A POWER STRUCTURE OVER THE GROTHENDIECK RING OF VARIETIES

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Abstract. Let \(\mathcal{R}\) be either the Grothendieck semiring (semigroup with multiplication) of complex quasi-projective varieties, or the Grothendieck ring of these varieties, or the Grothendieck ring localized by the class \(L\) of the complex affine line. We define a power structure over these (semi)rings. This means that, for a power series \(A(t) = 1 + \sum_{i=1}^{\infty} [A_i] t^i\) with the coefficients \([A_i]\) from \(\mathcal{R}\) and for \([M]\in\mathcal{R}\), there is defined a series \((A(t))^{[M]}\), also with coefficients from \(\mathcal{R}\), so that all the usual properties of the exponential function hold. In the particular case \(A(t) = (1 - t)^{-1}\), the series \((A(t))^{[M]}\) is the motivic zeta function introduced by M. Kapranov. As an application we express the generating function of the Hilbert scheme of points, 0-dimensional subschemes, on a surface as an exponential of the surface.

By a semiring we mean a semigroup with multiplication. If \(\mathcal{R}\) is a semiring, we have the semiring \(\mathcal{R}[[t]]\) of formal power series with coefficients in \(\mathcal{R}\). Let \(1+\mathcal{R}_+[[t]]\) denote the set of series of the form \(A(t) = 1 + \sum_{i=1}^{\infty} A_i t^i\) with \(A_i \in \mathcal{R}\). If \(\mathcal{R}\) is a ring, resp. a semiring, \(1+\mathcal{R}_+[[t]]\) is an abelian group, resp. a semigroup, with respect to the multiplication. The Grothendieck semiring \(S_0(\text{Var}_C)\) of complex quasi-projective varieties is the semigroup generated by isomorphism classes \([X]\) of such varieties modulo the relation \([X] = [X - Y] + [Y]\) for a closed subvariety \(Y \subset X\); the multiplication is defined by \([X_1] \cdot [X_2] = [X_1 \times X_2]\). The Grothendieck ring \(K_0(\text{Var}_C)\) is the group generated by these classes with the same relation and the same multiplication. Let \(L \in K_0(\text{Var}_C)\) be the class of the affine line, which itself also will be denoted by \(L\), and let \(M := K_0(\text{Var}_C)[L^{-1}]\) be the localization of Grothendieck ring \(K_0(\text{Var}_C)\) with respect to \(L\).

Power series \(\sum_{i=0}^{\infty} [A_i] t^i\) with coefficients from one of these (semi)rings are usual objects of study, in particular, in the framework of the theory of motivic integration, see for instance [3],[4], [9]. The main result of the paper shows that there exists a natural notion of the \([M]\)-th power of such a series with \([A_0] = 1\) for the exponent \([M]\) from the same ring or semiring.

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**Definition:** A power structure over a (semi)ring $\mathcal{R}$ is a map $(1 + \mathcal{R}[[t]]) \times \mathcal{R} \to 1 + \mathcal{R}[[t]] : (A(t), m) \mapsto (A(t))^m$ which possesses the properties:

1. $(A(t))^0 = 1$.
2. $(A(t))^1 = A(t)$.
3. $(A(t) \cdot B(t))^m = (A(t))^m \cdot (B(t))^m$.
4. $(A(t))^{m+n} = (A(t))^m \cdot (A(t))^n$.
5. $(A(t))^m = ((A(t))^n)^m$.

**Remark.** Some examples of power structures can be found in the theory of $\lambda$-rings (see, e.g., [1], [2], [5]). Note also that in the theory of $\lambda$-rings the constructions in [2] and [5] have sense only over rings which are $\mathbb{Q}$-algebras. The main point of our construction is that we use factorization by actions of permutation groups instead of division by integers. Using our approach a natural structure of $\lambda$-ring on the Grothendieck ring of complex algebraic varieties would be given by $\lambda_t([M]) = (1 + t)^{[M]}$. But in such a case the coefficient at $t^2$ in the power series $(1 + t)^{k^2}$ computed as $\lambda_t(L \times L)$ should be equal to $2L^4 - 2L^2$, while in our construction it is equal to $L^4 - L^2$.

