

# Asymptotic symmetries of massless QED in even dimensions

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We consider the scattering of massless particles coupled to an abelian gauge field in  $2n$ -dimensional Minkowski spacetime. Weinberg’s soft photon theorem is recast as Ward identities for infinitely many new nontrivial symmetries of the massless QED  $\mathcal{S}$ -matrix, with one such identity arising for each propagation direction of the soft photon. These symmetries are identified as large gauge transformations with angle-dependent gauge parameters that are constant along the null generators of null infinity. Almost all of the symmetries are spontaneously broken in the standard vacuum and the soft photons are the corresponding Goldstone bosons. Our result establishes a relationship between soft theorems and asymptotic symmetry groups in any even dimension.

## 1. Introduction

Recent work [1–6] has connected long understood soft theorems [7–11] for gauge theory and gravity scattering amplitudes to Ward identities for asymptotic symmetry groups of massless interacting theories coupled to gauge theory and gravity in four dimensions. While several of the soft theorems have been known and understood since the 1960’s, many of their associated asymptotic symmetry groups have only recently drawn attention. Conjectures stemming from the correspondence have led to the discovery and investigation of new soft theorems in four dimensions [12–20], many of which were subsequently identified with asymptotic symmetry groups [4, 5, 21]. The leading and subleading soft theorems have been investigated at loop level [22–25], in the context of string and (ambi)-twistor string theory [26–30], and have been shown to hold in higher dimensions [31–34]. Some of the asymptotic symmetry groups associated to these new soft theorems, such as the extended BMS group, were previously conjectured [35–38], while the nature of others [5] remains unknown.

Although much work has been done, many questions remain unanswered. The tree-level leading soft-theorem is universal among all theories in arbitrary dimensions. Such striking universality can only be a reflection of an underlying symmetry. Motivated by the established correspondence between soft theorems and asymptotic symmetries in four dimensions and the existence of the soft theorems in higher dimensions, we are led to consider the leading soft theorem in massless QED in even-dimensional Minkowski space-time. The odd-dimensional case is of course also of interest but has additional subtleties which require a separate investigation. We recast the soft theorem in the form of a Ward identity for a new group of asymptotic symmetries. The asymptotic symmetry group in  $d = 2m + 2$  dimensions is the subgroup of the local  $U(1)$  large gauge transformations with a gauge parameter given by an unconstrained function on the  $S^{2m}$ . Our result generalizes the analysis performed in the four dimensional case [6], further strengthening the relationship between soft theorems and asymptotic symmetries in all even dimensions.

We work in the semiclassical limit and therefore prove the result only at tree-level. However, given that the leading QED soft factor is not renormalized in four dimensions, the result may be exact. Although massless QED is not renormalizable in dimensions greater than four, we are interested in infrared effects where two derivative theories minimally coupled to matter fields serve as good low energy effective theories. The infrared structure of gauge theories is simpler in  $d > 4$  due to the absence of soft divergences, so we hope that studying the soft theorems and associated asymptotic symmetries in higher dimensions will help to clarify the fate of the new symmetries in the presence of loop corrections both in  $d = 4$  and  $d > 4$ .

This paper is organized as follows. In Section 2 we review the structure of massless QED in  $d = 2m + 2$  dimensions and establish our coordinates and conventions. In Section 3 we restrict our attention to six dimensions for illustrative purposes. We determine appropriate boundary conditions and introduce the gauge field mode expansions and the relevant soft photon operators. In Section 4 we rewrite Weinberg's soft theorem as a Ward identity for local charge operators involving the matter current and the soft photon operators. In Section 5 we demonstrate that these charges generate a new asymptotic symmetry group, which is a subgroup of the original  $U(1)$  gauge group. In Section 6 we discuss how to generalize our result to the arbitrary even-dimensional case.

