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A NOTE ON AKBULUT CORKS

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Abstract. We prove that the involution on the boundary Σ of the Akbulut cork relating blown up elliptic surfaces to completely decomposable manifolds acts non-trivially on the Floer homology of Σ. We also show that Σ provides an example of an irreducible manifold with non-zero boundary operator in its Floer chain complex.

Let X_0 and X_1 be smooth, closed, oriented, simply connected 4-manifolds. If X_0 is homeomorphic to X_1 then there is a compact contractible 4-manifold $W \subset X_0$ such that by cutting W out of X_0 and re-gluing it by an involution on its boundary, we obtain a smooth 4-manifold diffeomorphic to X_1, see [6] and [12]. This contractible piece W is called an Akbulut cork corresponding to the pair $(X_0,\, X_1)$.

Explicit examples of Akbulut corks were constructed in [1] and [11]. They correspond to pairs $(X_0(n),\, X_1(n))$ where $X_0(n) = E(n) \,\#\, (-\mathbb{C}P^2)$ is a blow up of the elliptic surface $E(n)$, and $X_1(n) = (2n-1)\cdot \mathbb{C}P^2 \,\#\, 10n \cdot (-\mathbb{C}P^2)$, with $n \geq 2$. For any given $n \geq 2$, the manifolds $X_0(n)$ and $X_1(n)$ are homeomorphic but not diffeomorphic. Their Akbulut cork W is independent of n and is obtained by attaching a two-handle to $S^1 \times D^3$ along its boundary as shown in Figure 1.

Observe that $\Sigma = \partial W$ is an integral homology sphere obtained by surgery on the link in Figure 1 both components of which are 0-framed, and that it is symmetric with respect to the involution $\tau : \Sigma \to \Sigma$ interchanging the two link components. The manifold $X_1(n)$ is obtained from $X_0(n)$ by cutting out W and re-gluing it using τ.

The goal of this paper is to study the homomorphism $\tau_* : I_*(\Sigma) \to I_*(\Sigma)$ which τ induces on the Floer homology of Σ, see [9].

Theorem 1. *Let Σ be the boundary of manifold W shown in Figure 1. Then*

(1) *$I_n(\Sigma) = 0$ if n is even, and $I_n(\Sigma) = \mathbb{Z}$ if n is odd, and*
(2) *the homomorphism $\tau_* : I_*(\Sigma) \to I_*(\Sigma)$ is a non-trivial involution.*

This result gives an insight into how re-gluing of W leads to different smooth structures, via the study of the effect that τ_* has on the Donaldson invariants, see Section 4.

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Our proof of Theorem 1 also implies the following result. As far as we know, this is the first example of an irreducible homology sphere having this property.

Theorem 2. *The Floer chain complex of* Σ *has a non-trivial boundary operator.*

It should be mentioned that in general constructing diffeomorphisms acting non-trivially on the Floer homology is not an easy task. This is due in part to a close relation between such actions and exotic smooth structures on 4–manifolds. We briefly discuss these and related issues in Section 5.

I am thankful to Selman Akbulut, whose papers [1] and [2] have motivated this research, and to Jim Bryan, Olivier Collin, Ronald Fintushel and Slawomir Kwasik for inspiring discussions concerning the matters in this paper. I am also thankful to the referee for pointing out an omission in the original argument. The `Maple` software package was used for calculations in Section 2.

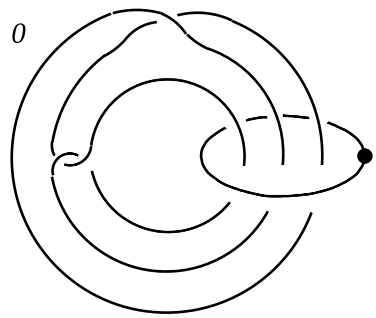

FIGURE 1

1. Floer homology of Σ

The Floer homology groups of Σ are not difficult to compute by using the Floer exact triangle and Kirby calculus.

Proposition 3. *The groups* $I_n(\Sigma)$ *vanish for even* n*, and* $I_n(\Sigma) = \mathbb{Z}$ *for odd* n*.*

Proof. For an integer p, define an integral homology sphere Σ_p as the boundary of a contractible 4-manifold obtained by surgery on the link shown in Figure 1 with the framing of the two-handle equal to p. Then $\Sigma_0 = \Sigma$ and Σ_3 is orientation preserving diffeomorphic to the Brieskorn homology sphere $\Sigma(2,5,7)$, see [3]. According to [8], we have $I_n(\Sigma(2,5,7)) = 0$ for even n and $I_n(\Sigma(2,5,7)) = \mathbb{Z}$ for odd n. Therefore, the proposition will follow as soon as we prove that $I_*(\Sigma_p)$ is independent of p.