**Definition:** Let $A(t) = 1 + [A_1] t + [A_2] t^2 + \ldots$ be a formal power series with coefficients from $S_0(\text{Var}_C)$ and $[M] \in S_0(\text{Var}_C)$. We define

$$(A(t))^{[M]} := 1 + \sum_{k=1}^{\infty} \left\{ \sum_{k: \Sigma_i k_i = k} \left[ \left( \prod_i M^{k_i} \right) \setminus \Delta \right] \times \prod_i A_i^{k_i} / \prod_i S_{k_i} \right\} \cdot t^k,$$

where $\Delta$ is the "large diagonal" in $M^{\Sigma k}$ which consists of $(\sum k_i)$-tuples of points of $M$ with at least two coinciding ones, the group $S_{k_i}$ acts by permuting corresponding $k_i$ factors in $\prod_i M^{k_i} \supset (\prod_i M^{k_i}) \setminus \Delta$ and the spaces $A_i$ simultaneously.

**Remarks. 1.** This definition can also be "read from" the following formula for the usual power $(1 + a_1 t + a_2 t^2 + \ldots)^m$ of a series with a natural exponent:

$$(1 + a_1 t + a_2 t^2 + \ldots)^m =$$

$$= 1 + \sum_{k=1}^{\infty} \left( \sum_{k: \Sigma_i k_i = k} m(m-1) \cdot \ldots \cdot (m - \Sigma k_i + 1) \cdot \prod_i a_i^{k_i} / \prod_i k_i ! \right) \cdot t^k,$$

see, for instance [12], page 40. In this formula one should understand the product $m(m-1) \cdot \ldots \cdot (m - \Sigma k_i + 1)$ as $M^{\Sigma k_i} \setminus \Delta$. If $M$ is a set which consists of $m$ points, this product is the number of elements of the indicated space. The division by $\prod_i k_i !$ should be understood as the factorization by the group $\prod_i S_{k_i}$.
2. For a complex algebraic variety $M$, its motivic zeta function $\zeta_{[M]}(t)$ was defined by M. Kapranov in [8] as the power series

$$
\zeta_{[M]}(t) := \sum_{k=0}^{\infty} [S^k M] \cdot t^k = 1 + [S^1 M] \cdot t + [S^2 M] \cdot t^2 + [S^3 M] \cdot t^3 + \ldots ,
$$

where $S^k M$ is the $k$-th symmetric power $M^k/S_k$ of the variety $M$, $S_k$ is the symmetric group of permutations on $k$ elements. It has the property $\zeta_{M+N}(t) = \zeta_M(t) \cdot \zeta_N(t)$.

**Theorem 1.** The equation (1) defines a power structure over the Grothendieck semiring $S_0(\text{Var}_C)$ of complex algebraic varieties such that

$$(1 + t + t^2 + \ldots)^{[M]} = \zeta_{[M]}(t).$$

**Proof.** The fact that $(1 + t + t^2 + \ldots)^{[M]} = \zeta_{[M]}(t)$ and the properties 1 and 2 from the definition of a power structure are obvious. Let us reformulate the definition of the series $(A(t))^{[M]}$ a little bit so that the properties 3 to 5 will be proved by establishing one-to-one correspondences between the sets representing the coefficients of the left hand side (LHS) and the right hand side (RHS) series.

The coefficient at the monomial $t^k$ in the series $(A(t))^{[M]}$ is represented by the set whose element is a finite subset $K$ of points of the variety $M$ with positive multiplicities such that the total number of points of the set $K$ counted with multiplicities is equal to $k$ plus a map $\varphi$ from $K$ to $\mathcal{A} = \prod_{i=0}^{\infty} A_i$, such that a point of multiplicity $s$ goes to $A_s \subset \mathcal{A}$. Here $\coprod$ means the disjoint union which is the sum in the semiring $S_0(\text{Var}_C)$. For short, instead of writing that "a coefficient of a series is represented by a set which consists of elements of the form ..." we shall write that "an element of the coefficient is ...".

