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STABILITY OF SYZYGY BUNDLES ON AN ALGEBRAIC SURFACE

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Abstract. We establish the stability of the syzygy bundle associated to any sufficiently positive embedding of an algebraic surface.

Introduction

The purpose of this paper is to prove the stability of the syzygy bundle associated to any sufficiently positive embedding of an algebraic surface.

Let X be a smooth projective algebraic variety over an algebraically closed field k, and let L be a very ample line bundle on X. The *syzygy* (or *kernel*) *bundle* M_L associated to L is by definition the kernel of the evaluation map

$$\mathrm{eval}_L : H^0(L) \otimes_k \mathcal{O}_X \longrightarrow L.$$

Thus M_L sits in an exact sequence

$$0 \longrightarrow M_L \longrightarrow H^0(L) \otimes_k \mathcal{O}_X \longrightarrow L \longrightarrow 0.$$

These vector bundles (and some analogues) arise in a variety of geometric and algebraic problems, ranging from the syzygies of X to questions of tight closure. Consequently, there has been considerable interest in trying to establish the stability of M_L in various settings. When X is a smooth curve of genus $g \geq 1$, the situation is well-understood thanks to the work of several authors [1, 3, 4, 8, 12, 13]; in particular, M_L is stable as soon as $\deg L \geq 2g+1$. When $X = \mathbf{P}^n$ and $L = \mathcal{O}_{\mathbf{P}^n}(d)$, the stability of M_L was established by Flenner [9, Cor. 2.2] in characteristic 0 and by Trivedi [14] in characteristic > 0 for many d. A more general statement, due to Coandă [6], treats the bundles associated to possibly incomplete linear subseries of $H^0\big(\mathbf{P}^n, \mathcal{O}_{\mathbf{P}^n}(d)\big)$. Motivated by questions of tight closure, the stability of syzygy bundles on $\mathbf{P}^n$ arising from a somewhat more general construction has been studied by Brenner [2] and by Costa, Marques and Miró–Roig [7, 11]. In dimension 2, Camere [5] recently proved that kernel bundles on $K3$ and abelian surfaces are stable.

We show here that if L is a sufficiently positive divisor on any smooth projective surface X, then M_L is stable with respect to a suitable hyperplane section of X. Specifically, fix an ample divisor A and an arbitrary divisor P on X. Given a large integer d, set

$$L_d \ = \ dA + P,$$

and write $M_d = M_{L_d}$. Our main result is

Theorem A. *If d is sufficiently large, then M_d is slope stable with respect to A.*

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Recall that the conclusion means that if $F \subseteq M_d$ is a subsheaf with $0 < \text{rank}\ (F) < \text{rank}\ (M_d)$, then

$$\frac{c_1(F) \cdot A}{\text{rank } F} \;<\; \frac{c_1(M_d) \cdot A}{\text{rank } M_d}.$$

Since a slope-stable bundle is also Gieseker stable, it follows that M_d is parameterized for $d \gg 0$ by a point on the moduli space of bundles on X with suitable numerical invariants. On the other hand, working over $\mathbf{C}$, Camere [5, Proposition 2] shows that if $H^1(X, \mathcal{O}_X) = 0$, and if the natural map

$$H^0(X, K_X) \otimes H^0(X, L) \longrightarrow H^0(X, K_X + L)$$

is surjective for some very ample line bundle L, then M_L is rigid. However, this surjectivity is automatic if K_X is globally generated and L is sufficiently positive. Hence, we deduce

Corollary B. *Let X be a complex projective surface with vanishing irregularity $q(X) = 0$, and assume that K_X is globally generated. Then M_d corresponds to an isolated point of the moduli space of stable vector bundles on X when $d \gg 0$.*

It is natural to suppose that the analogue of Theorem A holds also for varieties of dimension ≥ 3, but unfortunately our proof does not work in this setting. However, if $\text{Pic}(X) \cong \mathbf{Z}$, then the argument of Coandă [6] goes through with little change to establish:

Proposition C. *Assume that* $\dim X \geq 2$ *and that* $\text{Pic}(X) = \mathbf{Z} \cdot [A]$ *for some ample divisor A. Write $L_d = dA$. Then $M_d =_{\text{def}} M_{L_d}$ is A-stable for $d \gg 0$.*

