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Observational cosmology with semi-relativistic stars

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Galaxy mergers lead to the formation of massive black hole binaries which can accelerate background stars close to the speed of light. We estimate the comoving density of ejected stars with a peculiar velocity in excess of $0.1c$ or $0.5c$ to be $\sim 10^{10}$ and 10^{5} Gpc^{-3}, respectively, in the present-day Universe. Semi-relativistic giant stars will be detectable with forthcoming telescopes out to a distance of a few Mpc, where their proper motion, radial velocity, and age can be spectroscopically measured. In difference from traditional cosmological messengers, such as photons, neutrinos, or cosmic-rays, these stars shine and so their trajectories need not be directed at the observer for them to be detected. Tracing the stars to their parent galaxies as a function of speed and age will provide a novel test of the equivalence principle and the standard cosmological parameters. Semi-relativistic stars could also flag black hole binaries as gravitational wave sources for the future *eLISA* observatory.

1. Introduction

Mergers of galaxies produce binary systems of massive black holes embedded in a dense environment of stars [1]. Such binaries are capable of accelerating rare background stars to high speeds [2–5]. In this *Letter* we identify the conditions under which the accelerated stars reach a fraction of the speed of light, $\gtrsim 0.1c$, and calculate the cosmological implications of the resulting population of semi-relativistic hypervelocity stars (SHS) that fill the Universe.

2. Abundance of semi-relativistic stars

N-body simulations suggest that massive black hole binaries are excited to high eccentricities prior to merger [6–8]. Stars originally bound to the less massive black hole (secondary) can then be ejected at high speeds, in a manner similar to the production of hypervelocity stars from stellar binaries orbiting a single black hole [2]. The ejection speed can exceed the orbital speed of the stars [9], occasionally ejecting SHS near the speed of light. The

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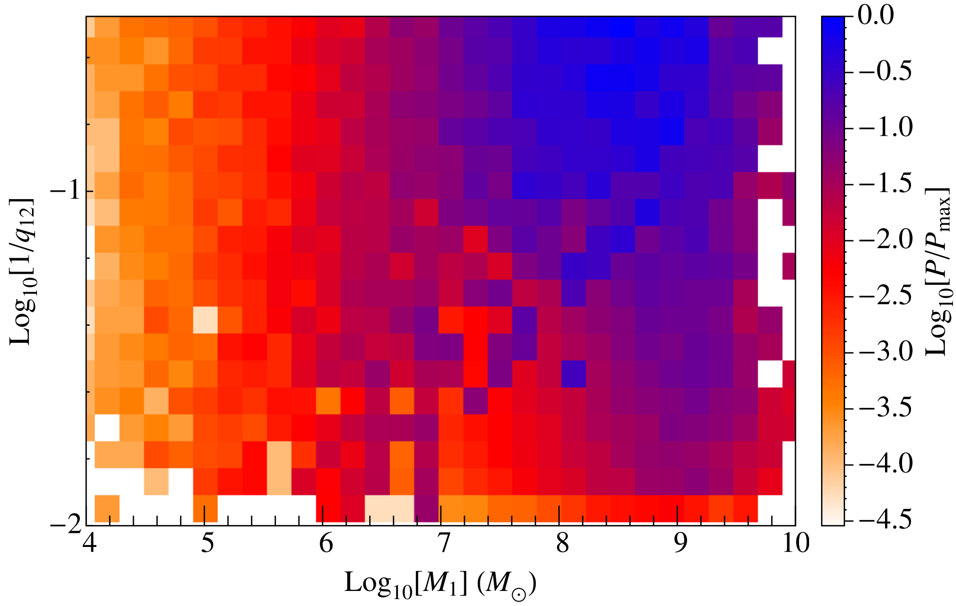

Figure 1: Fraction of SHS $P/P_{\max}$ that originate from binary black hole binaries of a given primary mass M_1 and primary-to-secondary ratio q_{12}, where $P_{\max}$ is the maximum probability amongst all mass combinations.

highest speeds are achieved when stars approach the secondary black hole as closely as possible (without being tidally disrupted) at the moment of its closest approach to the primary black hole. The likelihood of ejecting stars at different speeds depends on the galaxy merger rate, the mass ratio of the resulting binaries, and the distribution of stars around the two black holes during the mergers.

