

Penrose Limits of RG Fixed Points and PP-Waves with Background Fluxes

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Abstract

We consider a family of pp-wave solutions of IIB supergravity. This family has a non-trivial, constant 5-form flux, and non-trivial, (light-cone) time-dependent RR and NS-NS 3-form fluxes. The solutions have either 16 or 20 supersymmetries depending upon the time dependence. One member of this family of solutions is the Penrose limit of the solution obtained by Pilch and Warner as the dual of a Leigh-Strassler fixed point. The family of solutions also provides indirect evidence in support of a recent conjecture concerning a large N duality group that acts on RG flows of $\mathcal{N} = 2$ supersymmetric, quiver gauge theories.

1 Introduction

The remarkable observation in [2] that string theory is solvable in certain pp-wave backgrounds has opened up new opportunities to test and analyze string theory [1]. Combined with the observation that pp-waves are Penrose limits of $AdS_m \times S^n$ backgrounds, this has enabled extensive new testing to the AdS/CFT correspondence. Our purpose in this paper is to examine these issues for a supergravity solution that has already provided a sharp set of tests of the AdS/CFT correspondence: the $\mathcal{N} = 2$ supersymmetric supergravity solution [15, 14, 13] that corresponds to a non-trivial, $\mathcal{N} = 1$ supersymmetric, conformal fixed point [17] of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. This supergravity solution involves non-trivial fluxes for the 3-forms $H_{(3)}$, $F_{(3)}^{RR}$ and for the 5-form, $F_{(5)}^{RR}$. All of these fluxes remain non-zero in the Penrose limit, and indeed $H_{(3)}$ and $F_{(3)}^{RR}$ are non-constant. The solution of [15, 14, 13] has $\frac{1}{4}$ -supersymmetry, and we find that the corresponding pp-wave has 20 supersymmetries, which may be understood as the “universal” 16 plus one quarter of the supernumeraries. We also find that, in spite of a “light-cone time-dependence,” the string theory is still solvable in this background, and we compute the modes.

The Penrose limit of the solution [15, 14, 13] is, in fact, one point in a family of pp-wave solutions: one can make more general Ansätze for the fluxes by introducing arbitrary constant parameters, and one can even introduce a non-constant dilaton and axion. The result is a large, multi-parameter space of solutions that involves: a constant, symmetric, 8×8 , real matrix in the metric; a constant, complex, skew, 4×4 matrix in the 2-form fields, $B^{NS} + iB^{RR}$; an arbitrary complex constant, a , for the dilaton/axion; an arbitrary real constant f for $F_{(5)}^{RR}$; and an arbitrary real parameter, β , that determines the time dependence of the background fields. The only non-trivial equation is the R_{++} Einstein equation, and it yields one real constraint. In this paper we will focus for simplicity on the family of solutions that have trivial dilaton and axion. We will also specify a very particular form for the 2-form tensor gauge fields. This form is motivated by the Penrose limit of the solution [15, 14, 13], but we will retain an arbitrary complex constant, b , as the coefficient. The Einstein equations then reduce to $\frac{1}{4}\beta^2|b|^2 + f^2 = 1$, and the solution space is thus parametrized by $S^2 \times \mathbb{R}$. We find that these solutions have 20 supersymmetries if and only if the time-dependence parameter is fixed to $\beta = -2f$, and otherwise the solution only has 16 supersymmetries.

The fact that the background 3-form fields depend explicitly upon x^+ means that the induced string action, in light-cone gauge, depends explicitly upon (world-sheet) time. This time dependence naively suggests that

“energy” should not be conserved, however, as one sometimes finds in AdS backgrounds, it is natural to mix energy with angular momentum to arrive at a more natural conserved “energy.” For the backgrounds that we consider here we find that the apparent time dependence of the 3-form fields can be removed by shifting an angular coordinate, $\varphi \rightarrow \varphi + k x^+$, for some constant k , to go to a co-rotating frame. This induces some off-diagonal metric terms, but the metric remains stationary and the background fields become independent of x^+ . For the Penrose limit of the background in [13] one has $k = 3$ and the natural conserved energy is then:

$$\Delta = \frac{3}{2} \mathcal{J}, \quad (1)$$

where Δ is the canonical AdS energy dual to the conformal dimension of the operators on the brane, and \mathcal{J} is the R -charge of the $\mathcal{N} = 1$ field theory on the brane. The combination, (1), is, of course, the “topological” energy that vanishes on chiral, primary operators.

The family of theories described in this paper also provides some support for a conjecture made in [9]. The goal of [9] was to find the five-dimensional, gauged supergravity description of a class of holographic RG flows within $\mathcal{N} = 2$ quiver gauge theories. For the A_p quiver theory, this class of flows involves giving a mass to the chiral multiplets associated to the nodes of the quiver, and so there are $(p+1)$ (complex) mass parameters. The corresponding five-dimensional gauged supergravity theory has an $SU(p+1)$ symmetry acting on the mass parameters, and so all of the flows are equivalent under this symmetry. At the non-trivial RG fixed point, an overall scale and phase of these mass parameters disappears, leaving a fixed surface isomorphic to \mathbb{CP}^p . The symmetry and the fixed-point manifold is manifest in the five-dimensional supergravity, but is very surprising from the ten-dimensional perspective since it involves trading topologically trivial B -field fluxes for Kähler moduli of blow-ups of singularities. In particular, there should be an $SU(2)$ symmetry acting on an S^2 ’s worth of solutions that smoothly interpolate between the flow of [22] and the flow of [14], and even more simply, there should be an $SU(2)$ symmetry acting on an S^2 ’s worth of solutions that smoothly interpolate between the IIB solutions corresponding to the RG fixed points, that is between the $T^{(1,1)}$ solution of [21] and the solution of [13].

While there have been some attempts to construct this $SU(2)$ family of solutions directly, it is technically rather difficult [11]. Thus, a slightly more modest, but manageable, test would be to see if the Penrose limits of these solutions have the requisite $SU(2)$ symmetry. The Penrose limit of the $T^{(1,1)}$ compactification was obtained and analyzed in [18, 19, 20], and

it was found that the limit was the same as that of the maximally supersymmetric background of $AdS_5 \times S^5$, that is, the maximally supersymmetric pp-wave of IIB supergravity [2, 3]. Thus the Penrose limit seems to remove all of the supersymmetry-breaking effects. This does not happen with the Penrose limit of the solution of [13]: as described above, the Penrose limit has 20 supersymmetries, and is still in a sense, $\frac{1}{4}$ -supersymmetric. Thus, in the Penrose limit, the $SU(2)$ symmetry and the S^2 conjectured in [9] must smoothly interpolate between the Penrose limit of the solution of [13] and the maximally supersymmetric pp-wave of IIB supergravity [2, 3]. At a generic point on the S^2 there must be 20 supersymmetries, and this will increase to 32 when the background 3-form field vanishes. This is, indeed, exactly what we find in this paper. Moreover, it was also argued in [9] that the $SU(p+1)$ symmetry should only be a large N symmetry, and would be broken to a discrete symmetry at finite N . This means that it should be a symmetry of the supergravity, but not of the string spectrum. Again, this is confirmed by the results presented here.