Let us perform a (-1)–surgery along the unknot S shown in Figure 2. This surgery results in replacing framing p by framing $p+1$ while preserving the rest of the picture. On the other hand, 0–surgery along the same circle yields the

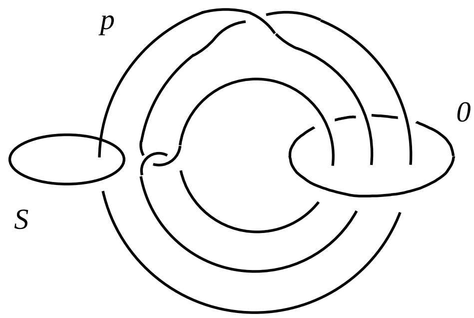

FIGURE 2

manifold $\overline{K} = S^1 \times S^2$ with the trivial Floer homology, $I_*(\overline{K}) = 0$. The Floer exact triangle [10]

$$\begin{array}{ccc} & I_*(\overline{K}) & \\ \swarrow & & \nwarrow \\ I_*(\Sigma_p) & \xrightarrow{\;Z_*\;} & I_*(\Sigma_{p+1}) \end{array}$$

now implies that the trace Z of the (-1)–surgery induces a degree zero isomorphism in Floer homology. This proves that $I_*(\Sigma_p)$ is independent of p. $\square$

The homomorphism $\tau_* : I_*(\Sigma) \to I_*(\Sigma)$ induced by the involution τ has degree zero. The fact that $\tau^2 = 1$ implies that $(\tau_*)^2 = 1$; therefore, on each of the factors $\mathbb{Z}$, the homomorphism τ_* is either identity or minus identity. A careful analysis of the Floer chain complex of Σ in the next section will help us sort this out.

2. The representation variety of $\pi_1\Sigma$

After due simplifications, the standard Wirtinger presentation of the fundamental group of Σ is of the form

$$\pi_1\Sigma = \langle\, a,\, b \mid\ (ba)^2(ab)^{-2}b^{-1}(ab)^2 = a^2,\ (ab)^2(ba)^{-2}a^{-1}(ba)^2 = b^2\,\rangle$$

where a and b are meridians of the two components of the link in Figure 1 exchanged by the involution τ. Specifying a representation $\alpha : \pi_1\Sigma \to SU(2)$ amounts to specifying two $SU(2)$–matrices, $A = \alpha(a)$ and $B = \alpha(b)$, satisfying the above relations. Since Σ is an integral homology sphere, its only reducible representation is the trivial one. We will assume therefore that α is irreducible, i.e. A and B do not commute. Conjugating if necessary, we may assume that

$$A = \begin{pmatrix} t + i\sqrt{1-t^2} & 0 \\ 0 & t - i\sqrt{1-t^2} \end{pmatrix} \quad \text{and}$$

$$B = \begin{pmatrix} u + ir\sqrt{1-u^2} & \sqrt{(1-r^2)(1-u^2)} \\ -\sqrt{(1-r^2)(1-u^2)} & u - ir\sqrt{1-u^2} \end{pmatrix}$$

for some real t, u and r such that $-1 < t, u, r < 1$. The relations on the matrices A and B can be rewritten as

$$(1) \qquad (AB)^{-2}B(AB)^2 = A^{-2}(BA)^2 \quad \text{and} \quad (BA)^{-2}A(BA)^2 = B^{-2}(AB)^2.$$

From this we conclude that $\operatorname{tr} B = \operatorname{tr}(A^{-1}BA \cdot B)$ and $\operatorname{tr} A = \operatorname{tr}(B^{-1}AB \cdot A)$ and, after simplification, that

$$2(u^2 + r^2 - u^2r^2)(t+1) = 1 \quad \text{and} \quad 2(t^2 + r^2 - t^2r^2)(u+1) = 1.$$

Solutions of these equations are of two types, $t + u = 1/2$ and $t = u$. In terms of matrices A and B, these two types correspond to $\operatorname{tr} A + \operatorname{tr} B = 1$ and $\operatorname{tr} A = \operatorname{tr} B$.