## 2. Maxwell's equations in even dimensional Minkowski spacetime

Abelian gauge theory in  $d = 2m + 2$  dimensional flat Minkowski spacetime is governed by Maxwell's equations

$$(2.1) \quad \nabla^\mu F_{\mu\nu} = e^2 J_\nu^M,$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ,  $J_\nu^M$  is the matter current density, and  $e$  is the coupling constant of the theory. The equations (2.1) are invariant under local gauge transformations of the form

$$(2.2) \quad A_\mu \rightarrow A_\mu + \partial_\mu \varepsilon, \quad \Psi_{\mathcal{Q}} \rightarrow e^{i\mathcal{Q}\varepsilon} \Psi_{\mathcal{Q}},$$

where  $\Psi_{\mathcal{Q}}$  is a matter field with electric charge  $\mathcal{Q}$ . It is useful to introduce retarded coordinates  $(u, r, z^a)$  given by

$$(2.3) \quad x^0 = u + r, \quad x^i = r \hat{x}^i(z),$$

where  $u$  is retarded time and  $\hat{x}^i(z)$  describes an embedding of the unit  $S^{2m}$  with coordinates  $z^a, a = 1, \dots, 2m$  into  $\mathbb{R}^{2m+1}$  with coordinates  $x^i, i = 1, \dots, 2m + 1$ . The flat Minkowski metric then takes the form

$$(2.4) \quad ds^2 = -(dx^0)^2 + (dx^i)^2 = -du^2 - 2dudr + r^2 \gamma_{ab} dz^a dz^b.$$

Here  $\gamma_{ab}$  is the metric on the unit radius  $S^{2m}$  with covariant derivative  $D_a$ . In the conformal compactification of Minkowski spacetime, we can identify future null infinity ( $\mathcal{I}^+$ ) as the null surface  $(r = \infty, u, z^a)$ . We also employ advanced coordinates

$$(2.5) \quad x^0 = v - r, \quad x^i = -r \hat{x}^i(z),$$

so that past null infinity ( $\mathcal{I}^-$ ) is identified as the surface  $(r = \infty, v, z^a)$  and

$$(2.6) \quad ds^2 = -dv^2 + 2dvdr + r^2 \gamma_{ab} dz^a dz^b.$$

The advanced  $S^{2m}$  coordinate  $z$  is antipodally related to the retarded  $S^{2m}$  coordinate  $z$  in such a way that null generators of  $\mathcal{I}$  passing through spatial infinity are labeled by the same value of  $z$  on  $\mathcal{I}^+$  and  $\mathcal{I}^-$ . We denote the  $u = \pm\infty$  boundaries of  $\mathcal{I}^+$  as  $\mathcal{I}_\pm^+$ , and the  $v = \pm\infty$  boundaries of  $\mathcal{I}^-$  as  $\mathcal{I}_\pm^-$ .

Maxwell's equations in retarded coordinates take the form

$$(2.7) \quad \begin{aligned} r^{-2m} \partial_r (r^{2m} F_{ru}) - \partial_u F_{ru} + r^{-2} D^a F_{au} &= e^2 J_u^M, \\ -r^{-2m} \partial_r (r^{2m} F_{ur}) + r^{-2} D^a F_{ar} &= e^2 J_r^M, \\ r^{-2m+2} \partial_r (r^{2m-2} (F_{ra} - F_{ua})) - \partial_u F_{ra} + r^{-2} D^b F_{ba} &= e^2 J_a^M. \end{aligned}$$

Similar expressions hold for the advanced coordinates. The constraint equation for the hypersurface at future null infinity is

$$(2.8) \quad \begin{aligned} n^\mu \nabla^\nu F_{\nu\mu} &= \frac{1}{2} r^{-2m} \partial_r (r^{2m} F_{ru}) - \partial_u F_{ru} + r^{-2} D^a \left( F_{au} - \frac{1}{2} F_{ar} \right) \\ &= e^2 n^\mu J_\mu^M, \end{aligned}$$

where the null normal vector is  $n = \partial_u - \frac{1}{2} \partial_r$ .

### 3. Six-dimensional Maxwell primer

In this section we consider six-dimensional abelian gauge theory at null infinity,<sup>1</sup> postponing the discussion of arbitrary even dimensions to Section 6. We determine appropriate boundary conditions for the gauge fields, determine matching conditions to link  $\mathcal{I}^-$  quantities to  $\mathcal{I}^+$  quantities, and isolate the gauge field zero-mode operators appearing in Weinberg's soft theorem.