Quite apart from the motivations of holographic field theory, the results presented here are interesting in that they are backgrounds with non-trivial, non-constant fluxes in which the supersymmetry is *partially* broken, and yet the string theory is still solvable. There have been quite a number of pp-wave solutions in which the string theory is solvable, and the supersymmetry is broken using the metric, or by using Penrose limits of intersecting branes (see, for example, [4, 5, 23]). In this paper the background B -field fluxes are intrinsically “dielectric”: they are not generated by a simple set of pure BPS branes. When combined with other families of solutions, the class presented here should generate a new, richer class of interesting, supersymmetric pp-wave solutions.

In section 2 we will introduce a general class of pp-wave solutions to IIB supergravity, and then examine the Penrose limit of the results of [13]. In section 3 we will discuss the supersymmetry of the $S^2 \times \mathbb{R}$ family of solutions described earlier. Section 4 contains an analysis of the modes of the Green-Schwarz string in an $S^2 \times \mathbb{R}$ family of pp-waves, and section 5 contains some final remarks. Throughout this paper our conventions will be those of [8], but with one minor exception: the complex dilaton/axion field called B in [8] will be re-labeled $D_{(0)}$.

Note added in proof

Since this paper was submitted to arXiv.org, two closely related papers [26, 27] appeared. Both of these papers discuss aspects of the dual field theory, such as RG flows and the operator spectrum, and they also take Penrose limits using other geodesics.

2 A class of pp-Wave solutions

2.1 The general class

Consider a general pp-wave with an $\mathbb{R}_t^4 \oplus \mathbb{R}_y^4$ split, null 5-form, complex 2-form potential, and a non-trivial dilaton and axion:

$$\begin{aligned} ds^2 &= 2 dx^- dx^+ + \sum_{i < j} \left(A_{ij}^r r_i r_j + A_{ij}^y y_i y_j + 2 A_{ij}^m r_i y_j \right) (dx^+)^2 \\ &\quad - d\vec{r}^2 - d\vec{y}^2, \\ F &= f dx^+ \wedge (dr_1 \wedge \cdots \wedge dr_4 + dy_1 \wedge \cdots \wedge dy_4), \\ \mathcal{B} &= \frac{1}{2} e^{i\beta x^+} \mathcal{C}^{jk} dy_j \wedge dy_k, \\ D_{(0)} &= a e^{2i\beta x^+}. \end{aligned} \tag{2}$$

Following [8], we are using a metric that is “mostly minus.” The tensor, $\mathcal{C}_{jk} = \mathcal{C}^{jk}$, is a constant, complex and skew symmetric (we could, of course, extend \mathcal{C} into the other transverse directions, but we are primarily interested in configurations that are Penrose limits of IIB solutions that have vanishing \mathcal{B} -components in the AdS_5 directions.) The parameters f and β are real constants, while a is a complex constant.

Owing to the x^- -independence of the metric and the fact that $g^{++} = 0$, the only non-trivial IIB equation of motion [8] is a component of the Einstein equation:

$$\begin{aligned} R_{++} &= \text{Tr} (A^r + A^y) \\ &= 8f^2 + (1 - |a|^2)^{-1} (8\beta^2 |a|^2 \\ &\quad + \frac{1}{4} (\mathcal{C}_{jk} - a \mathcal{C}_{jk}^*) (\mathcal{C}^{*jk} - a^* \mathcal{C}^{jk})). \end{aligned} \tag{3}$$

For simplicity, we take a trivial dilaton/axion background, and so set $a = 0$. For the string theory to be solvable in the background one needs to further restrict the matrix \mathcal{C}_{jk} . The choice that we will make here¹ is to take:

$$\mathcal{B} = b e^{i\beta x^+} d\zeta_1 \wedge d\zeta_2, \tag{4}$$

with $\zeta_1 = y_1 + iy_2, \zeta_2 = y_3 + iy_4$. We will also take the metric to be the most symmetric possibility, with:

$$A_{ij}^r = A_{ij}^y = \delta_{ij}, \quad A_{ij}^m = 0. \tag{5}$$

¹There are more general possible choices that lead to a solvable string spectrum.

The equations of motion then reduce to:

$$f^2 + \frac{1}{4}\beta^2|b|^2 = 1. \quad (6)$$

The pp-wave limit with maximal supersymmetry satisfies (6) with $f = \pm 1, b = 0$. Recall that this is the pp-wave limit of both $AdS_5 \times S^5$ and $AdS_5 \times T^{1,1}$. The solution we present below as the Penrose limit of the gravity dual of the Leigh-Strassler fixed point is also of the simplified form (5), satisfying (6), but with:

$$f = -\frac{1}{2}, \quad b = i\sqrt{3}, \quad \beta = 1.$$

The Ansatz (2), (4) and (5) with parameters satisfying (6), defines a 2-sphere of solutions that interpolate between the solution of defined below and the point of maximal supersymmetry. Thus this interpolating family is related to the Penrose limit of the family of solutions which interpolate between the orbifold of the solution of [13] and conifold fixed points in the supergravity duals of $\mathcal{N} = 2$ quiver gauge theories.

2.2 The $\mathcal{N} = 2$ supersymmetric AdS solution

Here we summarize the essential features of the $\mathcal{N} = 2$ supersymmetric supergravity dual of the $\mathcal{N} = 1$ “Leigh-Strassler” field theory fixed point in four dimensions. The five-dimensional gauged supergravity solution was found in [15], and this was subsequently “lifted” to ten-dimensional, IIB supergravity in [13]. The ten-dimensional metric takes the form:

$$ds_{10}^2 = \Omega^2 ds_{AdS_5}^2 - ds_5^2. \quad (7)$$

For the AdS_5 directions, we take

$$ds_{AdS_5}^2 = L^2 (\cosh^2 \rho dt^2 - d\rho^2 - \sinh^2 \rho d\Omega_3^2), \quad (8)$$

where $d\Omega_3^2$ is the metric on the unit 3-sphere. The parameter, L , is the “radius” of the AdS_5 . The internal metric is:

$$\begin{aligned} ds_5^2 = & \frac{\sqrt{3}}{8} a^2 (3 - \cos(2\theta))^{1/2} \left(d\theta^2 + \frac{\cos^2 \theta}{3 - \cos(2\theta)} ((\sigma_1)^2 + (\sigma_2)^2) \right. \\ & \left. + \frac{\sin^2(2\theta)}{(3 - \cos(2\theta))^2} (\sigma_3)^2 \right) \\ & + \frac{\sqrt{3}}{12} a^2 (3 - \cos(2\theta))^{1/2} \left(d\phi + \frac{2 \cos^2 \theta}{3 - \cos(2\theta)} \sigma_3 \right)^2, \end{aligned} \quad (9)$$

where the scale, a , is given by:

$$a = \frac{2^{13/6}}{3} L,$$

and the warp factor Ω in (7) is

$$\Omega^2 = 2^{1/3} \left(1 - \frac{1}{3} \cos(2\theta) \right)^{1/2}. \quad (10)$$