We have conducted numerical calculations of the statistics of SHS, whose details are reported in a companion paper [10]. Figure 1 shows the fraction of SHS originating from black hole binaries of a given primary mass M_1 and primary-to-secondary mass ratio q_{12}, and illustrates the fact that most of these stars originate from the mergers of the very largest black holes in the universe ($M_1 \sim 10^8$–$10^9 M_\odot$).

The resulting comoving density of SHS with different speeds in the present-day universe[1] is illustrated in Fig. 2, and shows that tens of tril-

[1]The statistics of stars very close to the speed of light requires an extension of our simplified Newtonian calculation to General Relativity, and should depend on the unknown distribution of black hole spins.

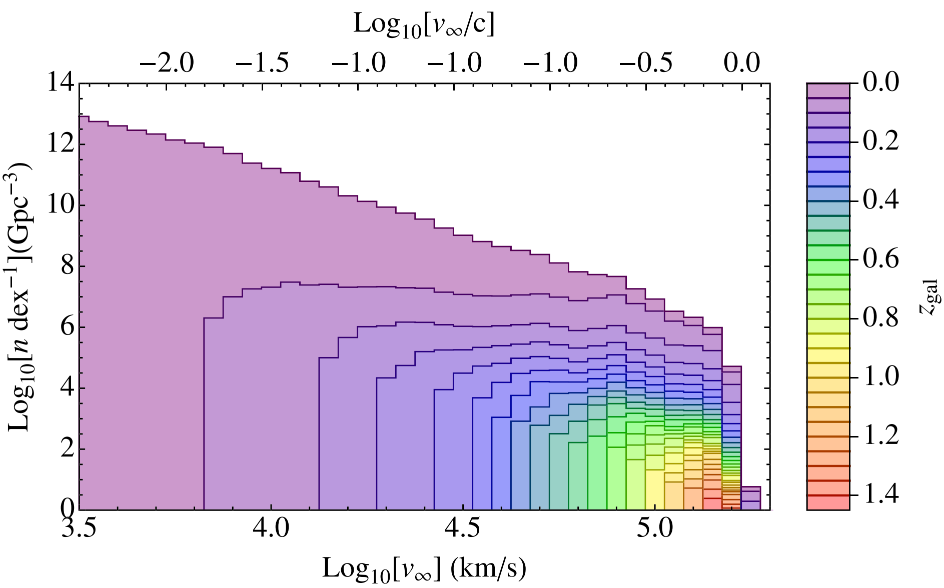

Figure 2: Number density of SHS n originally ejected at velocity v_∞, normalized per decade of $\log_{10} v_\infty$. The colored shading indicates the present-day redshift of the source galaxy z_{gal}, with higher velocity SHS typically originating from more distant galaxies.

lions of such stars lie within each cubic Gpc, many of which originate from galaxies that are at redshifts exceeding unity. In what follows, we focus on the implications of the existence of these stars for cosmological studies.

The intrinsic SHS flux per unit frequency F_ν would be observed to be boosted by a factor $D^{3-\alpha}$, where $D \equiv [\gamma(1-\beta\cos\theta)]^{-1}$ is the relativistic Doppler factor for a SHS propagating with a speed $\beta = (v/c) = \sqrt{1-\gamma^{-2}}$ at a angle θ relative to the line-of-sight, and $\alpha = d\ln F_\nu / d\ln\nu$ is the average spectral index in the observed frequency band [11]. For a blackbody stellar spectrum with an effective temperature T_{eff}, $\alpha(\nu) = [e^x(3-x)-3]/[e^x-1]$, where $x \equiv h\nu/kT_{\mathrm{eff}}$. In the Rayleigh-Jeans part of the spectrum ($x \ll 1$), $\alpha = 2$, whereas in the Wien tail ($x \gg 1$), $\alpha = 3 - x$. The apparent transverse speed [12] of a SHS, $\beta_{\perp,app} = (D\sin\theta)\gamma\beta$, peaks at a value of $\gamma\beta$ (which could exceed unity) for $\cos\theta = \beta$. Figure 3 shows the expected boost in the observed flux and proper motion of an M3III giant star as functions of the radial and tranverse components of its velocity. Different stellar spectra lead to different average values of α (depending on the observed frequency band) and hence different boost factors.