As in [5] the strategy for Theorem A is to reduce the question to the stability of syzygy bundles on curves, but we avoid the detailed calculations appearing in that paper. In order to explain the idea, we sketch a quick proof of Camere's result [5, Theorem 1] that if L is a globally generated ample line bundle on a K3 surface X, then M_L is L-stable. Supposing to the contrary, let $F \subseteq M_L$ be a saturated destabilizing subsheaf, and fix a general point $x \in X$. Consider now a general curve $C \in |L \otimes \mathfrak{m}_x|$; we may suppose that F sits as a sub-bundle of M_L along C. Restriction to C yields a diagram:

$$(*)\qquad \begin{array}{ccccccccc} & & & & F|C & & & & \\ & & & & \big\downarrow & & & & \\ 0 & \longrightarrow & \mathcal{O}_C & \longrightarrow & M_L|C & \longrightarrow & \overline{M}_{L|C} & \longrightarrow & 0, \end{array}$$

where $\overline{M}_{L|C}$ is the syzygy bundle on C associated to $\Omega_C = L|C$. However, $\overline{M}_{L|C}$ is semi-stable by [13], while

$$\mu\big(F|C\big) \;\geq\; \mu\big(M_L|C\big) \;>\; \mu\big(\overline{M}_{L|C}\big).$$

It follows that $F|C$ cannot inject into $\overline{M}_{L|C}$, and hence the two sub-bundles $F|_C$ and $\mathcal{O}_C$ of $M_L|C$ have a non-trivial intersection, which in turn implies that $\mathcal{O}_C$ is contained in $F|_C$. On the other hand, consider the fibres at x of the various bundles in play. The vertical map in (*) corresponds to a fixed subspace $F(x) \subsetneqq H^0(X, L \otimes \mathfrak{m}_x)$. So we would be asserting that the equation defining a general curve $C \in |L \otimes \mathfrak{m}_x|$

lies in this subspace, and this is certainly not the case. The proof of Theorem A in general proceeds in an analogous manner, the main complication being that we have to deal with a trivial vector bundle of higher rank appearing on the left in the bottom row of (*).

Concerning the organization of the paper, Section 1 is devoted to the proof of Theorem A. Proposition C appears in Section 2, where we also propose some open problems.

1. Proof of main theorem

This section is devoted to the proof of Theorem A from the Introduction.

We start by fixing notation and set-up. As in the Introduction, X is a smooth projective surface, and $L_d = dA + P$ where A is an ample divisor, and P is an arbitrary divisor on X. For the duration of the argument, we fix an integer $b \gg 0$ such that $(bA + P - K_X)$ is very ample, and so that $H^1(X, L_b) = 0$, and put

$$B \;=\; L_b \;=\; bA + P.$$

Observe that b and B are independent of d. We also assume henceforth that d is sufficiently large, so that it satisfies the following properties:

(i) L_d and L_{d-b} are very ample.

(ii) For all $x \in X$, the natural mapping

$$H^0\big(X, (d-b)A \otimes \mathfrak{m}_x\big) \otimes H^0\big(X, B\big) \longrightarrow H^0\big(X, L_d \otimes \mathfrak{m}_x\big) \tag{1.1}$$

is surjective (where $\mathfrak{m}_x$ denotes the ideal sheaf of x).

Fixing such an integer d, assume now that $M_d = M_{L_d}$ is not A-stable. Recall that this means that there exists a non-trivial subsheaf

$$F_d \;\subseteq\; M_d$$

such that

$$\frac{c_1(F_d) \cdot A}{\text{rank } F_d} \;\geq\; \frac{c_1(M_d) \cdot A}{\text{rank } M_d}.$$

Without loss of generality, we assume that $F_d \subseteq M_d$ is saturated, and we fix a point $x = x_d \in X$ at which F_d is locally free.

The plan is to use the stability of syzygy bundles on curves to show that if $d \gg 0$, then no such F_d can actually exist. To this end, consider a general curve

$$C_d \;\in\; |(d-b)A| \;=\; |L_d - B|$$

passing through the fixed point $x \in X$. We may assume that C_d is smooth and irreducible, and that M_d/F_d is locally free along C_d. Observe also that for any torsion-free sheaf $\mathcal{F}$ on X that is locally free along C_d, one has

$$\mu_A\big(\mathcal{F}\big) \;=\; \frac{1}{(d-b)} \cdot \mu\big(\mathcal{F}|C_d\big).$$

In particular, if $\mathcal{F}$ is A-unstable as a sheaf on X, then $\mathcal{F}|C_d$ is unstable on C_d.