This solution also has a 5-form flux:

$$F = -\frac{2^{5/3}}{3L\Omega^5} (e^1 \wedge \dots \wedge e^5 + e^6 \wedge \dots \wedge e^{10}), \quad (11)$$

and a 3-form flux which can be obtained from the complex 2-form potential

$$\mathcal{B} = -\frac{i}{\sqrt{3}} e^{-2i\phi} (e^6 + i e^9) \wedge (e^7 - i e^8). \quad (12)$$

In equation (11), we have included a factor of 2 to correct a typo in [13]².

It will be useful later to recall some further details of [13]. First, one can parametrize \mathbb{R}^4 using an $SU(2)$ transformation based upon Euler angles, φ_j :

$$\begin{aligned} \zeta_1 &= y_1 + i y_2 = y \cos\left(\frac{1}{2} \varphi_1\right) e^{-\frac{i}{2}(\varphi_3 + \varphi_2)}, \\ \zeta_2 &= y_3 + i y_4 = y \sin\left(\frac{1}{2} \varphi_1\right) e^{-\frac{i}{2}(\varphi_3 - \varphi_2)}. \end{aligned} \quad (13)$$

The left-invariant one-forms, σ_j , can then be written explicitly as:

$$\sigma_1 \pm i \sigma_2 = e^{\pm i \varphi_3} (d\varphi_1 \mp i \sin(\varphi_1) d\varphi_2), \quad \sigma_3 = d\varphi_3 + \cos(\varphi_1) d\varphi_2. \quad (14)$$

In [13] the S^5 was parametrized by taking:

$$u_1 = \zeta_1 e^{-i\phi/2}, \quad u_2 = \zeta_2 e^{-i\phi/2}, \quad u_3 = \sin(\theta) e^{-i\phi}. \quad (15)$$

with $y = \cos(\theta)$ so that $|u_1|^2 + |u_2|^2 + |u_3|^2 = 1$. Since the metric and the tensor gauge fields are constructed using the σ_j there is a manifest $SU(2)$ symmetry. The metric also has a $U(1) \times U(1)$ symmetry under φ_3 and ϕ translations, but the B -field (12) breaks this to

$$\phi \rightarrow \phi - \alpha, \quad \varphi_3 \rightarrow \varphi_3 + 2\alpha, \quad (16)$$

since $(e^7 - i e^8) \sim (\sigma_1 - i \sigma_2) \sim e^{-i \varphi_3}$. The residual ($\mathcal{N} = 1$) supersymmetry generators on the brane transform under this as $\epsilon_{\pm} \rightarrow e^{\pm i \alpha} \epsilon_{\pm}$, and

²The correct coefficient can be obtained from subsequent papers: [12, 10].

so (16) represents the canonically normalized \mathcal{R} -symmetry of the $\mathcal{N} = 1$ superconformal theory on the brane.

It is simple to modify the foregoing solution in order to describe an analogous $\mathcal{N} = 1$ fixed point in $\mathcal{N} = 2$ A_{n-1} quiver gauge theory. The UV dual is IIB on $\text{AdS}_5 \times S^5/\mathbb{Z}_n$, where the orbifold action identifies the Euler angle $\varphi_3 \sim \varphi_3 + 4\pi/n$. The $\mathcal{N} = 1$ orbifold IR fixed point is described by the above solution together with this identification on φ_3 .

2.3 The Penrose limit

After substituting all the factors into (9) one finds terms of the form:

$$L^2 \Omega^2 \left(\cosh \rho \, dt^2 - \frac{4}{9} \left(d\phi + \frac{2 \cos^2 \theta}{3 - \cos(2\theta)} \sigma_3 \right)^2 + \dots \right). \quad (17)$$

We thus consider geodesics that lie near $\rho = 0$, $\theta = \pi/2$, and have $x^- \sim L^2(t + \frac{2}{3}\phi)$. It is convenient to introduce some additional constants to clean up the result, and so we make the coordinate redefinitions:

$$t = x^+, \quad \rho = \frac{3^{1/4}}{2^{2/3}} \frac{r}{L}, \quad \theta = \frac{\pi}{2} - \frac{3^{3/4}}{2^{7/6}} \frac{y}{L}, \quad \phi = \frac{3}{2} \left(\frac{\sqrt{3}}{2^{4/3}} \frac{x^-}{L^2} - x^+ \right), \quad (18)$$

In the limit $L \rightarrow \infty$ we find the metric:

$$ds^2 = 2 \, dx^- \, dx^+ + (r^2 + y^2)(dx^+)^2 - d\vec{r}^2 - ds_4^2, \quad (19)$$

where

$$\begin{aligned} d\vec{r}^2 &= dr^2 + \frac{1}{4} r^2 d\Omega_3^2, \\ ds_4^2 &= dy^2 + \frac{1}{4} y^2 ((\sigma_1)^2 + (\sigma_2)^2 + (\sigma_3 - 2 \, dx^+)^2). \end{aligned} \quad (20)$$

The same limit of the 5-form yields:

$$F = -\frac{1}{2} dx^+ \wedge (dr_1 \wedge \dots \wedge dr_4 + dy_1 \wedge \dots \wedge dy_4). \quad (21)$$

while the complex 2-form potential (12) becomes

$$\mathcal{B} = \frac{i}{2\sqrt{3}} e^{3ix^+} y (dy - \frac{i}{2} y \sigma_3) \wedge (\sigma_1 - i \sigma_2) = -\frac{i}{\sqrt{3}} e^{3ix^+} d\zeta_1 \wedge d\zeta_2. \quad (22)$$

There are now two natural changes of variable: (i) $\varphi_3 \rightarrow \varphi_3 + 2x^+$, and (ii) $\varphi_3 \rightarrow \varphi_3 + 3x^+$. The former removes the cross-term in the metric

(20), while the latter removes the x^+ dependence in \mathcal{B} . This removal of the x^+ -dependence is an important step in computing the normal modes of the string, and so we will return to it in section 4.