**Proof of 3.** Consider another series $B(t) = \sum_{i=0}^{\infty} [B_i] t^i$ with coefficients from the same semiring. Let $[C_j] := \sum_{i=0}^{j} [A_i][B_{j-i}]$ be the coefficient at the monomial $t^j$ in the product $A(t) \cdot B(t)$. An element of the coefficient at the monomial $t^k$ in the LHS of the equation 3 is a $k$-point subset $K$ of $M$ with a map $\varphi$ from $K$ to $\coprod_{i=0}^{\infty} C_i = \coprod_{i,j\geq 0} A_iB_j$ such that a point of multiplicity $s$ goes to $C_s = \coprod_{i=0}^{s} A_i \times B_{s-i}$.

An element of the coefficient at the monomial $t^k$ in the RHS of this equation is: an $\ell$-point subset $K_1$ of the variety $M$ with $0 \leq \ell \leq k$, a map $\varphi_1$ from $K_1$ to $\mathcal{A} = \coprod_{i=0}^{\infty} A_i$ such that a point of multiplicity $s$ goes to $A_s$, a $(k-\ell)$-point subset $K_2$ of the variety $M$, a map $\varphi_2$ from $K_2$ to $\mathcal{B} = \coprod_{i=0}^{\infty} B_i$ such that a point of multiplicity $s$ goes to $B_s$. Suppose we have an element of the first set. Let us decompose the subset $K$ into two parts $K_1$ and $K_2$. A point $x$ of $K$
of multiplicity $s = s(x)$ goes to $C_s = \prod_{i=0}^{s} A_i \times B_{s-i}$. Let us include the point $x$ into the set $K_1$ with the multiplicity $i_0$ and into the set $K_2$ with the multiplicity $s - i_0$. If $i_0$ and/or $s - i_0$ is positive, define $\varphi_1(x)$ and/or $\varphi_2(x)$ as the corresponding projection, to $A_{i_0}$ and/or to $B_{s-i_0}$ respectively, of the point $\varphi(x) \in A_{i_0} \times B_{s-i_0}$.

In the other direction, from an element of the second set one can construct an element of the first one uniting the subsets $K_1$ and $K_2$ so that a multiplicity of a point $x$ in $K_1 \cup K_2$ is equal to the sum $s_1 + s_2$ of its multiplicities in $K_1$ and in $K_2$ and defining $\varphi(x)$ as $\varphi(x) \in A_{s_1} \times B_{s_2} \subset C_{s_1+s_2}$. One easily sees that these correspondences are inverse to each other and thus are one-to-one. 

Proof of 4. An element of the coefficient at the monomial $t^k$ in the LHS of the equation is a $k$-point subset $K$ of the variety $M \cup N$ with a map $\varphi$ from $K$ to $\mathcal{A} = \prod_{i=0}^{\infty} A_i$. An element of the coefficient at the monomial $t^k$ in the RHS is a pair: an $\ell$-point subset $K_1$ of the variety $M$ with $0 \leq \ell \leq k$ and a $(k-\ell)$-point subset $K_2$ of the variety $N$ with their maps to $\mathcal{A}$. The union of these two subsets with the corresponding map to $\mathcal{A}$ gives a subset in the union $M \cup N$ with a map to $\mathcal{A}$. This correspondence is obviously one-to-one.

Proof of 5. An element of the coefficient at the monomial $t^k$ in the LHS of the equation is a $k$-point subset $K$ of the product $M \times N$ with a map $\varphi : K \to \mathcal{A} = \prod_{i=0}^{\infty} A_i$. An element of the coefficient at the monomial $t^k$ in the RHS is a $k$-point subset $K_M$ of the variety $M$ with a map from it which sends a point of multiplicity $s$ to an $s$ point subset of $N$ with a map from it to $\mathcal{A}$. To establish a one-to-one correspondence between these coefficients, rather between the variety representing them, one should define $K_M$ as the projection $\text{pr}_M K$ of the subset $K \subset M \times N$ to the first factor and, for $x \in K_M$, the corresponding subset in $N$ as $\text{pr}_N \text{pr}_M^{-1}(x)$ with the natural map to $\mathcal{A}$.

Example.

\[(1 + t)^{[M]} = 1 + \sum_{k=1}^{\infty} \left[ (M^k \setminus \Delta)/S_k \right] \cdot t^k,\]

where $(M^k \setminus \Delta)/S_k$ is the configuration space of $k$ distinct unlabeled points in $M$.