We now consider the restriction of M_d and F_d to C_d. Writing $\overline{M}_d = M_{\overline{L}_d}$ for the syzygy bundle on C_d of the restriction $\overline{L}_d = L_d|C_d$, a straightforward analysis of the exact sequence $0 \to B \to L_d \to \overline{L}_d \to 0$ yields an exact sequence

$$0 \longrightarrow H^0(B)_{C_d} \longrightarrow M_d|C_d \longrightarrow \overline{M}_d \longrightarrow 0,$$

where the term on the left is the trivial bundle on C_d with fibre $H^0(X,B)$. We complete this to a diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \overline{K}_d & \longrightarrow & F_d \mid C_d & \longrightarrow & \overline{N}_d & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & H^0(B)_{C_d} & \longrightarrow & M_d|C_d & \longrightarrow & \overline{M}_d & \longrightarrow & 0 \end{array} \tag{1.2}$$

of vector bundles on C_d, where $\overline{N}_d$ denotes the image of $F_d|C_d$ in $\overline{M}_d$, and $\overline{K}_d$ is the kernel of the resulting map $F_d|C_d \longrightarrow \overline{N}_d$.

Observe now that

$$L_d|C_d \equiv_{\text{lin}} \big(C_d + B\big)|C_d \equiv_{\text{lin}} \big(K_X + C_d + Q)|C_d$$

where $Q = bA + P - K_X$ is very ample by assumption. In particular, $\deg(L_d|C_d) > 2g(C_d) + 1$, and hence $\overline{M}_d$ is stable on C_d thanks to [8]. On the other hand, it follows from the bottom row of (1.2) that $\mu(M_d|C_d) > \mu(\overline{M}_d)$, and hence

$$\mu\big(F_d|C_d\big) \ \geq \ \mu\big(M_d|C_d\big) \ > \ \mu\big(\overline{M}_d\big). \tag{1.3}$$

Therefore $F_d|C_d$ cannot be a subsheaf of $\overline{M}_d$, and hence $\overline{K}_d \neq 0$.

The following two lemmas constitute the heart of the proof. The first asserts that the destabilizing subsheaf F_d must have large rank.

Lemma 1.1. *One has*

$$\operatorname{rank}(F_d) \geq h^0\big((L_d - B) \otimes \mathfrak{m}_x\big) = h^0\big(L_d - B\big) - 1.$$

The second lemma shows that if d is sufficiently large, then the vertical inclusion on the left of (1.2) is the identity.

Lemma 1.2. *If $d \gg 0$, then $\overline{K}_d = H^0(B)_{C_d}$.*

Granting these assertions for now, we give the

Proof of Theorem A. We need to show that if $d \gg 0$ then the picture introduced above cannot occur. To this end, we consider the fibres at the fixed point $x \in X$ of the vector bundles appearing in the left hand square of (1.2). Since the fibre of the map eval_L at x is the natural surjection $H^0(L_d) \to H^0(L_d \otimes \mathcal{O}_x)$, the fibre $M_d(x)$ of M_d at x is canonically identified with $H^0(X, L_d \otimes \mathfrak{m}_x)$, so these take the form

$$\begin{array}{ccc} & & F_d(x) \\ & & \downarrow \\ H^0(X,B) & \xrightarrow{C_d} & H^0(X, L_d \otimes \mathfrak{m}_x). \end{array} \tag{1.4}$$

Here the bottom map is the natural inclusion determined by a local equation for $C_d \in |(L_d - B) \otimes \mathfrak{m}_x|$. It follows from Lemma 1.2 that $H^0(X,B)$ maps into the subspace

$$F_d(x) \ \subsetneqq \ H^0(X, L_d \otimes \mathfrak{m}_x).$$

So for the required contradiction, it is enough to show that as C_d varies over an open subset of $|(L_d - B) \otimes \mathfrak{m}_x|$, the images of the corresponding embeddings of $H^0(X,B)$ span all of $H^0(X, L_d \otimes \mathfrak{m}_x)$. But this follows from the surjectivity of the map (1.1). □

Proof of Lemma 1.1. We continue to work with the diagram (1.4), and we write $\mathbf{P}_{\text{sub}}(W)$ for the projective space of one-dimensional subspaces of a vector space W. Multiplication of sections gives rise to a finite morphism:

$$\mu_d : \mathbf{P}_{\text{sub}}\big(H^0(X, (L_d - B) \otimes \mathfrak{m}_x)\big) \times \mathbf{P}_{\text{sub}}\big(H^0(X, B)\big) \longrightarrow \mathbf{P}_{\text{sub}}\big(H^0(X, L_d \otimes \mathfrak{m}_x)\big).$$