To get to the Ansatz (4) one first makes the shift: $\varphi_3 \rightarrow \varphi_3 + 2x^+$. This generates an extra term in \mathcal{B} that is proportional to $dx^+ \wedge (\sigma_1 - i\sigma_2)$ and which may be removed by adding a pure gauge term, $d\mathcal{A}$, where:

$$\mathcal{A} = \frac{i}{\sqrt{3}} e^{ix^+} y^2 (\sigma_1 - i\sigma_2). \quad (23)$$

The final result is:

$$\begin{aligned} ds_4^2 &= d\vec{y}^2 = dy^2 + \frac{1}{4} y^2 ((\sigma_1)^2 + (\sigma_2)^2 + (\sigma_3)^2), \\ \mathcal{B} &= i\sqrt{3} e^{ix^+} d\zeta_1 \wedge d\zeta_2. \end{aligned} \quad (24)$$

Thus the pp-wave metric is the simplest possible form: on the transverse coordinates $\mathbb{R}_r^4 \oplus \mathbb{R}_y^4$ the symmetric matrix, A , in (2) is the identity matrix. It fits the Ansatz above with $\beta = 1, f = -\frac{1}{2}$ and $b = i\sqrt{3}$. Note that $\beta = -2f$.

Finally, note that for the orbifold, S^5/Γ , the φ_3 coordinate of \mathbb{R}_y^4 is still identified under the orbifold action. So after taking the scaling limit of $\text{AdS}_5 \times S^5/\Gamma$, the space of the \vec{y} is \mathbb{R}_y^4/Γ , for the choice of geodesic in (18).

3 Supersymmetry and Killing Spinors

For vanishing dilaton and axion, the supersymmetry conditions are

$$\begin{aligned} D_M \epsilon + \frac{i}{480} F_{PQRST} \gamma^{PQRST} \gamma_M \epsilon + \frac{1}{96} G_{PQR} (\gamma_M{}^{PQR} \\ - 9 \delta_M{}^P \gamma^{QR}) \epsilon^* = 0, \end{aligned} \quad (25)$$

and

$$\gamma^{MNP} G_{MNP} \epsilon = 0. \quad (26)$$

3.1 Integrability conditions

The easiest way to count the number of supersymmetries is to compute the integrability, or zero-curvature condition for the differential operator in (25). The result of doing this is an algebraic condition of the form:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \epsilon \\ \epsilon^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (27)$$

We consider a metric of the form given in (2) and (5), and we take F to be as in (2), with \mathcal{B} given by (4). We recall that the only non-trivial field equation is (6). For this background it is straightforward to solve (27) and (26).

Introduce frames e^A for the metric in (2) with $e^j = dr_j$, $e^{j+4} = dy_j$, $j = 1, \dots, 4$, and with e^0 and e^9 as the remaining time-like and space-like components. Define $\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^9)$. As is by now familiar, there are always 16 solutions to these conditions given by the zeroes of:

$$\gamma^+ \epsilon = \gamma^+ \epsilon^* = 0. \quad (28)$$

One finds four more non-trivial solutions to (26) and (27) *if and only if* $\beta = -2f$. The solutions are present for all values of b and f satisfying (6), and the solutions are precisely those that solve the projection conditions:

$$\gamma^+(\gamma^5 + i\gamma^6)\epsilon = \gamma^+(\gamma^7 + i\gamma^8)\epsilon = 0. \quad (29)$$

The integrability conditions are necessary conditions for supersymmetry, and are usually, but not always, sufficient. To arrive at a sufficient condition one must check higher order integrability using further commutators of the supercovariant derivative [16]. For the background we are considering here, it turns out that these extra integrability conditions are trivially satisfied. Alternatively, one can see that there must be at least four additional supersymmetries by boosting the solutions of [13].

4 The modes of the Green-Schwarz string

It is convenient to introduce a scale μ by rescaling $x^- \rightarrow x^-/\mu$, $x^+ \rightarrow \mu x^+$. This changes the metric to:

$$ds^2 = 2 dx^- dx^+ + \mu^2(r^2 + y^2)(dx^+)^2 - d\vec{r}^2 - d\vec{y}^2, \quad (30)$$

and it also introduces factors of μ into F and \mathcal{B} .

4.1 The bosonic modes

Other than the metric, the only background field that couples to the bosonic string is the NS-NS two-form, which is the real part of \mathcal{B} . After the rescaling, this is given by

$$B = \frac{1}{2} \left(b e^{i\beta\mu x^+} d\zeta_1 \wedge d\zeta_2 + \bar{b} e^{-i\beta\mu x^+} d\bar{\zeta}_1 \wedge d\bar{\zeta}_2 \right). \quad (31)$$

The bosonic part of the Green-Schwarz Lagrangian is:

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \partial_\alpha x^\mu \partial^\alpha x^\nu + B_{\mu\nu} \partial_\tau x^\mu \partial_\sigma x^\nu, \quad (32)$$

In the light-cone gauge, one sets $x^+ = \tau$, and then (32) becomes

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}(\partial_\alpha r_i)^2 - \frac{1}{2}|\partial_\alpha \zeta_I|^2 - \frac{\mu^2}{2}((r_i)^2 + |\zeta_I|^2) \\ & + \frac{\mu}{2} \left(b e^{i\beta\mu\tau} (\partial_\tau \zeta_1 \partial_\sigma \zeta_2 - \partial_\sigma \zeta_1 \partial_\tau \zeta_2) \right. \\ & \left. + \bar{b} e^{-i\beta\mu\tau} (\partial_\tau \bar{\zeta}_1 \partial_\sigma \bar{\zeta}_2 - \partial_\sigma \bar{\zeta}_1 \partial_\tau \bar{\zeta}_2) \right), \end{aligned} \quad (33)$$

The four bosons, r_i , form a set of four oscillator towers with the same spectrum as in the pp-wave background without any 3-form flux:

$$\omega_0^2 = \mu^2 + \frac{n^2}{(\alpha' p^+)^2}. \quad (34)$$

The ζ_I system has a set of coupled equations of motion

$$\begin{aligned} (-\partial_\tau^2 + \partial_\sigma^2) \zeta_1 - \mu^2 \zeta_1 - \bar{b}\mu \left(\partial_\tau (e^{-i\beta\mu\tau} \partial_\sigma \bar{\zeta}_2) - e^{-i\beta\mu\tau} \partial_\tau \partial_\sigma \bar{\zeta}_2 \right) &= 0, \\ (-\partial_\tau^2 + \partial_\sigma^2) \bar{\zeta}_2 - \mu^2 \bar{\zeta}_2 + b\mu \left(\partial_\tau (e^{i\beta\mu\tau} \partial_\sigma \zeta_1) - e^{i\beta\mu\tau} \partial_\tau \partial_\sigma \zeta_1 \right) &= 0, \end{aligned} \quad (35)$$

and their complex conjugates.

