) - h^{3,1}(X)),$$

if $h^{1,0}(X) = h^{2,0}(X) = h^{3,0}(X) = 0$.

There are a lot of examples of Calabi-Yau varieties X having at worst Gorenstein canonical singularities which are hypersurfaces and complete intersections in Gorenstein toric Fano varieties [1, 3]. It has been shown in [5] that for all these singular Calabi-Yau varieties X one can define so called *stringy Hodge numbers* $h^{p,q}_{\rm st}(X)$ [2]. Moreover, the stringy Hodge numbers of Calabi-Yau complete intersections in Gorenstein toric varieties agree with the topological mirror duality test [4]. It is natural to expect that one has the same kind of identity for stringy Hodge numbers of singular Calabi-Yau varieties as for usual Hodge numbers of smooth Calabi-Yau manifolds, i.e.,

$$(1) \quad \sum_{p,q} (-1)^{p+q} h^{p,q}_{\rm st}(X) \left(p - \frac{n}{2}\right)^2 = \frac{n}{12} \sum_{p,q} (-1)^{p+q} h^{p,q}_{\rm st}(X) = \frac{n}{12} e_{\rm st}(X).$$

This paper is to show that the formula (1) holds true. Moreover, one can define a rational number $c^{1,n-1}_{\rm st}(X)$, a stringy version $c_1(X)c_{n-1}(X)$, such that the stringy analogue of the equation in 1.2

$$(2) \quad \sum_{p,q} (-1)^{p+q} h^{p,q}_{\rm st}(X) \left(p - \frac{n}{2}\right)^2 = \frac{n}{12} e_{\rm st}(X) + \frac{1}{6} c^{1,n-1}_{\rm st}(X),$$

holds true provided the stringy Hodge numbers of X exist.

2. Stringy Hodge numbers

Recall our general approach to the notion of stringy Hodge numbers $h^{p,q}_{\rm st}(X)$ for projective algebraic varieties X with canonical singularites (see [2]). Our main definition in [2] can be reformulated as follows:

Definition 2.1. Let X be an arbitrary n-dimensional projective variety with at worst log-terminal singularities, $\rho : Y \to X$ a resolution of singularities whose exceptional locus D is a divisor with normally crossing components $D_1, \dots, D_r$. We set $I := \{1, \dots, r\}$ and $D_J := \bigcap_{j \in J} D_j$ for all $J \subseteq I$, where $D_J = Y$ if $J = \emptyset$. Define the **stringy E-function** of X to be

$$E_{\rm st}(X; u, v) := \sum_{\emptyset \subseteq J \subseteq I} E(D_J; u, v) \prod_{j \in J} \left(\frac{uv - 1}{(uv)^{a_j + 1} - 1} - 1 \right),$$

where the rational numbers $a_1, \dots, a_r$ are determined by the equality

$$K_Y = \rho^* K_X + \sum_{i=1}^{r} a_i D_i.$$

Then the **stringy Euler number** of X is defined as

$$e_{\rm st}(X) := \lim_{u,v \to 1} E_{\rm st}(X; u, v) = \sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J) \prod_{j \in J} \left(\frac{-a_j}{a_j + 1} \right),$$

where $c_{n-|J|}(D_J)$ is the Euler number of D_J (we set $c_{n-|J|}(D_J) = 0$ if D_J is empty).

Remark 2.2. It is important that the above definitions do not depend on the choice of a desingularization $\rho : Y \to X$ [2].

Definition 2.3. Let X be an arbitrary n-dimensional projective variety with at worst Gorenstein canonical singularites. We say that **stringy Hodge numbers of X exist**, if $E_{\rm st}(X; u, v)$ is a polynomial, i.e.,

$$E_{\rm st}(X; u, v) = \sum_{p,q} a_{p,q}(X) u^p v^q.$$

Under the assumption that $E_{\rm st}(X; u, v)$ is a polynomial, we define the **stringy Hodge numbers** $h^{p,q}_{\rm st}(X)$ to be $(-1)^{p+q} a_{p,q}(X)$.

Remark 2.4. In the above definitions, the condition that X has at worst log-terminal singularities means that $a_i > -1$ for all $i \in I$; the condition that X has at worst Gorenstein canonical singularities is equivalent for a_i to be nonnegative integers for all $i \in I$ (see [10]).