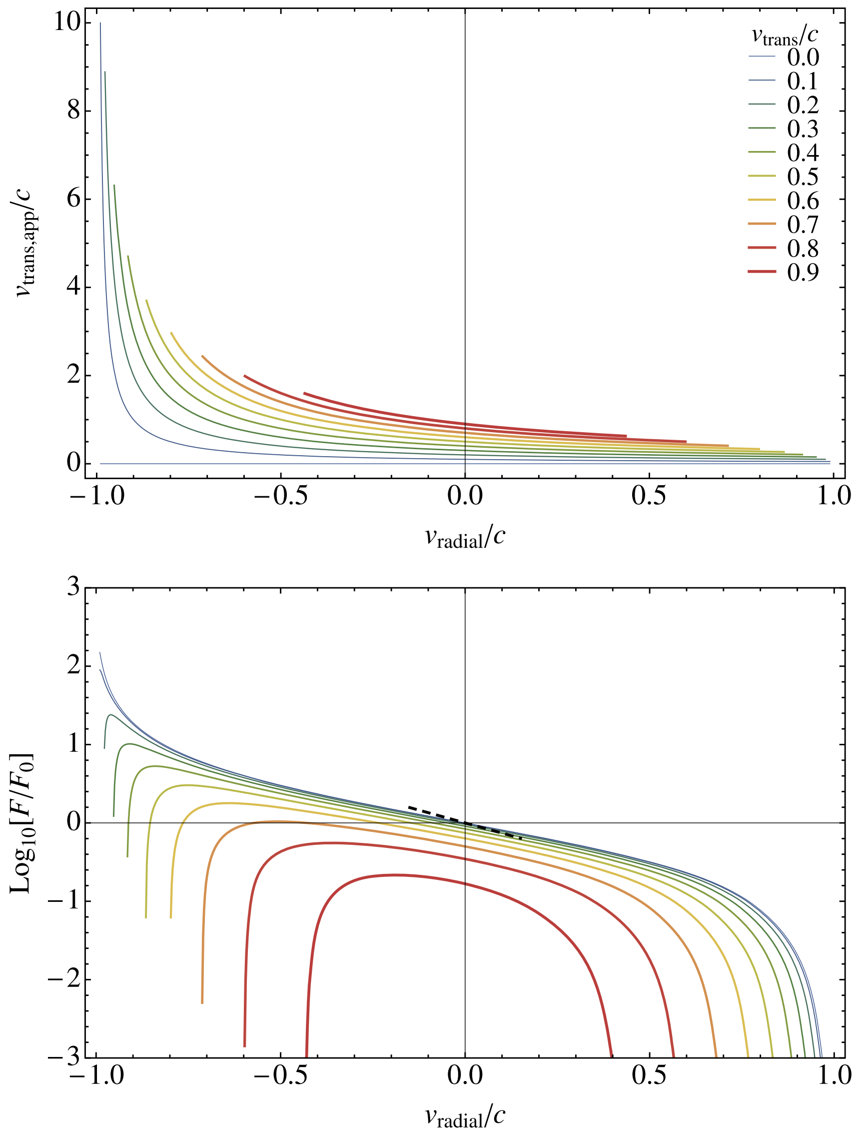

Figure 3: *Upper panel:* The apparent transverse speed of an SHS as a function of its actual transverse velocity $v_{\rm trans}$ and radial velocity $v_{\rm radial}$ (in units of c). *Lower panel:* Boost in total flux F compared to its value at zero velocity F_0 for an M3III giant SHS as a function of the radial ($v_{\rm radial}$) and transverse ($v_{\rm trans}$) components of its velocity. The dashed line segment shows the expected change in flux for a flat spectrum and purely radial motion when $v \ll c$.

3. Trajectory in a homogeneous Universe

Next, we consider the trajectory of a SHS that reached the vicinity of the Milky-Way galaxy at the present time t_0 with a measured speed v_0. The peculiar momentum per unit mass of any object in excess of the Hubble flow γv declines inversely with the scale-factor $a(t) = (1+z)^{-1}$ in an expanding universe (since the de Broglie wavelength is stretched like any other physical length scale). In the non-relativistic regime, this implies that the peculiar velocity of the star at earlier times $t < t_0$ was,

$$v = v_0/a(t). \tag{1}$$

On large spatial scales the Universe is filled with a nearly uniform distribution of SHS, with the velocity dispersion declining as a^{-2}, as expected for a collisionless gas of non-relativistic particles.