Let $\chi(X)$ be the Euler characteristic of the space $X$ (the alternating sum of ranks of the cohomology groups with compact support). For a series $A(t) = 1 + [A_1]t + [A_2]t^2 + \ldots$, one defines its Euler characteristic as the series

\[\chi(A(t)) = 1 + \chi([A_1])t + \chi([A_2])t^2 + \ldots \in \mathbb{Z}[[t]].\]
Statement 1.

\[ \chi \left( (A(t))^{[M]} \right) = (\chi (A(t)))^{\chi([M])}. \]

The proof follows either from direct calculations or from the fact that the coefficients at the monomials \( t^k \) in the LHS of the equation are polynomials in the Euler characteristics of the varieties \( M \) and \( A_i \) \((i = 1, 2, \ldots)\) and the equation holds for "natural numbers", i.e., for the case when all the varieties \( M \) and \( A_i \) are finite sets of points. □

Theorem 2. There exists a unique power structure over the Grothendieck ring \( K_0(\text{Var}_C) \) of complex algebraic varieties which extends the one defined over the semiring \( S_0(\text{Var}_C) \).

Proof. To define the operation notice that for any series \( A(t) \in 1+K_0(\text{Var}_C)[[t]] \) there exists a series \( B(t) = \sum_{i=0}^{\infty} [B_i] t^i \in 1 + K_0(\text{Var}_C)[[t]] \) with the coefficients from the image of the natural map \( S_0(\text{Var}_C) \to K_0(\text{Var}_C) \) such that all the coefficients of the product \( C(t) = A(t) \cdot B(t) \) are from the same image as well. Then one puts:

\[ (A(t))^{[M]} := (C(t))^{[M]} / (B(t))^{[M]} . \]

To define the power of a series with the exponent \([M]\) from the Grothendieck ring \( K_0(\text{Var}_C) \), one puts

\[ (A(t))^{-[M]} := 1 / (A(t))^{[M]} . \]

The properties of the definition of a power structure obviously hold. □

Theorem 3. There exists a unique power structure over the ring \( \mathcal{M} = K_0(\text{Var}_C)[[L^{-1}]] \) which extends the one defined over the ring \( K_0(\text{Var}_C) \).

Proof. First let us define the operation for a series \( A(t) \) with the coefficients \([A_i]\) from the ring \( \mathcal{M} \) and with the exponent \([M]\) from the "non-localized" ring \( K_0(\text{Var}_C) \). This is possible because of the following statement.

Statement 2. Let \([A_i]\) and \([M]\) be from the Grothendieck ring of complex algebraic varieties. Then, for any integer \(s \geq 0\), \( (A(L^s t))^{[M]} = (A(t)^{[M]})_{|t=t-L^{-s}t} . \)

Proof. It is sufficient to prove this equation for a series with the coefficients from \( S_0(\text{Var}_C) \). The coefficient at the monomial \( t^k \) in the power series \( (A(t))^{[M]} \) is a sum of the classes of varieties of the form

\[ V = \left( \prod_i M^{k_i} \right) \Delta \times \prod_i A_i^{k_i} / \prod S_{k_i} \]

with \( \sum i k_i = k \). The corresponding summand \( [\tilde{V}] \) in the coefficient at the monomial \( t^k \) in the power series \( (A(L^s t))^{[M]} \) has the form

\[ \tilde{V} = \left( \prod_i M^{k_i} \right) \Delta \times \prod_i (L^{k_i} \times A_i)^{k_i} / \prod S_{k_i} . \]
There is a natural map $\hat{V} \rightarrow V$ which from the point of view of differential geometry is a complex analytic vector bundle of rank $sk$. It is locally trivial over a neighbourhood of each point in the usual topology. According to [11] this is a vector bundle in the "algebraic sense" and then it is locally trivial over a Zariski open neighbourhood of each point. This implies that $[\hat{V}] = L^s k \cdot [V].$

For a series $A(t) = \sum_{i=0}^{\infty} [A_i] t^i$, let $J^r A(t)$ denote its truncation $\sum_{i=0}^{r} [A_i] t^i$ up to terms of degree $r$. Statement 2 implies that, for $[A_i] \in \mathcal{M}$, $i \geq 1$ and $[M] \in K_0 (\text{Var}_C)$, one can define the power $(A(t))^[[M]]$ by the formula

$$J^r \left( (A(t))^[[M]] \right) := (J^r A(L^s t))[[M]] \big|_{t \mapsto t/L^r} \mod t^{r+1}$$

for $s$ large enough so that all the coefficients of $J^r A(L^s t)$ belong to the image of the map $K_0 (\text{Var}_C) \rightarrow \mathcal{M}$. One can easily see that the properties 3–5 hold.