The equations (35) are separable, and so we can expand into Fourier modes in the σ coordinate:

$$\begin{aligned} \zeta_1 &= \sum_{n \geq 0} \alpha_n^{(1)}(\tau) e^{-in\sigma/(\alpha' p^+) - i(\beta\mu\tau + \arg(b))/2}, \\ \zeta_2 &= \sum_{n \geq 0} \alpha_n^{(2)}(\tau) e^{in\sigma/(\alpha' p^+) - i(\beta\mu\tau + \arg(b))/2}, \end{aligned} \quad (36)$$

and we then find coupled ODE's for the $\alpha_n^{(k)}(\tau)$:

$$\begin{aligned} \ddot{\alpha}_n^{(1)} - i\beta\mu \dot{\alpha}_n^{(1)} + \left(\omega_0^2 - \frac{\beta^2 \mu^2}{4} \right) \alpha_n^{(1)} - \frac{n\beta|b|\mu^2}{2\alpha' p^+} \bar{\alpha}^{(2)} &= 0, \\ \ddot{\bar{\alpha}}_n^{(2)} + i\beta\mu \dot{\bar{\alpha}}_n^{(2)} + \left(\omega_0^2 - \frac{\beta^2 \mu^2}{4} \right) \bar{\alpha}_n^{(2)} - \frac{n\beta|b|\mu^2}{2\alpha' p^+} \alpha^{(1)} &= 0, \end{aligned} \quad (37)$$

where ω_0 is given by (34). Note that the shifts by $\exp(\pm \frac{i}{2}\beta\mu\tau)$ in (36) were used to remove the factors of $e^{i\beta\mu\tau}$ from the system. This was possible as a consequence of the special form (4) of \mathcal{B} . These shifts also correspond to sending $\varphi_3 \rightarrow \varphi_3 + \beta x^+$, which removes the x^+ -dependence from \mathcal{B} .

Finally, we obtain the frequencies of the string modes from the eigenvalues of this linear system:

$$\omega_{\pm}^2 = \left[\omega_0^2 + \left(\frac{\beta \mu}{2} \right)^2 \pm \beta \mu \sqrt{\omega_0^2 + \left(\frac{n|b|}{2\alpha' p^+} \right)^2} \right]. \quad (38)$$

It is interesting to observe that at least this part of the string spectrum is not invariant under the $SU(2)$ symmetry of the family of supergravity solutions parametrized by b and f . Indeed, one could have seen this *ab initio* because the bosonic string action does not depend upon $F_{(5)}^{RR}$ and so cannot depend directly upon f : it is thus impossible for the $SU(2)$ invariant combination (6) to appear in the spectrum³. So string theory does indeed break the $SU(2)$, confirming that it can only be a large N symmetry of the field theory on the brane. As one should also expect, the particle-like excitations with $n = 0$ are independent of both f and b , and thus respect the $SU(2)$ symmetry.

4.2 The fermionic modes

There are several expressions of the quadratic fermionic part of the Green-Schwarz action in a general background [23, 24, 6, 7], but sadly there are several inconsistencies in signs, factors and normalizations. Fortunately, there is one clear principle that defines the relevant part of the action [6]: The differential operator is precisely the supercovariant derivative that appears in the gravitino variation. To be more precise, the part of the Green-Schwarz action that is quadratic in fermions is [6]:

$$\mathcal{L}_{2F} = i(\eta^{ab}\delta_{IJ} - \epsilon^{ab}\rho_{3IJ})\partial_a x^M \bar{\theta}^I \gamma_M \mathcal{D}_b \theta^J, \quad (39)$$

where \mathcal{D}_a is the pull-back of the supercovariant derivative:

$$\begin{aligned} \mathcal{D}_a \equiv \partial_a + \partial_a x^M \Big[& \left(\frac{1}{4} \omega_{MAB} - \frac{1}{8} H_{MAB} \rho_3 \right) \gamma^{AB} \\ & - \frac{1}{48} F_{ABC} \gamma^{ABC} \rho_1 \gamma_M \\ & + \frac{1}{480} F_{ABCDE} \gamma^{ABCDE} \rho_0 \gamma_M \Big]. \end{aligned} \quad (40)$$

The matrices, ρ_j , are defined in [6], and following this reference, we take $\eta^{00} = -\eta^{11} = -1$, $\epsilon^{01} = 1$.

³It might be possible for f to appear in these equations via the supersymmetry condition $\beta = -2f$, but even with this, (38) is not $SU(2)$ invariant.

While it is not immediately obvious, (40) agrees with the results of [6]. The 5-form in [6] is twice that of [8] and hence twice that of this paper. There is also an apparent discrepancy of a factor of two in the normalization of the RR 3-form, however, this normalization depends upon a choice of the constant value of the dilaton. In this paper we are taking the dilaton to be zero ($e^\phi = 1$), whereas⁴ [6] takes $e^\phi = 2$. We have thus adjusted the normalizations in (40) so as to be consistent with our conventions for the RR forms. One can confirm that (40) is properly normalized by checking that it is consistent with the gravitino variation, (25) of [8].

The first step is to pass between the real and complex bases by taking $\epsilon = \theta^1 + i\theta^2$, and writing $G_{(3)} = H_{(3)} + iF_{(3)}$. Beyond this, it seems that (40) and (25) are very different: The former manifestly preserves an $SU(1, 1)$ duality group and, in particular, preserves the $U(1)$ that rotates $H_{(3)}$ into $F_{(3)}$. The latter formula apparently breaks this symmetry. The key to understanding the difference is simply that the formulæ of [8] are all in the Einstein frame, whereas the string actions must, of course, be in the string frame. The passage to the string frame also involves mixing the gravitino with the dilatino (see, for example, Appendix B of [25]):

$$\psi_M \rightarrow \psi_M - \frac{i}{4} \gamma_M \lambda.$$

(There is a sign difference here compared to [25] because of our different γ -matrix conventions.) This means that to pass to the string frame one must add a multiple of (26) to (25). If one does this then one does, indeed, arrive at (40).