#### 3.1. Asymptotic analysis at $\mathcal{I}^+$

We work in retarded radial gauge. The gauge-fixing conditions are

$$(3.1) \quad A_r = 0, \quad A_u|_{\mathcal{I}^+} = 0.$$

This leaves unfixed a residual large gauge symmetry parameterized by an unconstrained function  $\varepsilon(z)$  on the  $S^4$  at  $\mathcal{I}^+$ . Under such a large gauge transformation

$$(3.2) \quad \delta A_a(z) = \partial_a \varepsilon(z).$$

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<sup>1</sup>Janis and Newman studied the null Cauchy problem for Maxwell's equations in four dimensions in [39] with similar conclusions.

In order to analyze the field equations near  $\mathcal{I}^+$  we assume an asymptotic expansion for the gauge field:

$$(3.3) \quad A_a = \sum_{n=0} \frac{A_a^{(n)}}{r^n}, \quad A_u = \sum_{n=1} \frac{A_u^{(n)}}{r^n}.$$

The  $\mathcal{O}(r^{-2}, r^{-3}, r^{-4})$  orders of the constraint equation are (in the absence of matter currents)

$$(3.4) \quad \partial_u(A_u^{(1)} - D^a A_a^{(0)}) = 0,$$

$$(3.5) \quad (D^2 - 1)A_u^{(1)} + \partial_u(2A_u^{(2)} - D^a A_a^{(1)}) = 0,$$

$$(3.6) \quad -A_u^{(2)} - \partial_u F_{ru}^{(4)} + D^a(F_{au}^{(2)} - \frac{1}{2}F_{ar}^{(2)}) = 0.$$

In six dimensions, a plane wave has transverse field strength behaving as  $F_{ab} \sim \frac{1}{r}$ . Finiteness of the energy flux at each point on  $\mathcal{I}^+$  and finiteness of the total energy evaluated on a space-like Cauchy surface requires

$$(3.7) \quad \begin{aligned} F_{ab}^{(0)} &= \partial_a A_b^{(0)} - \partial_b A_a^{(0)} = 0, \\ F_{ub}^{(0)} &= \partial_u A_b^{(0)} = 0, \end{aligned}$$

which implies

$$(3.8) \quad A_a^{(0)} = \partial_a \phi(z).$$

Here  $\phi(z)$  is a free, unconstrained function on  $S^4$  which will later be identified as the Goldstone mode of the spontaneously broken large gauge symmetry.

The subleading term  $A_a^{(1)}(u, z)$  represents the free radiative data. Finiteness of the total radiated energy requires that at large values of  $|u|$

$$(3.9) \quad A_a^{(1)}|_{\mathcal{I}^{\pm}} = 0.$$

Finiteness of the Coulombic energy and integration of (3.4) then imply

$$(3.10) \quad A_u^{(1)} = 0.$$

Demanding that the electric field fall off like  $\frac{1}{r^4}$  near  $\mathcal{I}^+$  together with (3.5) then imply

$$(3.11) \quad A_u^{(2)}|_{\mathcal{I}^+} = 0,$$

and interior values of  $A_u^{(2)}$  are determined by integrating (3.5). At the next order we must specify the boundary data for the electric field

$$(3.12) \quad F_{ru}^{(4)}|_{\mathcal{I}^\pm} = -3A_u^{(3)}|_{\mathcal{I}^\pm} \equiv E_r.$$

We are interested in scattering processes that revert to the vacuum at  $u = \infty$ , so we require  $F_{ru}^{(4)}|_{\mathcal{I}^\pm} = 0$ . We additionally require  $A_a^{(2)}|_{\mathcal{I}^\pm} = 0$ . A full perturbative solution of course requires the equations of motion as well as the constraints.

### 3.2. Asymptotic analysis at $\mathcal{I}^-$

Similar analysis can be applied to a Maxwell field  $B_\mu$  in advanced coordinates near  $\mathcal{I}^-$ . We label the corresponding field strength tensor  $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ , and denote  $G_{rv}^{(4)}|_{\mathcal{I}^\pm} = E_r^\pm$ . Advanced radial gauge

$$(3.13) \quad A_r = 0, \quad A_v|_{\mathcal{I}^-} = 0$$

leaves unfixed a residual large gauge symmetry parameterized by an unconstrained function  $\varepsilon^-(z)$ . The various finiteness conditions applied in the previous section lead to a similar set of boundary conditions for  $B_\mu$ . In particular, we have  $B_a^{(0)}(z) = \partial_a \psi(z)$ , with  $\psi(z)$  an unconstrained function on  $S^4$ .