Now we have to extend the operation to the exponent $[M]$ from the localized ring $\mathcal{M}$. First let us do it for one particular series, namely for $(1 + t + t^2 + \ldots)$. In other words, we define the zeta function $\zeta_{[M]}(t)$ for $[M] \in \mathcal{M}$.

**Statement 3.** For $M \in K_0 (\text{Var}_C)$, $s \geq 0$, one has

$$\zeta_{L^s [M]}(t) = \zeta_{[M]}(L^s t).$$

**Proof.** It is sufficient to prove the equation for $s = 1$. One has $\zeta_L (t) = 1 + L t + L^2 t^2 + \ldots$ and therefore, using Statement 2, gets

$$\zeta_{L[M]}(t) = (1 + L t + L^2 t^2 + \ldots)^[[M]] = (1 + t + t^2 + \ldots)^[[M]] \big|_{L \mapsto L^s}.$$

This statement permits to define $\zeta_{[M]}(t)$ for $[M] \in \mathcal{M}$ by the formula

$$\zeta_{[M]}(t) = \zeta_{L^s [M]}(L^{-s} t)$$

for $s$ large enough so that $L^s [M]$ belongs to the image of the map $K_0 (\text{Var}_C) \rightarrow \mathcal{M}$.

For $A(t) \in 1 + K_0 (\text{Var}_C)_+ ([t])$, $[M] \in K_0 (\text{Var}_C)$, $s > 0$, one has

$$(A(t)^s)^[[M]] = (A(t)^[[M]]) \big|_{t \mapsto L^s}.$$ 

Since $\zeta_{[M]}(t) = 1 + [M] t + \ldots$, for any series $A(t) \in 1 + \mathcal{M}_+ ([t])$ and for any $r > 0$, the truncated series $J^r A(t)$ can be represented in a unique way as

$$J^r \left( (\zeta_{[A_1]}(t) \cdot \zeta_{[A_2]}(t^2) \cdot \zeta_{[A_3]}(t^3) \cdot \ldots \cdot \zeta_{[A_r]}(t^r)) \right)$$

with $[A_i] \in \mathcal{M}$. One defines $(A(t))^[[M]]$ by the formula

$$(2) \quad J^r \left( (A(t))^[[M]] \right) := J^r \left( (\zeta_{[M][A_1]}(t) \cdot \zeta_{[M][A_2]}(t^2) \cdot \ldots \cdot \zeta_{[M][A_r]}(t^r)) \right).$$

Properties 3–5 of the definition of a power structure obviously hold.

Since the formula (2) can be used to define the power structure over the rings $K_0 (\text{Var}_C)$ and $\mathcal{M}$, such a structure over them is **unique**.

\[ \square \]
Remarks. 1. Yu.I.Manin has informed us that his calculations in [10] resemble a particular case of our construction.

2. The following construction was inspired by the paper of E.Getzler [6]. Let $\mathcal{R}$ be either the ring $K_0(\text{Var}_\mathbb{C})$ or the ring $\mathcal{M}$. Consider $1 + \mathcal{R}[[t]]$ and $\mathcal{R}_+[\![t]\!] = (t)\mathcal{R}[[t]]$ as groups with respect to the multiplication and addition respectively. The construction described here permits to define group isomorphisms $\text{Exp} : \mathcal{R}_+[\![t]\!] \to 1 + \mathcal{R}[[t]]$ and $\text{Log} : 1 + \mathcal{R}_+[\![t]\!] \to \mathcal{R}_+[\![t]\!]$. Namely,