For the particular class of background that we are considering here one obtains the following:

$$\begin{aligned} \mathcal{D}_a \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix} &= \partial_a \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix} \\ &+ (\partial_a x^+) \left[\frac{1}{4} \omega_{+AB} \gamma^{AB} \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix} - \frac{1}{8} H_{+AB} \gamma^{AB} \begin{pmatrix} \theta^1 \\ -\theta^2 \end{pmatrix} \right. \\ &\left. - \frac{1}{8} F_{+AB} \gamma^{AB} \begin{pmatrix} \theta^2 \\ \theta^1 \end{pmatrix} - \frac{f}{2} (\gamma^{1234} + \gamma^{5678}) \begin{pmatrix} -\theta^2 \\ \theta^1 \end{pmatrix} \right]. \end{aligned} \quad (41)$$

In spite of the string frame, the special form of the background preserves the $U(1)$ symmetry between $H_{(3)}$ and $F_{(3)}$. Using this, the fermion equations

⁴We are grateful to A. Tseytlin for clarifying this.

reduce to:

$$\begin{aligned} \gamma_+ \begin{pmatrix} (\partial_\tau + \partial_\sigma) \theta^1 \\ (\partial_\tau - \partial_\sigma) \theta^2 \end{pmatrix} = & + \frac{1}{4} \gamma_+ \mathcal{H}_{(3)} \begin{pmatrix} \theta^1 \\ -\theta^2 \end{pmatrix} + \frac{1}{4} \gamma_+ \mathcal{F}_{(3)} \begin{pmatrix} \theta^2 \\ \theta^1 \end{pmatrix} \\ & + \frac{1}{2} \gamma_+ \mathcal{F}_{(5)} \begin{pmatrix} \theta^2 \\ -\theta^1 \end{pmatrix}, \end{aligned} \quad (42)$$

where

$$\mathcal{H}_{(3)} = \frac{1}{2} H_{+\mu\nu} \gamma^{\mu\nu}, \quad \mathcal{F}_{(3)} = \frac{1}{2} F_{+\mu\nu} \gamma^{\mu\nu}, \quad \mathcal{F}_{(5)} = \frac{1}{24} F_{+\mu\nu\sigma\lambda} \gamma^{\mu\nu\sigma\lambda}. \quad (43)$$

As with the bosonic equations, the fermionic equations are separable. The detailed solution may be found in Appendix A, but we will summarize the key points here.

Half of the modes do not couple to the background B -field and so the mode analysis is elementary. We thus find eight sets of fermionic oscillators with frequencies:

$$\omega = \pm \sqrt{(f\mu)^2 + \left(\frac{n}{\alpha' p^+}\right)^2}, \quad (44)$$

with each sign having multiplicity 4. The remaining fermions couple to the B -field and so we have to shift them by phases, $\exp(\pm i(\beta\mu\tau + \arg(b))/2)$, much as we did for the bosons. We then obtain two copies of the following first order system of linear differential equations:

$$\begin{aligned} \left(\partial_\tau \mp \frac{1}{2} i\beta\mu + \left(\frac{in}{\alpha' p^+}\right) \right) \begin{pmatrix} \alpha_{(n)} \\ \tilde{\alpha}_{(n)} \end{pmatrix} \\ = -\frac{1}{2} |b| \beta\mu \begin{pmatrix} i\tilde{\alpha}_{(n)} - \tilde{\beta}_{(n)} \\ i\alpha_{(n)} + \beta_{(n)} \end{pmatrix} - f\mu \begin{pmatrix} \beta_{(n)} \\ \tilde{\beta}_{(n)} \end{pmatrix}, \\ \left(\partial_\tau \mp \frac{1}{2} i\beta\mu - \left(\frac{in}{\alpha' p^+}\right) \right) \begin{pmatrix} \beta_{(n)} \\ \tilde{\beta}_{(n)} \end{pmatrix} \\ = \frac{1}{2} |b| \beta\mu \begin{pmatrix} i\tilde{\beta}_{(n)} + \tilde{\alpha}_{(n)} \\ i\beta_{(n)} - \alpha_{(n)} \end{pmatrix} + f\mu \begin{pmatrix} \alpha_{(n)} \\ \tilde{\alpha}_{(n)} \end{pmatrix}. \end{aligned} \quad (45)$$

The normal modes of this system are:

$$\begin{aligned} \pm \left[\left(\frac{1}{2} |b|^2 \beta^2 \mu^2 + f^2 \mu^2 + p^2 + \frac{1}{4} \beta^2 \mu^2 \right) \right. \\ \left. \pm \frac{1}{2} \beta\mu \sqrt{(2f - |b|^2 \beta)^2 \mu^2 + 4p^2(1 + |b|^2)} \right]^{1/2} \end{aligned} \quad (46)$$

with all four permutations of the \pm signs, and where $p \equiv \frac{n}{\alpha' p^+}$. We thus find eight fermions having each of these frequencies with multiplicity two.

If one uses the equations of motion, (6), and the supersymmetry condition $\beta = -2f$, then these frequencies reduce to the much simpler form:

$$\omega = \pm \mu^2 \pm \sqrt{\mu^2 + \left(\frac{n}{\alpha' p^+}\right)^2}, \quad (47)$$

with all four permutations of the \pm signs, each with a multiplicity of two.

It is interesting to note that the modes (47) do not depend upon the values of f and b , whereas the modes (44) do, and thus this part of the string spectrum is not $SU(2)$ invariant.

4.3 Frequencies, energy and charge

Having found the eigenmodes of the string excitations, we would like to relate the frequencies to the physical quantum numbers, and most particularly, we would like to know how this works for the Penrose limit considered in section 2.

To find the normal modes we had to make shifts by $\exp(\pm \frac{i}{2}\beta\mu\tau)$, and this corresponds to going to a coordinate system in which the \mathcal{B} field is independent of x^+ . For the general family considered in section 2, this means redefining:

$$\varphi_3 = \tilde{\varphi}_3 + \beta x^+, \quad (48)$$

Thus time translations in our systems of differential equations are associated to a combination of $x^+ \rightarrow x^+ + \alpha$ and $\varphi_3 \rightarrow \varphi_3 + \beta\alpha$ (so that $\tilde{\varphi}_3$ remains fixed). Thus the frequencies, ω , computed above are associated to the corresponding mix of Nöther charges.