Representations with $\operatorname{tr} A + \operatorname{tr} B = 1$ can be found in a completely explicit form by solving the rest of the equations (1). There are two such representations, one with $\operatorname{tr} A = (1-\sqrt{5})/2$ and $\operatorname{tr} B = (1+\sqrt{5})/2$, and the other with $\operatorname{tr} A = (1+\sqrt{5})/2$ and $\operatorname{tr} B = (1-\sqrt{5})/2$. The parameter r for both representations equals $1/\sqrt{5}$. We call these representations β_1 and β_2, respectively. They are permuted by the involution τ^* and, in particular, they have the same Floer index.

Since $\operatorname{tr} A = \operatorname{tr} B$ for the representations of the second type, the matrix B is conjugate to A and hence can be written as $B = UAU^{-1}$ for some $SU(2)$–matrix U with $U^2 = -I$; one may assume that

$$U = \begin{pmatrix} i\rho & \sqrt{1-\rho^2} \\ -\sqrt{1-\rho^2} & -i\rho \end{pmatrix}$$

with $0 < \rho < 1$. Equations (1) then reduce to the single equation

$$(UA)^4(AU)^{-4}U^{-1}A^{-1}U(AU)^4 = A^2,$$

which is equivalent to a system of three polynomial equations in t and ρ. These equations have four solutions corresponding to the following values of t and ρ. The parameter $2t$ is a real solution of the equation

$$(2) \qquad z^7 + z^6 - 5z^5 - 6z^4 + 6z^3 + 5z^2 - 2z - 1 = 0.$$

This equation has five real solutions but only four of them lie between -2 and 2. Every t uniquely determines ρ by the formula

$$(3) \qquad \rho = (1/5)\sqrt{19 - 68t - 212t^2 + 600t^3 + 1024t^4 - 384t^5 - 832t^6}.$$

We name these four representations α_1 through α_4. They are preserved by the involution τ^*.

Proposition 4. *The $SU(2)$–representation variety of $\pi_1\Sigma$ consists of a trivial representation θ and six non-degenerate irreducible representations, α_1, α_2, α_3, α_4, β_1 and β_2. The involution induced by τ keeps the representations θ and α_1, α_2, α_3, α_4 fixed, and permutes β_1 and β_2.*

Proof. We only need to check that the representations are non-degenerate, that is, the group cohomology $H^1_\gamma(\pi_1\Sigma, \mathfrak{su}(2))$ with coefficients in the adjoint representation vanishes for all $\gamma = \alpha_1$, α_2, α_3, α_4, β_1 and β_2. This can be seen as follows.

The dimension of the space of 1–coboundaries equals the rank of the operator

$$\begin{pmatrix} I - \mathrm{Ad}_A \\ I - \mathrm{Ad}_B \end{pmatrix} : \mathbb{R}^3 \to \mathbb{R}^6, \tag{4}$$

where I is the identity matrix and $\mathrm{Ad}_A(x) = AxA^{-1}$. Let us consider the three-by-three minor of (4) consisting of the rows with numbers 3, 4, and 6. A direct calculations shows that its determinant equals

$$8t\,(1-r^2)(1-u^2)\,\sqrt{1-t^2}.$$

Since $t \neq 0$ for any of the representations γ, this determinant is not zero, and hence the rank of (4) is three.

The cocycles are crossed homomorphisms $\xi : \pi_1\Sigma \to \mathfrak{su}(2)$, which can be identified with vectors $(\xi_A, \xi_B) \in \mathbb{R}^6$ satisfying the linear system of equations

$$\begin{cases} U_A\,\xi_A + U_B\,\xi_B = 0 \\ V_A\,\xi_A + V_B\,\xi_B = 0, \end{cases}$$

arising from the two relations in $\pi_1\Sigma$. Here, U_A, U_B, V_A, and V_B are three-by-three matrices which can be explicitly written in terms of t, u, and r. The dimension of the space of cocycles thus equals the co-rank of the block matrix

$$X = \begin{pmatrix} U_A & U_B \\ V_A & V_B \end{pmatrix}. \tag{5}$$

For the representation β_1, consider the three-by-three minor of (5) at the intersection of the rows and columns numbered 1, 2, and 4. A direct calculation shows that it equals the matrix

$$\begin{pmatrix} \sqrt{5}/5 & -1+2\sqrt{5}/5 & 1+\sqrt{5}/5 \\ 1+2\sqrt{5}/5 & -5/4+11\sqrt{5}/20 & 1+2\sqrt{5}/5 \\ 2-6\sqrt{5}/5 & 1/2+\sqrt{5}/10 & -1-4\sqrt{5}/5 \end{pmatrix}$$

with determinant $(15 - 5\sqrt{5})/4$. Therefore, the cocycles at β_1 have dimension three, so that β_1 is non-degenerate. An argument for β_2 is similar.