### 3.3. Scattering

Given asymptotic data  $A_\mu$  on  $\mathcal{I}^+$  and  $B_\mu$  on  $\mathcal{I}^-$ , we must specify a matching condition for the boundary values of the two gauge fields in order to properly define the scattering problem. In doing so, we also single out a diagonal subgroup of the large gauge transformations acting separately at  $\mathcal{I}^+$  and  $\mathcal{I}^-$ , which can then be interpreted as a symmetry of the  $\mathcal{S}$ -matrix.

The boundary condition (3.9) provides a trivial matching condition for the radiative data. The only nontrivial component of the gauge field strength on the boundaries of  $\mathcal{I}^+$  and  $\mathcal{I}^-$  are the quantities  $E_r$  and  $E_r^-$ . As in four dimensions [6], we impose the matching condition

$$(3.14) \quad E_r(z) = E_r^-(z).$$

Here  $z$  labels a null generator, so that the coordinate argument of  $E_r^-$  is antipodally related to the argument of  $E_r$ . The corresponding matching condition for the Goldstone modes is

$$(3.15) \quad \phi(z) = \psi(z).$$

The diagonal subgroup of large gauge transformations acting at  $\mathcal{I}^+$  and  $\mathcal{I}^-$  is therefore obtained by imposing

$$(3.16) \quad \varepsilon(z) = \varepsilon^-(z).$$

### 3.4. Mode expansions

The radiative modes of the gauge field in the plane wave basis take the form

$$(3.17) \quad A_\mu(x) = e \sum_\alpha \int \frac{d^5q}{(2\pi)^5} \frac{1}{2\omega_q} \left[ \varepsilon_\mu^{*\alpha}(\vec{q}) a_\alpha(\vec{q}) e^{iqx} + \varepsilon_\mu^\alpha(\vec{q}) a_\alpha(\vec{q})^\dagger e^{-iqx} \right],$$

where  $\omega_q = |\vec{q}|$ ,  $\varepsilon_\mu^\alpha$  are the four independent polarization vectors for the photon in six dimensions, and

$$(3.18) \quad [a_\alpha(\vec{p}), a_\beta(\vec{q})^\dagger] = 2\omega_q \delta_{\alpha\beta} (2\pi)^5 \delta^5(\vec{p} - \vec{q}).$$

The free radiative data in this basis is of the form

$$(3.19) \quad A_a^{(1)}(u, z^a) = -\frac{2\pi^2 e}{(2\pi)^5} \partial_a \hat{x}^i \sum_\alpha \int \omega_q d\omega_q [\varepsilon_i^{*\alpha} a_\alpha(\omega_q \hat{x}) e^{-i\omega_q u} + \varepsilon_i^\alpha a_\alpha(\omega_q \hat{x})^\dagger e^{i\omega_q u}].$$

We can define a Fourier image for the radiative modes

$$(3.20) \quad \begin{aligned} A_a^{\omega(1)}(z) &= -\frac{e\omega}{8\pi^2} \partial_a \hat{x}^i(z) \sum_\alpha \varepsilon_i^{*\alpha} a_\alpha(\omega \hat{x}(z)), \\ A_a^{-\omega(1)}(z) &= -\frac{e\omega}{8\pi^2} \partial_a \hat{x}^i(z) \sum_\alpha \varepsilon_i^\alpha a_\alpha(\omega \hat{x}(z))^\dagger, \end{aligned}$$

with  $\omega > 0$  assumed for both expressions. We can define the corresponding zero mode operator

$$(3.21) \quad A_a^{0(1)} \equiv \frac{1}{2} \lim_{\omega \rightarrow 0} (A_a^{\omega(1)} + A_a^{-\omega(1)}).$$