The following statement has been proved in [2]:

Theorem 2.5. *Let X be an arbitrary n-dimensional projective variety with at worst Gorenstein canonical singularites. Assume that stringy Hodge numbers of X exist. Then they have the following properties:*

(i) $h^{0,0}_{\rm st}(X) = h^{n,n}_{\rm st}(X) = 1$;

(ii) $h^{p,q}_{\rm st}(X) = h^{n-p,n-q}_{\rm st}(X)$ *and* $h^{p,q}_{\rm st}(X) = h^{q,p}_{\rm st}(X)$ $\forall p,q$;
(iii) $h^{p,q}_{\rm st}(X) = 0$ $\forall p,q > n$.

3. The number $c^{1,n-1}_{\rm st}(X)$

Definition 3.1. Let X be an arbitrary n-dimensional projective variety X having at worst log-terminal singularities and $\rho : Y \to X$ is a desingularization with normally crossing irreducible components $D_1, \ldots, D_r$ of the exceptional locus. We define the number

$$c^{1,n-1}_{\rm st}(X) := \sum_{\emptyset \subseteq J \subseteq I} \rho^* c_1(X) c_{n-|J|-1}(D_J) \prod_{j \in J} \left(\frac{-a_j}{a_j+1} \right),$$

where $\rho^* c_1(X) c_{n-|J|-1}(D_J)$ is considered as the intersection number of the 1-cycle $c_{n-|J|-1}(D_J) \in A_1(D_J)$ with the ρ-pullback of the class of the anticanonical $\mathbb{Q}$-divisor of X.

Remark 3.2. It is not clear a priori that the number $c^{1,n-1}_{\rm st}(X)$ in the above the definition does not depend on the choice of a desingularization ρ. Later we shall show this independence.

The proof of the next obvious statement is left to the reader:

Proposition 3.3. *For any smooth n-dimensional projective variety V, one has*

$$\frac{d}{du} E(V; u, 1)|_{u=1} = \frac{n}{2} c_n(V).$$

Proposition 3.4. *For any n-dimensional projective variety X having at worst log-terminal singularities, one has*

$$\frac{d}{du} E_{\rm st}(X; u, 1)|_{u=1} = \frac{n}{2} e_{\rm st}(X).$$

Proof. By definition 2.1, we have

$$E_{\rm st}(X; u, 1) = \sum_{\emptyset \subseteq J \subseteq I} E(D_J; u, 1) \prod_{j \in J} \left(\frac{u-1}{u^{a_j+1}-1} - 1 \right).$$

Applying 3.3 to every smooth submanifold $D_J \subset Y$, we obtain

$$\begin{aligned} \frac{d}{du} E_{\rm st}(X; u, 1)|_{u=1} &= \sum_{\emptyset \subseteq J \subseteq I} \frac{(n-|J|)}{2} c_{n-|J|}(D_J) \prod_{j \in J} \left(\frac{-a_j}{a_j+1} \right) + \\ &\quad + \sum_{\emptyset \subseteq J \subseteq I} \frac{|J|}{2} c_{n-|J|}(D_J) \prod_{j \in J} \left(\frac{-a_j}{(a_j+1)} \right) \\ &= \frac{n}{2} \sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J) \prod_{j \in J} \left(\frac{-a_j}{a_j+1} \right) = \frac{n}{2} e_{\rm st}(X). \end{aligned}$$

□

Proposition 3.5. *Let V be a smooth projective algebraic variety of dimension n and $W \subset V$ a smooth irreducible divisor on V or empty divisor (the latter means that $\mathcal{O}_V(W) \cong \mathcal{O}_V$). Then*

$$c_1(\mathcal{O}_V(W))c_{n-1}(V) = c_{n-1}(W) + c_1(\mathcal{O}_W(W))c_{n-2}(W),$$

where $c_{n-1}(W)$ is considered to be zero if $W = \emptyset$.