If γ is one of the representations α, it is more convenient to work with parameters t and ρ. We consider the minor of (5) at the intersection of rows 1, 2, and 4, and columns 1, 2, and 5. Its determinant is a polynomial in t and ρ, where t and ρ satisfy equations (2) and (3). Setting the determinant equal to zero gives us three polynomial equations, which do not have common solutions. Therefore, the co-rank of (5) in this case is also three, and all the α_i are non-degenerate. □

Corollary. *There exist non-zero boundary operators in the Floer chain complex of* Σ.

Proof. Since the representation variety of Σ is non-degenerate, its Floer chain complex $IC_*(\Sigma)$ is generated by the six irreducible representations. Since only four Floer homology groups are non-trivial, and each of them is isomorphic to $\mathbb{Z}$, not all boundary operators in the chain complex vanish. The same conclusion can be drawn even more easily from the fact that $IC_*(\Sigma)$ has two generators, β_1 and β_2, of the same Floer index. □

3. The boundary operators

The boundary operators in the Floer chain complex of Σ deserve a closer attention.

Let μ denote the Floer index. From the knowledge of the Floer homology groups $I_*(\Sigma)$ and the representation variety of $\pi_1\Sigma$ we conclude that $\mu(\beta_1) = \mu(\beta_2) = 1 \mod 2$ (keeping in mind that β_1 and β_2 are permuted by the operator τ_* which preserves Floer index) and that exactly one of the representations α_1 through α_4 has even Floer index; let us call it α.

By an obvious dimensional count, the only non-zero incidence numbers can be those between α and β_1 and between α and β_2. These are determined by signed counts $\#\hat{\mathcal{M}}_g(\alpha,\beta_1)$ and $\#\hat{\mathcal{M}}_g(\alpha,\beta_2)$ of isolated points in the instanton moduli spaces, where g is a generic metric on Σ. Due to the lack of equivariant transversality, in general, it is not clear if one can choose a generic metric g so that τ is an isometry. Therefore, we cannot claim that τ identifies $\hat{\mathcal{M}}_g(\alpha,\beta_1)$ with $\hat{\mathcal{M}}_g(\alpha,\beta_2)$; however, the following weaker result will suffice for our purposes.

Proposition 5. *Let g be a generic metric then $\#\hat{\mathcal{M}}_g(\alpha,\beta_1) = \#\hat{\mathcal{M}}_g(\alpha,\beta_2)$.*

Proof. We begin by showing that the signed counts $\#\hat{\mathcal{M}}_g(\alpha,\beta_i)$, $i = 1,2$, are independent of the choice of generic metric g. Given two such metrics, g_0 and g_1, consider the product cobordism $W = \Sigma \times I$ with metric extending g_0 and g_1 at the two boundary components. This cobordism induces a degree zero chain map

$$W_* : IC_*(\Sigma, g_0) \to IC_*(\Sigma, g_1).$$

An easy calculation shows that $W_*(\alpha) = \#\mathcal{M}_W(\alpha,\alpha)\cdot\alpha = \alpha$, since the only isolated instanton in $\mathcal{M}_W(\alpha,\alpha)$ is flat. Similarly, $W_*(\beta_1) = \#\mathcal{M}_W(\beta_1,\beta_1)\cdot\beta_1 + \#\mathcal{M}_W(\beta_1,\beta_2)\cdot\beta_2 = \beta_1$ and $W_*(\beta_2) = \beta_2$. The last two formulas use the observation that all isolated instantons in $\mathcal{M}_W(\beta_1,\beta_2)$ and $\mathcal{M}_W(\beta_2,\beta_1)$ are flat, which is due to the fact that the Chern-Simons functional takes the same value on both β_1 and β_2. However, there are no flat connections on W interpolating between $\beta_1 \neq \beta_2$. The fact that W_* is a chain map now easily implies that $\#\hat{\mathcal{M}}_{g_0}(\alpha,\beta_i) = \#\hat{\mathcal{M}}_{g_1}(\alpha,\beta_i)$ for $i = 1,2$.