$$\text{Exp}([A_1]t + [A_2]t^2 + \ldots) := \prod_{k \geq 1} (1 - t^k)^{-[A_k]} = \prod_{k \geq 1} \zeta_{[A_k]}(t^k);$$

for a series $M(t) = 1 + [M_1]t + [M_2]t^2 + \ldots \in 1 + \mathcal{R}_+[\![t]\!]$ ($\mathcal{R}$ is either $K_0(\text{Var}_\mathbb{C})$ or $\mathcal{M}$),

$$\text{Log}(M(t)) := \sum_{k \geq 1} [A_k'] t^k,$$

where $[A_k'] \in \mathcal{R}$ are defined by the equation $M(t) = \zeta_{[A_1]}(t) \cdot \zeta_{[A_2]}(t^2) \cdot \zeta_{[A_3]}(t^3) \cdot \ldots$ (such a representation is unique). Obviously $\text{Exp}$ and $\text{Log}$ are inverse to each other. The properties of the power structure imply that the maps $\text{Exp}$ and $\text{Log}$ possess the usual properties of the log and exp functions: $\text{Exp}(A(t) + B(t)) = \text{Exp}(A(t)) \text{Exp}(B(t))$, $\text{Log}(M(t) \cdot N(t)) = \text{Log}(M(t)) + \text{Log}(N(t))$, $\text{Exp}([M]t) = \zeta_{[M]}(t)$, $\ldots$.

An application. For a smooth quasi-projective surface $M$, let $M^{[n]} = \text{Hilb}^n M$ be the Hilbert scheme of 0-dimensional subschemes of length $n$ on $M$.

Statement 4. In the Grothendieck ring $K_0(\text{Var}_\mathbb{C})$ one has:

$$1 + \sum_{n \geq 1} [M^{[n]}] t^n = \prod_{k \geq 1} \zeta_{L^{-1}[M]}((Lt)^k) = \left( \prod_{k \geq 1} \frac{1}{1 - L^{k-1} t^k} \right)^{[M]} = \prod_{k \geq 1} \left( \frac{1}{1 - t^k} \right)^{L^{k-1}[M]} = \left( \text{Exp} \left( \frac{Lt}{1 - Lt} \right) \right)^{L^{-1}[M]}.$$

The proof follows from the following result of L. Göttsche [7]:

$$[M^{[n]}] = \sum_{\sum k_i = n} [s_{k_i} M] \cdot \mathbb{L}^{n-|k|};$$
where $|k| = \sum k_i$, $S^k M = S^{k_1} M \times \cdots \times S^{k_n} M$. Therefore
\[
1 + \sum_{n \geq 1} [M^{[n]}] t^n = \sum_{n \geq 0} \left( \sum_{k: \Sigma k_i = n} [S^k M] \cdot \mathbb{L}^{n-|k|} \right) \cdot t^n
\]
\[
= \sum_{n \geq 0} \left( \sum_{k: \Sigma k_i = n} [S^{k_1} M] \cdots [S^{k_n} M] \cdot \mathbb{L}^{n-(k_1+\cdots+k_n)} \right) \cdot t^{\Sigma k_i}
\]
\[
= \sum_{n \geq 0} \left( \sum_{k: \Sigma k_i = n} ([S^{k_1} M] \cdot \mathbb{L}^{-k_1}) \cdots ([S^{k_n} M] \cdot \mathbb{L}^{-k_n}) \right) \cdot (\mathbb{L}t)^{\Sigma k_i}
\]
\[
= \prod_{k \geq 1} \left( \sum_{r \geq 0} [S^r M] \cdot \mathbb{L}^{-r} (\mathbb{L}t)^{kr} \right) = \prod_{k \geq 1} \zeta_{\mathbb{L}^{-1}[M]} \left( (\mathbb{L}t)^k \right). \quad \square
\]

Remarks. 1. D. van Straten told us that the fact that the generating function of the Hilbert scheme of 0-dimensional subschemes of a surface is in some sense an exponent was conjectured by several people.

2. All the results of this paper can be extended to the Grothendieck ring of algebraic varieties over an algebraically closed field of characteristic zero. In this case Statement 2 follows from a version of Lemma 4.4 in [7]. In Statement 1, the Euler characteristic should be replaced by the $\ell$-adic Euler characteristic with compact support.

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