Proof. Consider the short exact sequnce

$$0 \to T_W \to T_V|_W \to \mathcal{O}_W(W) \to 0,$$

where T_W and T_V are tangent sheaves on W and V. It gives the following the relation between Chern polynomials

$$(1 + c_1(\mathcal{O}_W(W))t)(1 + c_1(W)t + c_2(W)t^2 + \cdots + c_{n-1}(W)t^{n-1}) = \\ 1 + c_1(T_V|_W)t + c_2(T_V|_W)t^2 + c_{n-1}(T_V|_W)t^{n-1}.$$

Comparing the coefficients by t^{n-1} and using $c_{n-1}(T_V|_W) = c_1(\mathcal{O}_V(W))c_{n-1}(V)$, we come to the required equality. □

Corollary 3.6. *Let Y be a smooth projective variety, $D_1, \dots, D_r$ smooth irreducible divisors with normal crossings, $I := \{1, \dots, r\}$. Then for all $J \subseteq I$ and for all $j \in J$ one has*

$$c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_j))c_{n-|J|}(D_{J\setminus\{j\}}) - c_{n-|J|}(D_J) = c_1(\mathcal{O}_{D_J}(D_j))c_{n-|J|-1}(D_J),$$

where D_J is the complete intersection $\bigcap_{j\in J} D_j$.

Proof. One sets in 3.5 $V := D_{J\setminus\{j\}}$ and $W := D_J$. □

Proposition 3.7. *Let $\rho : Y \to X$ be a desingularization as in* 3.1*. Then*

$$\sum_{\emptyset\subseteq J\subseteq I} c_1(D_J)c_{n-|J|-1}(D_J)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) = \\ c_{\mathrm{st}}^{1,n-1}(X) + \sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in J}(a_j+1)c_{n-|J|}(D_J)\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right).$$

Proof. Using the formula

$$c_1(Y) = \rho^* c_1(X) + \sum_{i\in I} -a_i c_1(\mathcal{O}_Y(D_i)),$$

and the adjunction formula for every complete intersection D_J ($J \subseteq I$), we obtain

$$c_1(D_J) = \rho^* c_1(X)|_{D_J} + \sum_{j\in J}(-a_j-1)c_1(\mathcal{O}_{D_J}(D_j)) + \sum_{j\in I\setminus J}(-a_j)c_1(\mathcal{O}_{D_J}(D_j)).$$

Therefore

$$(3)\qquad \sum_{\emptyset\subseteq J\subseteq I} c_1(D_J)c_{n-|J|-1}(D_J)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) = c_{\rm st}^{1,n-1}(X) +$$
$$\sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in J}(-a_j-1)c_1(\mathcal{O}_{D_J}(D_j))c_{n-|J|-1}(D_J)\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) +$$
$$\sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in I\setminus J}(-a_j)c_1(\mathcal{O}_{D_J}(D_j))c_{n-|J|-1}(D_J)\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right).$$

Using 3.6, we obtain

$$(4)\qquad \sum_{j\in J}(-a_j-1)c_1(\mathcal{O}_{D_J}(D_j))c_{n-|J|-1}(D_J) =$$
$$\sum_{j\in J}(-a_j-1)\left(c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_j))c_{n-|J|}(D_{J\setminus\{j\}}) - c_{n-|J|}(D_J)\right).$$

By substitution (4) to (3), we come to the required equality, because

$$\sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in J}(-a_j-1)c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_j))c_{n-|J|}(D_{J\setminus\{j\}})\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) +$$
$$\sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in I\setminus J}(-a_j)c_1(\mathcal{O}_{D_J}(D_j))c_{n-|J|-1}(D_J)\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) = 0.$$

□

Theorem 3.8. *Let X be an arbitrary n-dimensional projective variety variety with at worst log-terminal singularities. Then*

$$\frac{d^2}{du^2}E_{\rm st}(X;u,1)|_{u=1} = \frac{3n^2-5n}{12}e_{\rm st}(X) + \frac{1}{6}c_{\rm st}^{1,n}(X).$$

Proof. Using the equalities

$$\frac{d}{du}\left(\frac{u-1}{u^{a+1}-1}-1\right)_{u=1} = \frac{-a}{2(a+1)},\quad \frac{d^2}{du^2}\left(\frac{u-1}{u^{a+1}-1}-1\right)_{u=1} = \frac{a(a+2)}{6(a+1)},$$

together with the identities in 1.1 and 3.3 for every submanifold $D_J \subset Y$, we obtain

$$
\begin{aligned}
\frac{d^2}{du^2}E_{\mathrm{st}}(X;u,1)|_{u=1} = \sum_{\emptyset\subseteq J\subseteq I} \frac{c_1(D_J)c_{n-|J|-1}(D_J)}{6}\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) + \\
c_{n-|J|}(D_J)\frac{3(n-|J|)^2-5(n-|J|)}{12}\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) + \\
\sum_{\emptyset\subseteq J\subseteq I}\frac{(n-|J|)|J|c_{n-|J|}(D_J)}{2}\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) + \\
\sum_{\emptyset\subseteq J\subseteq I}\frac{c_{n-|J|}(D_J)(|J|-1)|J|}{4}\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right) + \\
\sum_{\emptyset\subseteq J\subseteq I}\frac{c_{n-|J|}(D_J)(-\sum_{j\in J}(a_j+2))}{6}\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right).
\end{aligned}
$$