In the AdS/CFT correspondence one should recall that time translations, $t \rightarrow t + \alpha$ yield a Nöther charge that is the quantum number, Δ , representing the conformal dimension of the operator on the brane. In [13] the $U(1)_{\mathcal{R}}$ symmetry was identified as (16), at least for $\beta = 1$. In taking the Penrose limit we had to shift $\varphi_3 \rightarrow \varphi_3 + 2x^+$, and to diagonalize the system of eigenmodes we had to make a further shift to the x^+ -independent configuration. We are thus led to the following coordinates that describe the x^+ -independent configuration:

$$x^+ = t, \quad x^- \sim L^2(t + \frac{2}{3}\phi), \quad \hat{\varphi}_3 = \varphi_3 - 3t. \quad (49)$$

Shifting $t \rightarrow t + \alpha$ and keeping the new coordinates fixed requires $\varphi_3 \rightarrow \varphi_3 + 3\alpha$, $\phi \rightarrow \phi - \frac{3}{2}\alpha$. Hence the time translations in our system of differential

equations is associated to the conserved quantity:

$$\Delta - \frac{3}{2} \mathcal{J}.$$

5 Final Comments

We have found a new family of pp-wave solutions of IIB supergravity. The family has constant 5-form flux, non-constant 3-form fluxes, and time varying dilaton and axion. There are obviously many generalizations of the solution presented here, both within IIB supergravity and in \mathcal{M} -theory. We have not attempted to classify the broad class of possibilities, but instead we have focused on the solutions that are most closely related to interesting results within the AdS/CFT correspondence. Even within this context there are more general possibilities: one can presumably find Penrose limits of any of the flow solutions.

There are several important features of our solutions: first, the string theory is exactly solvable in these backgrounds. Secondly, our solutions are Penrose limits of AdS solutions that are interesting as holographic duals of field theory fixed points. Finally, our backgrounds *are not* maximally supersymmetric. Thus our results enable one to perform some deeper “stringy” tests of the holographic duals of $\mathcal{N} = 1$ supersymmetric flows.

Acknowledgments

This work was supported in part by funds provided by the DOE under grant number DE-FG03-84ER-40168. NH is supported by a Fletcher Jones Graduate Fellowship. We would like to thank D. Berenstein and J. Gomis for discussions and are particularly grateful A. Tseytlin for helpful correspondence.

A Fermionic Spectrum

The equations of motion for the fermion system are:

$$\begin{aligned} \gamma_+ \begin{pmatrix} (\partial_\tau + \partial_\sigma) \theta^1 \\ (\partial_\tau - \partial_\sigma) \theta^2 \end{pmatrix} = & + \frac{1}{4} \gamma_+ \mathcal{H}_{(3)} \begin{pmatrix} \theta^1 \\ -\theta^2 \end{pmatrix} + \frac{1}{4} \gamma_+ \mathcal{F}_{(3)} \begin{pmatrix} \theta^2 \\ \theta^1 \end{pmatrix} \\ & + \frac{1}{2} \gamma_+ \mathcal{F}_{(5)} \begin{pmatrix} \theta^2 \\ -\theta^1 \end{pmatrix}, \end{aligned} \tag{50}$$

where

$$\mathcal{H}_{(3)} = \frac{1}{2} H_{+\mu\nu} \gamma^{\mu\nu}, \quad \mathcal{F}_{(3)} = \frac{1}{2} F_{+\mu\nu} \gamma^{\mu\nu}, \quad \mathcal{F}_{(5)} = \frac{1}{24} F_{+\mu\nu\sigma\lambda} \gamma^{\mu\nu\sigma\lambda}. \quad (51)$$

One could solve this by using the explicit form of the spacetime Dirac matrices, or as we will, proceed by realizing these matrices as creation and destruction operators.

$$a_j^\dagger = \frac{i}{2}(\gamma^{2j-1} + i\gamma^{2j}), \quad a_j = \frac{i}{2}(\gamma^{2j-1} - i\gamma^{2j}), \quad j = 1, \dots, 4. \quad (52)$$

Note that these operators have a factor of i in front of them since we are using anti-hermitian γ -matrices as in [8]. For $\mathbf{8}_s$ we take the spinor basis to be:

$$\begin{pmatrix} |0\rangle \\ a_1^\dagger a_2^\dagger |0\rangle \\ a_1^\dagger a_3^\dagger |0\rangle \\ a_1^\dagger a_4^\dagger |0\rangle \\ a_2^\dagger a_3^\dagger |0\rangle \\ a_2^\dagger a_4^\dagger |0\rangle \\ a_3^\dagger a_4^\dagger |0\rangle \\ a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger |0\rangle \end{pmatrix} \quad (53)$$

We can now express the matrices in / as:

$$\begin{aligned} \mathcal{H}_{(3)} &= -2ib\beta\mu e^{i\beta\mu\tau} a_3^\dagger a_4^\dagger + 2i\bar{b}\beta\mu e^{-i\beta\mu\tau} a_3 a_4 \\ \mathcal{F}_{(3)} &= -2b\beta\mu e^{i\beta\mu\tau} a_3^\dagger a_4^\dagger - 2\bar{b}\beta\mu e^{-i\beta\mu\tau} a_3 a_4 \\ \mathcal{F}_{(5)} &= -f\mu(2 - 2\mathcal{N}_{1234} - 4(a_3^\dagger a_4^\dagger a_3 a_4 + a_1^\dagger a_2^\dagger a_1 a_2)) \end{aligned} \quad (54)$$

where \mathcal{N}_{1234} is the fermion number operator. Since we are studying type IIB theory, both $\begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}$ are in $\mathbf{8}_s$. Using the foregoing, (50) becomes:

$$\gamma_+(\partial_\tau + \partial_\sigma) \begin{pmatrix} \theta_1^1 \\ \theta_2^1 \\ \theta_3^1 \\ \theta_4^1 \\ \theta_5^1 \\ \theta_6^1 \\ \theta_7^1 \\ \theta_8^1 \end{pmatrix} = -\frac{1}{2} \gamma_+ \beta \mu \begin{pmatrix} \bar{b} e^{-i\beta\mu\tau} (i\theta_7^1 - \theta_7^2) + \frac{2f}{\beta} \theta_1^2 \\ \bar{b} e^{-i\beta\mu\tau} (i\theta_8^1 - \theta_8^2) + \frac{2f}{\beta} \theta_2^2 \\ -\frac{2f}{\beta} \theta_3^2 \\ -\frac{2f}{\beta} \theta_4^2 \\ -\frac{2f}{\beta} \theta_5^2 \\ -\frac{2f}{\beta} \theta_6^2 \\ b e^{i\beta\mu\tau} (i\theta_1^1 + \theta_1^2) + \frac{2f}{\beta} \theta_7^2 \\ b e^{\beta\mu\tau} (i\theta_2^1 + \theta_2^2) + \frac{2f}{\beta} \theta_8^2 \end{pmatrix}. \quad (55)$$