To leading order, we assume a homogeneous, isotropic and geometrically flat Universe described by the metric, $ds^2 = c^2dt^2 - a^2(t)(dr^2 + r^2 d\Omega)$, where r is the comoving (present-day) radius relative to the observer. For simplicity, we consider the regime where the source galaxy from where the star was ejected at time t_{ej} is much farther away ($r \sim$ Gpc) than the distance of the star from the observer ($\sim$ Mpc) which is itself much larger than the eventual distance of closest approach relative to the observer. In this regime, the trajectory of the star is nearly radial towards the observer, with a time-dependent velocity $v(t) = a(t)dr/dt$. By substituting this relation in Eq. (1), we get $dr = v_0 dt/a^2(t)$, and after integrating both sides of this equation we find the comoving distance of the source galaxy,

$$r = v_0 \int_{t_{ej}}^{t_0} \frac{dt}{a^2(t)}. \tag{2}$$

Note that SHS with different v_0 offer the opportunity to measure the distances of a source galaxy at multiple look-back times, in difference from photons which provide only a single snapshot due to their fixed propagation speed. The value of r as a function of the star's travel time $t_\star = (t_0 - t_{ej})$ is shown by the upper curve in Fig. 4 for the standard LCDM cosmology. The SHS-producing merger of galaxies often funnels gas to the galaxy centers and results in star formation there. Many of the newly formed stars are tightly bound to their black holes, and could have been slingshot ejected when they were much younger than their age today. In general, the measured age of each SHS only represents the maximum travel time that it could have had.

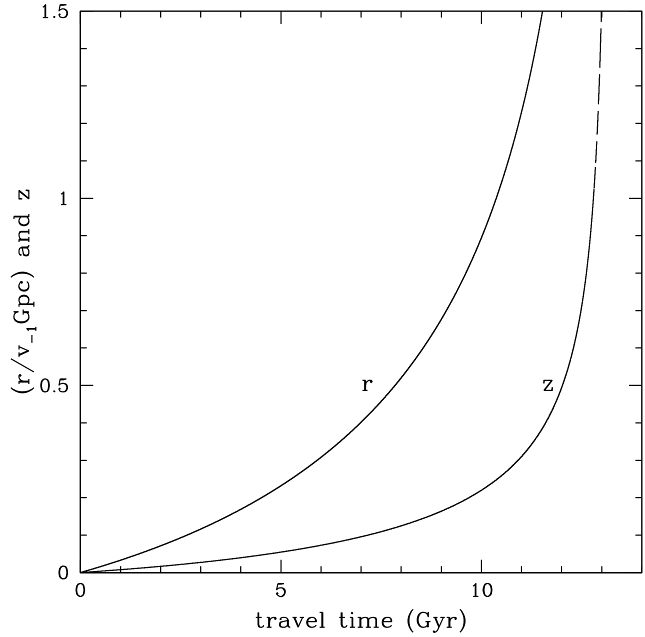

Figure 4: The upper curve shows the comoving distance r of the source galaxy in units of v_{-1}Gpc [where $v_{-1} \equiv (v_0/0.1c)$] as a function of the star's travel time $t_\star$ in Gyr for the standard LCDM cosmology [13] with $\Omega_m = 0.32$, $\Omega_\Lambda = 0.68$ and $H_0 = 67$ km s^{-1} Mpc^{-1}. The lower curve shows the corresponding source redshift in the particular case of $v_0 = 0.1c$; lower v_0 would translate to lower redshifts. At short distances, $z \propto r \propto v_0$.

But among all the stars that move at the same speed and have the same age, those that traverse the largest distance are most likely to have been ejected at youth. As long as multiple such stars provide consistent cosmological constraints, their use as rulers would be reliable.