By 3.7, the first term of the above equals

$$
\frac{1}{6}c_{\mathrm{st}}^{1,n-1}(X) + \frac{1}{6}\sum_{\emptyset\subseteq J\subseteq I}\left(\sum_{j\in J}(a_j+1)c_{n-|J|}(D_J)\right)\prod_{j\in J}\left(\frac{-a_j}{a_j+1}\right).
$$

Now the required statement follows from the equality

$$
\begin{aligned}
\frac{\sum_{j\in J}(a_j+1)}{6} + \frac{3(n-|J|)^2-5(n-|J|)}{12} + \frac{(n-|J|)|J|}{2} + \\
\frac{(|J|-1)|J|}{4} + \frac{-\sum_{j\in J}(a_j+2)}{6} = \frac{3n^2-5n}{12}.
\end{aligned}
$$

□

Corollary 3.9. *The number $c_{\mathrm{st}}^{1,n}(X)$ does not depend on the choice of the desingularization $\rho : Y \to X$.*

Proof. By 3.4 and 3.8, $c_{\mathrm{st}}^{1,n}(X)$ can be computed in terms of derivatives of the stringy E-function of X. But the stringy E-function does not depend on the choice of a desingularization [2]. □

Corollary 3.10. *Let X be a projective variety with at worst Gorenstein canonical singularities. Assume that the stringy Hodge numbers of X exist. Then*

$$
\sum_{p,q}(-1)^{p+q}h_{\mathrm{st}}^{p,q}(X)\left(p-\frac{n}{2}\right)^2 = \frac{n}{12}e_{\mathrm{st}}(X) + \frac{1}{6}c_{\mathrm{st}}^{1,n-1}(X).
$$

Proof. The equality follows immediately from 3.8 using the properties of the stringy Hodge numbers 2.5. □

Corollary 3.11. *Assume that the canonical class of X is numerically trivial. Then $c_{\rm st}^{1,n-1}(X) = 0$. In particular, for Calabi-Yau varieties with at worst Gorenstein canonical singularities we have*

$$\frac{d^2}{du^2}E_{\rm st}(X;u,1)|_{u=1} = \frac{3n^2-5n}{12}e_{\rm st}(X),$$

and therefore stringy Hodge numbers of X satisfy the identity (1) *provided that stringy numbers exist.*

Example 3.12. Let Δ be an n-dimensional reflexive polyhedron and Δ^* its polar polyhedron [1]. We denote by $\mathbb{P}_\Delta$ the Gorenstein toric Fano variety associated with Δ. If Θ a convex lattice polyhedron of dimension k then we denote by $v(\Theta)$ the integer $k!\,{\rm vol}_k(\Theta)$ where ${\rm vol}_k(\Theta)$ is the k-dimensional volume of Θ with respect to the lattice. It is known that $e_{\rm st}(\mathbb{P}_\Delta) = v(\Delta^*)$ (see Cor. 7.7 [5]). One can ask about the meaning of the new invariant $c_{\rm st}^{1,n}$ in terms of Δ. Assume that $\mathbb{P}_\Delta$ has a smooth crepant toric desingularization $\rho : \widehat{\mathbb{P}_\Delta} \to \mathbb{P}_\Delta$ which is defined by a triangulation $\mathcal{T}$ of the boundary $\partial\Delta^*$ by regular simplices. By 3.1 we obtain that

$$c_{\rm st}^{1,n}(\mathbb{P}_\Delta) = c_1(\widehat{\mathbb{P}_\Delta})c_{n-1}(\widehat{\mathbb{P}_\Delta}).$$