and

$$\gamma_+(\partial_\tau - \partial_\sigma) \begin{pmatrix} \theta_1^2 \\ \theta_2^2 \\ \theta_3^2 \\ \theta_4^2 \\ \theta_5^2 \\ \theta_6^2 \\ \theta_7^2 \\ \theta_8^2 \end{pmatrix} = \frac{1}{2} \gamma_+ \beta \mu \begin{pmatrix} \bar{b} e^{-i\beta\mu\tau} (i\theta_7^2 + \theta_7^1) + \frac{2f}{\beta} \theta_1^1 \\ \bar{b} e^{-i\beta\mu\tau} (i\theta_8^2 + \theta_8^1) + \frac{2f}{\beta} \theta_2^1 \\ -\frac{2f}{\beta} \theta_3^1 \\ -\frac{2f}{\beta} \theta_4^1 \\ -\frac{2f}{\beta} \theta_5^1 \\ -\frac{2f}{\beta} \theta_6^1 \\ b e^{i\beta\mu\tau} (i\theta_1^2 - \theta_1^1) + \frac{2f}{\beta} \theta_7^1 \\ b e^{i\beta\mu\tau} (i\theta_2^2 - \theta_2^1) + \frac{2f}{\beta} \theta_8^1 \end{pmatrix}, \quad (56)$$

One can solve this system of first order ordinary differential equations by Fourier expanding in the σ coordinate and shifting each mode by a constant:

$$\begin{aligned} \theta_\mu^1 &= \exp\left(\frac{1}{2} \varepsilon (i\beta\mu\tau + \arg(b))\right) \sum_{n=1}^{\infty} \alpha_{(n)}^\mu(\tau) \exp\left(\frac{in\sigma}{\alpha'p^+}\right) \\ \theta_\mu^2 &= \exp\left(\frac{1}{2} \varepsilon (i\beta\mu\tau + \arg(b))\right) \sum_{n=1}^{\infty} \beta_{(n)}^\mu(\tau) \exp\left(\frac{in\sigma}{\alpha'p^+}\right), \end{aligned} \quad (57)$$

where $\varepsilon = -1$ for $\mu = 1, 2$; $\varepsilon = +1$ for $\mu = 7, 8$; and $\varepsilon = 0$ for all other μ . Shifting the phase by this and including the constant means that in the equations we get $b, \bar{b} \rightarrow |b|$ and all of the $e^{\pm i\beta\mu\tau}$ dependence cancels.

The equations break into the three obvious groups. Eight of these fermions are not affected by the B-field at all, while the others group as $(\theta_1^1, \theta_7^1, \theta_1^2, \theta_7^2)$, $(\theta_2^1, \theta_8^1, \theta_2^2, \theta_8^2)$. The mode analysis is trivial for the fermions that do not couple to the B -field, and we thus find eight sets of fermionic oscillators with frequencies:

$$\omega = \pm \sqrt{(f\mu)^2 + \left(\frac{n}{\alpha'p^+}\right)^2}, \quad (58)$$

with each sign having multiplicity 4.

The groups $(\theta_1^1, \theta_7^1, \theta_1^2, \theta_7^2)$, $(\theta_2^1, \theta_8^1, \theta_2^2, \theta_8^2)$ produce identical sets of equations so we need only consider one of them:

$$\begin{aligned} &\left(\partial_\tau \mp \frac{1}{2} i\beta\mu + \frac{in}{\alpha'p^+}\right) \begin{pmatrix} \alpha_{(n)}^1 \\ \alpha_{(n)}^7 \end{pmatrix} \\ &= -\frac{1}{2} |b| \beta \mu \begin{pmatrix} i\alpha_{(n)}^7 - \beta_{(n)}^7 \\ i\alpha_{(n)}^1 + \beta_{(n)}^1 \end{pmatrix} - f\mu \begin{pmatrix} \beta_{(n)}^1 \\ \beta_{(n)}^7 \end{pmatrix}, \end{aligned} \quad (59)$$

$$\begin{aligned} & \left(\partial_\tau \mp \frac{1}{2} i \beta \mu - \frac{in}{\alpha' p^+} \right) \begin{pmatrix} \beta_{(n)}^1 \\ \beta_{(n)}^7 \end{pmatrix} \\ &= \frac{1}{2} |b| \beta \mu \begin{pmatrix} i \beta_{(n)}^7 + \alpha_n^7 \\ i \beta_{(n)}^1 - \alpha_{(n)}^1 \end{pmatrix} + f \mu \begin{pmatrix} \alpha_{(n)}^1 \\ \alpha_{(n)}^7 \end{pmatrix}, \end{aligned} \quad (60)$$

Doing normal mode analysis we get,

$$\begin{aligned} & \begin{pmatrix} -ip + \frac{i\beta\mu}{2} & -\frac{i|b|\beta\mu}{2} & -f & \frac{|b|\beta\mu}{2} \\ -\frac{i|b|\beta\mu}{2} & -ip - \frac{i\beta\mu}{2} & -\frac{|b|\beta\mu}{2} & -f \\ f & \frac{|b|\beta\mu}{2} & ip + \frac{i\beta\mu}{2} & \frac{i|b|\beta\mu}{2} \\ -\frac{|b|\beta\mu}{2} & f & \frac{i|b|\beta\mu}{2} & ip - \frac{i\beta\mu}{2} \end{pmatrix} \begin{pmatrix} \alpha_{(n)}^1 \\ \alpha_{(n)}^7 \\ \beta_{(n)}^1 \\ \beta_{(n)}^7 \end{pmatrix} \\ &= i \omega \begin{pmatrix} \alpha_{(n)}^1 \\ \alpha_{(n)}^7 \\ \beta_{(n)}^1 \\ \beta_{(n)}^7 \end{pmatrix}, \end{aligned} \quad (61)$$

where $p \equiv \frac{n}{\alpha' p^+}$. The four normal modes of this system therefore have frequencies:

$$\begin{aligned} & \pm \left[\left(\frac{1}{2} |b|^2 \beta^2 \mu^2 + f^2 \mu^2 + p^2 + \frac{1}{4} \beta^2 \mu^2 \right) \right. \\ & \quad \left. \pm \frac{1}{2} \beta \mu \sqrt{(2f - |b|^2 \beta)^2 \mu^2 + 4p^2(1 + |b|^2)} \right]^{1/2} \end{aligned} \quad (62)$$

If one uses the equations of motion for the supergravity background and the condition for enhanced supersymmetry:

$$f^2 + \frac{1}{4} \beta^2 |b|^2 = 1, \quad \beta = -2f, \quad (63)$$

then the frequencies simplify significantly:

$$\omega = \pm \mu \pm \sqrt{\left(\frac{n}{\alpha' p^+} \right)^2 + \mu^2}, \quad (64)$$

with all four permutations of the \pm signs.

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