In difference from traditional cosmological observations in which $v \approx c$ messengers (such as photons, neutrinos, or cosmic rays) must intersect the collecting area of a telescope in order to be detected, SHS can be detected at a distance. While photons must follow radial trajectories from the source to the observer, SHS emit their own light and therefore can be detected even if they miss the observer with a finite impact parameter or pass beyond the observer's location. This implies that SHS could possess substantial proper motion near their point of closest approach. On average, one expects equal numbers of redshifted and blueshifted SHS that move at a fraction of c; the

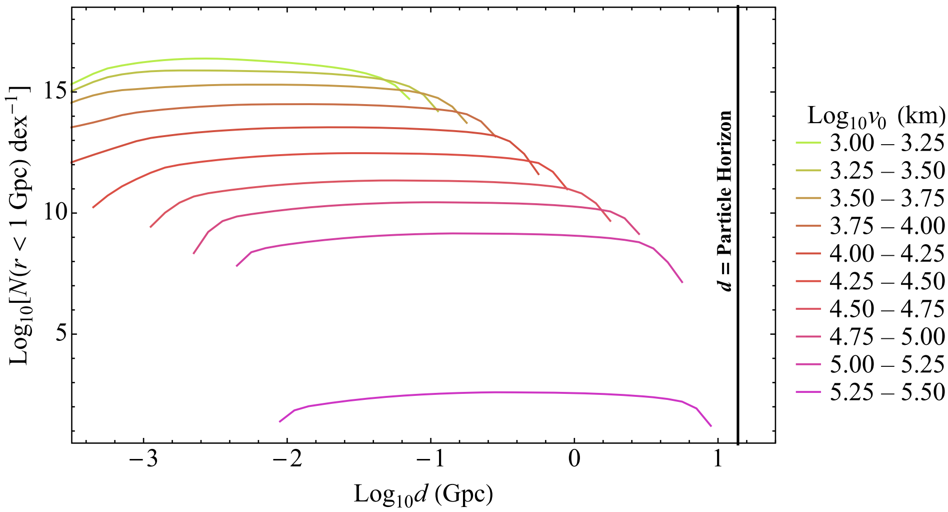

Figure 5: Histograms of distances traveled d by SHS observed within 1 Gpc as a function of their present-day speed v_0.

former class involving stars that have already passed through (or were just ejected out of) the observer's region. Figure 5 shows the distances traveled by an SHS observed within 1 Gpc, as a function of its observed speed.

4. Gravitational deflection

A non-relativistic star passing within an impact parameter b from a projected (cylindrically-integrated) concentration of mass $M(b)$ along its path, would be deflected by an angle [14],

$$\theta = \frac{2GM(b)}{v^2 b} = \frac{2V_c^2}{v^2} = 27.5'' \left[\frac{(V_c/200 \text{ km s}^{-1})}{(v/0.1c)}\right]^2, \tag{3}$$

where $V_c \equiv \sqrt{GM(b)/b}$ [2]. The deflection would also lead to a slight delay in arrival time relative to the relation between r and $t_\star$ in Fig. 4. If the parent galaxy is identified, measuring the deflection angle of SHS as a function of speed can be used to determine the projected mass profile of the deflector, be it a galaxy or a cluster of galaxies.

[2]Note that the deflection angle of a non-relativistic particle deviates from that of a photon by a factor of 2 aside from the substitution of v for c.

The angular cross-section A for deflecting a star ejected from a source galaxy behind a singular isothermal sphere with a 1D velocity dispersion σ, can be derived in analogy with gravitational lensing of photons [15],

$$A = 4\pi^3 \left(\frac{\sigma}{v}\right)^4 \left(1 - \frac{r_d}{r_s}\right)^2, \tag{4}$$

where the subscripts d and s label the deflector and source, respectively. This cross-section is larger by a factor of $\frac{1}{4}(c/v)^4$ than the Einstein cross-section for gravitationally-lensed photons [16]. At low v, the maximum value of A is limited by the finite virial radius of the deflector, R_{vir}. In the following, we consider a sufficiently large v such that $b < R_{vir}$. For short travel times $t_\star \ll H_0^{-1} = 14$ Gyr, the optical depth for deflection τ is obtained by integrating the local density of deflectors times A over the path length, giving

$$\tau = 4\pi^3 \Gamma n_\star \left(\frac{\sigma_\star}{v_0}\right)^4 r_s^3 \approx 0.2 \left(\frac{v_0}{0.1c}\right)^{-4} \left(\frac{r_s}{\mathrm{Gpc}}\right)^3, \tag{5}$$

where for the local galaxy population we have adopted the Faber-Jackson relation $L/L_\star = (\sigma/\sigma_\star)^\gamma$ and a Schechter luminosity function to describe the differential number density of galaxies per luminosity L bin, $dn/dL = (n_\star/L_\star)(L/L_\star)^\alpha \exp\{-L/L_\star\}$, with the parameters $n_\star = 5\times10^{-3}$ Mpc^{-3}, $\sigma_\star = 130$ km s^{-1}, $\alpha = -1.05$, $\gamma = 4.1$ [17, 18], and $\Gamma \equiv \Gamma(1+\alpha+4/\gamma)$. The likelihood for SHS deflection by galaxies and large-scale structure [19] exceeds unity at low velocities, $v_0 \lesssim 0.07(r_s/\mathrm{Gpc})^{3/4}c$. For long travel times, it is necessary to incorporate Eq. (1) into the deflection cross-section in calculating the optical depth.