Now we use that fact that in the nonsigular case the Chern class $c_{n-1}(\widehat{\mathbb{P}_\Delta})$ is the sum of all 1-dimensional strata in $\widehat{\mathbb{P}_\Delta}$ [7]. These 1-dimensional strata correspond to $(d-2)$-dimensional simplices in the triangulation $\mathcal{T}$ of $\partial\Delta^*$. Let τ be such a $(d-2)$-dimensional simplex which is a common boundary of two $(d-1)$-dimensional ones σ_1 and σ_2. Denote by Z_τ the 1-cycle on $\widehat{\mathbb{P}_\Delta}$ corresponding to τ. It follows from the description of c_1 in terms of a canonical piecewise linear function having value 1 on $\partial\Delta^*$ that the intersection number of Z_τ with $c_1(\widehat{\mathbb{P}_\Delta})$ is zero unless σ_1 and σ_2 belong to different faces γ_1^* and γ_2^* of codimension 1 of Δ^* . The faces γ_1^* and γ_2^* are dual to two vertices $\gamma_1, \gamma_2 \in \Delta$ and the intersection number $c_1(\widehat{\mathbb{P}_\Delta})Z_\tau$ equals just the integral length $v([\gamma_1,\gamma_2])$ of the segment $[\gamma_1,\gamma_2]$. Let $[\gamma_1,\gamma_2]^* \subset \Delta^*$ be the dual to $[\gamma_1,\gamma_2]$ $(d-2)$-dimensional face. Then $[\gamma_1,\gamma_2]^*$ contains exactly $v([\gamma_1,\gamma_2]^*)$ simplices from the triangulation $\mathcal{T}$, because for all such simplices one has $v(\tau) = 1$. The above arguments show that the number

$$c_{\rm st}^{1,n}(\mathbb{P}_\Delta) = c_1(\widehat{\mathbb{P}_\Delta})\sum_{\tau\in\mathcal{T}} Z_\tau$$

equals

$$\sum_{[\gamma,\gamma']\subset\Delta} v([\gamma,\gamma'])\cdot v([\gamma,\gamma']^*).$$

So we have

$$c_{\rm st}^{1,n-1}(\mathbb{P}_\Delta) = \sum_{\theta\subset\Delta,\dim\theta=1} v(\theta)\cdot v(\theta^*).$$

4. Virasoro algebra

Recall that the Virasoro algebra with the central charge c consists of operators L_n ($m \in \mathbb{Z}$) satisfying the relations

$$[L_n, L_m] = (n-m)L_{n+m} + c\frac{n^3-n}{12}\delta_{n+m,0} \quad n, m \in \mathbb{Z}.$$

For arbitrary compact Kähler manifold X, Eguchi et. al have proposed in [8, 9] a new approach to its quantum cohomology and to its Gromov-Witten invariants for all genera g using so called the Virasoro condition:

$$L_n Z = 0, \forall n \geq -1,$$

where

$$Z = \exp F = \exp\left(\sum_{g\geq 0} \lambda^{2g-2} F_g\right)$$

is the partition function of the topological σ-model with the target space X and F_g the free energy function corresponding to the genus g. In this approach, the central charge c acts as the multiplication by $c_n(X)$. Moreover, all Virasoro operators L_n can be explicitly written in terms of elements of a basis of the cohomology of X, their gravitational descendants and the action of $c_1(X)$ on the cohomology by the multiplication. In particular the commutator relation

$$[L_1, L_{-1}] = 2L_0$$

implies precisely the identity of Libgober and Wood in the form

$$\sum_{p,q} (-1)^{p+q} h^{p,q}(X) \left(\frac{n+1}{2} - p\right)\left(p - \frac{n-1}{2}\right) = \frac{1}{6}\left(\frac{3-n}{2} c_n(X) - c_1(X)c_{n-1}(X)\right).$$

Now let X be a projective algebraic variety with at worst log-terminal singularities. We conjecture that there exists an analogous approach to the quantum cohomology as well as to the Gromov-Witten invariants of X for all genera using the Virasoro algebra in such a way that for any resolution of singularities $\rho : Y \to X$ the corresponding Virasoro operators can be explicitly computed via the numbers a_i appearing in the formula

$$K_X = \rho^* K_X + \sum_{i=1}^{r} a_i D_i,$$

and bases in cohomology of all complete intersections D_J together with the multiplicative actions of $c_1(D_J)$ in them. We consider our main result 3.8 as an evidence in favor of this conjecture.

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