Set

$$Z \;=\; \mu_d^{-1}\left(\mathbf{P}_{\text{sub}}\big(F_d(x)\big)\right).$$

Then

$$(*) \qquad \dim \mathbf{P}_{\text{sub}}\big(F_d(x)\big) \;\geq\; \dim Z$$

thanks to the finiteness of μ_d. On the other hand, for general $C_d \in |(L_d - B) \otimes \mathfrak{m}_x|$, the image of the corresponding inclusion

$$H^0(X, B) \;\subseteq\; H^0(X, L_d \otimes \mathfrak{m}_x)$$

must meet the subspace $F_d(x) \subseteq H^0(X, L_d \otimes \mathfrak{m}_x)$ non-trivially: indeed, this follows from (1.2) and the fact that $\overline{K_d}(x) \neq 0$. However, this means that the projection

$$(**) \qquad \text{pr}_2 : Z \longrightarrow \mathbf{P}_{\text{sub}}\big(H^0(X, (L_d - B) \otimes \mathfrak{m}_x)\big)$$

is dominant. The Lemma follows upon combining (*) and (**). □

Proof of Lemma 1.2. Since M_d/F_d is locally free along C_d, it follows from (1.2) that $\overline{K}_d$ is a saturated subsheaf of $H^0(B)_{C_d}$, so it suffices to show that rank $\overline{K}_d = h^0(B)$. The argument is numerical. First, note from (1.2) and the stability of $\overline{M}_d$ that

$$\begin{aligned} \mu\big(F_d|C_d\big) \;&=\; \frac{\deg \overline{K}_d + \deg \overline{N}_d}{\text{rank } F_d} \;\leq\; \frac{\deg \overline{N}_d}{\text{rank } F_d} \\ &= \mu\big(\overline{N}_d\big) \cdot \left(\frac{\text{rank } \overline{N}_d}{\text{rank } F_d}\right) \qquad (1.5) \\ &< \mu\big(\overline{M}_d\big) \cdot \left(1 - \frac{\text{rank } \overline{K}_d}{\text{rank } F_d}\right). \end{aligned}$$

Now $\deg(M_d|C_d) = \deg(\overline{M}_d)$, and since

$$\mu\big(M_d|C_d\big) \;\leq\; \mu\big(F_d|C_d\big),$$

equation (1.5) yields:

$$\frac{\deg(M_d|C_d)}{\text{rank } \overline{M}_d + h^0(B)} \;<\; \frac{\deg(M_d|C_d)}{\text{rank } \overline{M}_d} \cdot \left(1 - \frac{\text{rank } \overline{K}_d}{\text{rank } F_d}\right).$$

Observing that $\deg(M_d|C_d) < 0$, this is equivalent to the inequality

$$\frac{1}{\text{rank } \overline{M}_d + h^0(B)} \;>\; \frac{1}{\text{rank } \overline{M}_d} \cdot \left(1 - \frac{\text{rank } \overline{K}_d}{\text{rank } F_d}\right),$$

i.e.,

$$\frac{\text{rank } \overline{M}_d}{\text{rank } \overline{M}_d + h^0(B)} \;>\; 1 - \frac{\text{rank } \overline{K}_d}{\text{rank } F_d}.$$

Thus,

$$\frac{\text{rank } \overline{K}_d}{\text{rank } F_d} \;>\; 1 - \frac{\text{rank } \overline{M}_d}{\text{rank } \overline{M}_d + h^0(B)} \;=\; \frac{h^0(B)}{\text{rank } M_d},$$

and hence

$$(*) \qquad \operatorname{rank} \overline{K}_d \ > \ h^0(B) \cdot \left(\frac{\operatorname{rank} F_d}{\operatorname{rank} M_d} \right).$$

However, by the previous lemma, rank $F_d \geq h^0(X, L_d - B) - 1$. Furthermore, B is independent of d, and rank $M_d = h^0(X, L_d) - 1$. Thus as d grows, the fraction on the right in (*) becomes arbitrarily close to 1.[1] It follows that

$$\operatorname{rank} \overline{K}_d \ > \ h^0(B) - 1$$

provided that $d \gg 0$, and hence rank $\overline{K}_d = h^0(B)$, as required. □

2. Complements

In this section, we first of all prove Proposition C by adapting the method of proof of Theorem 1.1 in [6]. Then we propose some open problems.

2.1. Coandă's argument. We begin by stating (without proof) two preliminary results on which the method rests; the first of these is a cohomological characterization of stability, and the second is a vanishing theorem of Green.