To finish the proof, choose a generic metric g on Σ. The metric $\tau_* g$ is then also generic, and we have a natural bijective correspondence $\hat{\mathcal{M}}_g(\alpha,\beta_1) = \hat{\mathcal{M}}_{\tau_* g}(\alpha,\beta_2)$. This correspondence is orientation preserving, since τ preserves both orientation and homology orientation. Therefore,

$$\#\hat{\mathcal{M}}_g(\alpha,\beta_1) = \#\hat{\mathcal{M}}_{\tau_* g}(\alpha,\beta_2) = \#\hat{\mathcal{M}}_g(\alpha,\beta_2).$$

□

Since there is no torsion in $I_*(\Sigma)$, the above proposition implies that $\#\hat{\mathcal{M}}_g(\alpha,\beta_1) = \#\hat{\mathcal{M}}_g(\alpha,\beta_2) = \pm 1$. If $n = \mu(\beta_1) = \mu(\beta_2)$ then $I_n(\Sigma) = \mathbb{Z}$ is generated by β_1, and $\tau_* : I_n(\Sigma) \to I_n(\Sigma)$ is minus identity.

4. Gluing formulas and equivariant Floer homology

Let X be a smooth closed simply connected 4–manifold split as $X = U \cup V$ with U and V smooth compact simply connected 4–manifolds such that $\partial U = \Sigma$ and $\partial V = -\Sigma$, where Σ is an integral homology 3–sphere. Let $D(X)$ be a degree d Donaldson polynomial corresponding to a bundle P over X, and let $u_1, \dots, u_r \in H_2(U,\mathbb{Z})$ and $v_1, \dots, v_{d-r} \in H_2(V,\mathbb{Z})$. In favorable circumstances, there exist well-defined relative Donaldson polynomials $D(U)(u_1,\dots,u_r)$ and $D(V)(v_1,\dots,v_{d-r})$ with coefficients in equivariant Floer homology $I_*^{\mathcal{G}}(\Sigma)$ and $I_*^{\mathcal{G}}(-\Sigma)$, respectively, such that $D(X)(u_1,\dots,u_r,v_1,\dots,v_{d-r})$ is obtained from them by pairing $I_*^{\mathcal{G}}(\Sigma)$ with $I_*^{\mathcal{G}}(-\Sigma)$, see [4].

The equivariant Floer homology $I_*^{\mathcal{G}}(\Sigma)$ of Austin and Braam [4] is the homology of a chain complex built from all representations of $\pi_1\Sigma$, including reducible. When the representation variety of Σ is non-degenerate, which is the case for $\Sigma = \partial W$, the boundary of the Akbulut cork W, this equivariant Floer homology is roughly described as follows. The group $SO(3)$ acts by conjugation on the representation space $R(\Sigma) = \operatorname{Hom}(\pi_1\Sigma, SU(2))$ with quotient the representation variety in Proposition 4. Let us consider the equivariant cohomology

$$H^*_{SO(3)}(R(\Sigma),\mathbb{Z}) = H^*(ESO(3) \times_{SO(3)} R(\Sigma),\mathbb{Z})$$

where $ESO(3) \to BSO(3)$ is the universal $SO(3)$–bundle. The connected components of $R(\Sigma)$ correspond to the points in the representation variety of Σ. Let R_α be the component containing a representation α. If α is irreducible, we have $R_\alpha = SO(3)$ and

$$H^*_{SO(3)}(R_\alpha,\mathbb{Z}) = H^*(R_\alpha/SO(3),\mathbb{Z}) = \mathbb{Z}.$$

The component R_θ is a point so that $H^*_{SO(3)}(R_\theta,\mathbb{Z}) = H^*(BSO(3),\mathbb{Z})$. These cohomology groups together form the E_1 term of a Morse-Bott spectral sequence which converges to $I_*^{\mathcal{G}}(\Sigma)$, see [4].

If the intersection forms of both U and V have maximal positive subspaces of dimensions $b_+(U) > 0$ and $b_+(V) > 0$, the instantons restricting to reducible flat connections on Σ can be perturbed away, and the regular Floer homology $I_*(\Sigma)$ can be used in the gluing formula instead of $I_*^{\mathcal{G}}(\Sigma)$.