If the star was ejected when it was much younger than its present age, then one could infer $t_{ej} = t_0 - t_\star$ from a spectroscopic measurement of the star's age. Forthcoming surveys, such as GAIA[3] or LSST[4], and future observatories, such as JWST[5] or large-aperture ground-based telescopes[6,7,8], could detect SHS during the giant phase of their evolution out to distances of order a few Mpc [10]. Follow-up spectroscopy could measure the radial velocity and age of SHS, which when combined with proper motion, can be used to relate SHS to their parent galaxies.

[3] http://sci.esa.int/gaia/
[4] http://www.lsst.org
[5] http://www.jwst.nasa.gov
[6] http://www.eso.org/sci/facilities/eelt
[7] http://www.tmt.org
[8] http://www.gmto.org

5. Identifying the parent galaxy

If deflections along the star's trajectory can be corrected for, and the source galaxy can be identified from a spectroscopic determination of the age of the star ($\gtrsim t_\star$), its spectroscopically-measured radial velocity (v_0) and its observed proper motion, then one might be able to uniquely identify the source galaxy and measure its redshift, z. The cosmological evolution of the scale factor, $a(t) = (1+z)^{-1}$, depends on cosmological parameters through the Friedmann equation. In principle, if the measurement accuracy is sufficiently high, it may be possible to constrain cosmological parameters (such as the equation of state of dark energy) by requiring that the source redshift (as measured by detecting photons at t_0) would agree with the comoving distance traveled by a star with a present-day velocity v_0 over its travel time $t_\star$ (cf. Eq. 2). The photon redshift $z = a^{-1}(t_\gamma) - 1$ is obtained from the equation,

$$r = c\int_{t_\gamma}^{t_0} \frac{dt}{a(t)} = \int_{1/(1+z)}^{1} \frac{da}{a^2 H}, \tag{6}$$

where $H = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}$ is the Hubble parameter for the redshifts of interest, with Ω_m and Ω_Λ being the present-day cosmological parameters of the matter and vacuum. By comparing Eqs. (2) and (6), it is clear that the photons must have been emitted after the SHS left the galaxy, in order for them to arrive to the observer at the same time. The lower curve in Fig. 4 shows the source redshift as a function the star's travel time $t_\star$ for the particular case of $v_0 = 0.1c$. This curve (as a function of v_0) provides a new variant of the conventional "Hubble Diagram" which traditionally uses photons as messengers. Comparing SHS with photons from the same source galaxy would constitute a novel test of the equivalence principle and the standard model of cosmology.

6. Intergalactic light

In addition to individual SHS passing relatively close to the observer, the combined effect of all SHS in the observable universe can contribute a significant fraction of the light in the voids between galaxy clusters. Most void light originates from within a few hundred kpc of the galaxies occupying those voids [20], and thus away from these void galaxies SHS are likely to be a signifcant source of light. We calculate that the intergalactic interhalo light from all SHS in the Universe (which constitute $\sim 10^{-5}$ of all stars at $z = 0$) is $\sim 10^{-3}$ nW m^{-2} sr^{-1} at a wavelength of 1μm. This is comparable to the minimum value expected in some regions of the sky [21].

7. Broader implications

The existence of SHS has multi-disciplinary implications. In the context of astro-biology, SHS could spread life beyond the boundaries of their host galaxies [22, 23]. In the context of gravitational-wave astrophysics, the abundance of SHS can be used to calibrate the coalescence rate of tight black holes binaries, especially the more-massive examples which produce the majority of SHS (see Figure 1), and for which some examples ($M_1 \sim 10^8 M_\odot$) have been discovered recently [24, 25]. If the parent galaxies are identified, SHS could flag additional binary black holes systems that would be strong gravitational wave sources for the future *e-LISA* observatory[9].

Acknowledgments

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