Lemma 2.1. *Let E be a vector bundle on X. If for every r with $0 < r < \operatorname{rk}(E)$ and for every line bundle N on X with $\mu_L(\Lambda^r E \otimes N) \leq 0$ one has $H^0(\Lambda^r E \otimes N) = 0$, then E is $L-$stable.* □

Lemma 2.2 ([10, 3.a.1])**.** *Let N, N' be line bundles on X and assume N is very ample. Then for $r \geq h^0(N')$, we have $H^0(\Lambda^r M_N \otimes N') = 0$.* □

Proof of Proposition C. Let X be a smooth projective variety of dimension $n \geq 2$ for which $\operatorname{Pic}(X) \cong \mathbb{Z} \cdot [A]$ for an ample line bundle A. Consider the function $q : \mathbb{N} \to \mathbb{Q}$ defined by $q(t) = \frac{h^0(tA)-1}{t}$. Since $q(t) = \frac{A^n}{n!} t^{n-1} + O(t^{n-2})$ for $t \gg 0$ by asymptotic Riemann–Roch, there exists a positive integer d_0 satisfying the following properties:

(1) For all integers a satisfying $1 \leq a \leq d_0 - 1$, we have $q(a) < q(d_0)$.
(2) For all integers $d \geq d_0$, we have that dA is very ample and $q(d) < q(d+1)$.

An immediate consequence is that $q(a) < q(d)$ whenever $d \geq d_0$ and $1 \leq a \leq d-1$. For the rest of the proof, we fix an integer $d \geq d_0$.

Recalling that $\operatorname{Pic}(X) = \mathbf{Z} \cdot [A]$ by assumption, it suffices by Lemmas 2.1 and 2.2 to show that given integers a and $0 < r < h^0(dA) - 1$, one has the implication

$$(2.1) \qquad \mu_A(\Lambda^r M_d \otimes \mathcal{O}_X(aA)) \ \leq \ 0 \implies r \ \geq \ h^0(aA),$$

where as before $M_d = M_{dA}$. This is automatic for $a \leq 0$, so we assume $a \geq 1$ throughout. We have that

$$(2.2) \quad \mu_A(\Lambda^r M_d \otimes \mathcal{O}_X(aA)) \ = \ r \cdot \mu_A(M_d) + a \cdot (A^n) \ = \ (A^n) \cdot \left(a - \frac{dr}{h^0(dA) - 1} \right)$$

Our assumption that $\mu_A(\Lambda^r M_d \otimes \mathcal{O}_X(aA)) \leq 0$ then implies that $a \leq \frac{d \cdot r}{h^0(dA)-1}$, or

$$(2.3) \qquad r \ \geq \ a \cdot \left(\frac{h^0(dA) - 1}{d} \right).$$

[1]In fact, $h^0(L_d) - h^0(L_d - B) = O(d)$, whereas $h^0(L_d)$ grows quadratically in d.

In particular, $a < d$, so $1 \leq a \leq d-1$. We will be done once we verify that

$$a \cdot \left(\frac{h^0(dA) - 1}{d} \right) \;>\; h^0(aA) - 1, \tag{2.4}$$

for $1 \leq a \leq d-1$. However, (2.4) is equivalent to $q(a) < q(d)$, so this follows from our assumption on d. □

Remark 2.3 (**Rigidity of M_L**). Let L be a very ample line bundle on a smooth complex projective variety X of dimension ≥ 3 with $H^1(X, \mathcal{O}_X) = 0$. Then arguing as in the proof of [5, Proposition 1], one sees that M_L is rigid, i.e., $\text{Ext}^1(M_L, M_L) = 0$. Consequently, in the situation of Proposition C, M_d again represents an isolated point of the moduli space of bundles when $\dim_{\mathbf{C}} X \geq 3$ and $d \gg 0$.

2.2. Some open problems. Recall that if X is a smooth curve of genus $g \geq 1$, then M_L is stable as soon as $\deg L \geq 2g + 1$. This suggests

Problem 2.4. Find an effective version of Theorem A.

Presumably one would want to work with divisors of the sort $L = K_X + B + N$ with B satisfying a suitable positivity hypothesis, and N nef.

It is also interesting to ask whether M_d satisfies some stronger stability properties:

Problem 2.5. As before, let $L_d = dA + P$, and put $M_d = M_{L_d}$. Is M_d slope stable with respect to *any* polarization on X when $d \gg 0$? In characteristic $p > 0$, is it strongly stable?

Finally, we conjecture that our main result extends to all dimensions.

Conjecture 2.6. Let X be a smooth projective variety of dimension n, and define M_d as above. Then M_d is A-stable for every $d \gg 0$.

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