Let now W be the Akbulut cork shown in Figure 1, and consider the splittings

$$X_0(n) = W \cup_{\mathrm{id}} Q \quad \text{and} \quad X_1(n) = W \cup_\tau Q$$

where $Q = X_0(n) \setminus \operatorname{int} W$ is a smooth compact simply connected 4-manifold with boundary $-\Sigma$. The manifold W is contractible so that $b_+(W) = 0$. An attempt to use $I_*(\Sigma)$ instead of $I_*^{\mathcal{G}}(\Sigma)$ in the gluing formula leads to a contradiction as follows.

The only relative Donaldson polynomial of W has degree zero and an easy index calculation shows that $D(W) \in I_5(\Sigma) = \mathbb{Z}$. Since $X_0(n)$ is an algebraic surface, there exist homology classes $v_1, \dots, v_d \in H_2(Q, \mathbb{Z}) = H_2(X_0(n), \mathbb{Z})$ such that $D(X_0(n))(v_1, \dots, v_d) \neq 0$, see [7]. Then

$$D(X_0(n))(v_1, \dots, v_d) = D(W) \cdot D(Q)(v_1, \dots, v_d) \neq 0$$

is a product of two non-zero numbers, and the involution $\tau_* : I_5(\Sigma) \to I_5(\Sigma)$ can only change sign of this product. On the other hand, we know that re-gluing by τ makes $X_0(n)$ into $X_1(n)$. Since $X_1(n)$ is completely decomposable, all its Donaldson polynomials vanish.

This contradiction shows that there exist instantons on $X_0(n)$ which do not factor through irreducible flat connections on Σ, and the full group $I_*^{\mathcal{G}}(\Sigma)$ should be taken into account. This last remark clarifies the statement of [2].

5. Concluding remarks

The fact that the action $\tau_* : I_*(\Sigma) \to I_*(\Sigma)$ is non-trivial implies that the mapping cylinder C_τ of the involution τ is not diffeomorphic to the product $\Sigma \times I$ rel boundary (although manifolds C_τ and $\Sigma \times I$ are in fact diffeomorphic).

Proposition 6. *The manifolds C_τ and $\Sigma \times I$ are not homeomorphic (or homotopy equivalent) rel boundary.*

Proof. Let us identify the two boundary components of each of C_τ and $\Sigma \times I$ by using identity maps. We end up with closed manifolds M_τ, which is the mapping torus of τ, and $\Sigma \times S^1$, respectively. If C_τ and $\Sigma \times I$ were homeomorphic (or homotopy equivalent) rel boundary, we would have that $\pi_1(M_\tau) = \pi_1(\Sigma \times S^1)$. However, the latter is not the case, which can be seen as follows.

The fundamental groups of both $\Sigma \times S^1$ and M_τ are HNN–extensions of $\pi_1(\Sigma)$. This easily implies that irreducible $SU(2)$–representations of $\pi_1(\Sigma \times S^1)$, respectively, $\pi_1(M_\tau)$, are in two-to-one correspondence with irreducible $SU(2)$–representations of $\pi_1(\Sigma)$, respectively, irreducible $SU(2)$–representations of $\pi_1(\Sigma)$ equivariant with respect to τ. Since there exist irreducible $SU(2)$–representations of $\pi_1(\Sigma)$ which are not τ–equivariant, we conclude that $\pi_1(M_\tau) \neq \pi_1(\Sigma \times S^1)$. □

The above proposition shows that the mapping cylinder of τ fails to give an example of an exotic smooth structure on $\Sigma \times I$. No such examples are currently known, although exotic smooth structures do exist on all non-compact manifolds $M \times \mathbb{R}$ where M is a closed oriented 3–manifold, see [5].

Any orientation preserving diffeomorphism $f : \Sigma \to \Sigma$ of an integral homology sphere Σ induces an automorphism $f_* : I_*(\Sigma) \to I_*(\Sigma)$ in its Floer homology. This automorphism is often an identity. Theorem 1 gives an example of $\tau : \Sigma \to \Sigma$ with $\tau_* \neq \mathrm{id}$; the manifold Σ is in fact hyperbolic. Examples of irreducible graph homology spheres with involutions acting non-trivially on their Floer homology can be found in [13] and [14].

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