In a similar way we can introduce the in-modes

$$(3.22) \quad B_a^{(1)}(v, z) \equiv \lim_{r \rightarrow \infty} r \partial_a x^i(r, z) A_i(v - r, r \hat{x}^i(z))$$

so that

$$(3.23) \quad \begin{aligned} B_a^{\omega(1)}(z) &= -\frac{e\omega}{8\pi^2} \partial_a \hat{x}^i(z) \sum_{\alpha} \varepsilon_i^{*\alpha} a_{\alpha}(-\omega \hat{x}(z)), \\ B_a^{-\omega(1)}(z) &= -\frac{e\omega}{8\pi^2} \partial_a \hat{x}^i(z) \sum_{\alpha} \varepsilon_i^{\alpha} a_{\alpha}(-\omega \hat{x}(z))^{\dagger}. \end{aligned}$$

The corresponding zero mode operator is

$$(3.24) \quad B_a^{0(1)} \equiv \frac{1}{2} \lim_{\omega \rightarrow 0} (B_a^{\omega(1)} + B_a^{-\omega(1)}).$$

#### 4. Soft theorem as a Ward identity

In this section we recast Weinberg's soft theorem as a Ward identity for charges constructed out of the matter and gauge fields. In the following section, we demonstrate that this Ward identity is associated to a new group of asymptotic symmetries of massless QED.

##### 4.1. Soft theorem

Weinberg's soft theorem takes the same form in any dimension:

$$(4.1) \quad \begin{aligned} &\lim_{\omega \rightarrow 0} \omega \langle z_{n+1}, \dots | a_{\alpha}(q) \mathcal{S} | z_1, \dots \rangle \\ &= e\omega \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} - \sum_{k=1}^n \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} \right] \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle. \end{aligned}$$

Here  $a_{\alpha}(q)$  is a creation operator for an outgoing on-shell photon with polarization  $\varepsilon_{\alpha}$  and momentum  $q$ . A null momentum vector in 6 dimensions is completely characterized by its energy  $\omega$  and a point  $z$  on the  $S^4$ . This allows us to express the soft photon's momenta as

$$(4.2) \quad q^{\mu} = \omega [1, \hat{x}^i(z)].$$

Here  $\hat{x}(z)$  is the embedding of the unit  $S^4$  into  $\mathbb{R}^5$ . We use the same parametrization for the momenta of the massless external particles:

$$(4.3) \quad p_k^{\mu} = E_k [1, \hat{x}^i(z_k)].$$

In states and out states are then determined by the energy  $E_k$ , electric charge  $\mathcal{Q}_k$ , and  $\mathcal{I}$ -crossing point  $z_k$  for each external particle. We denote the in and



out states by

$$(4.4) \quad |z_1, \dots, z_n\rangle, \quad \langle z_{n+1}, \dots, z_{n+n'}|,$$

respectively. In what follows, we assume that the incoming and outgoing states do not include soft photons.

Motivated by the expression for the radiative modes (3.20), we define the function

$$(4.5) \quad \begin{aligned} F_a^{out}(z, z_1, \dots, z_{n+n'}) &\equiv \partial_a \hat{x}^i(z) \omega \sum_{\alpha} \varepsilon_i^{*\alpha} \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} - \sum_{k=1}^n \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} \right] \\ &= \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \partial_a \log(1 - P(z, z_k)) - \sum_{k=1}^n \mathcal{Q}_k \partial_a \log(1 - P(z, z_k)). \end{aligned}$$

Here we have used the completeness relation for polarization vectors

$$(4.6) \quad \sum_{\alpha} \varepsilon_{\alpha}^{*i}(\vec{q}) \varepsilon_{\alpha}^j(\vec{q}) = \delta^{ij} - \frac{q^i q^j}{q^2}$$

and defined a function<sup>2</sup>

$$(4.7) \quad P(z, z_k) \equiv \hat{x}_i(z) \hat{x}^i(z_k).$$

$F_a^{out}(z, z_1, \dots, z_{n+n'})$  (abbreviated  $F_a(z; z_k)$ ) is simply related to the zero mode insertion:

$$(4.8) \quad \langle z_{n+1}, \dots | A_a^{0(1)}(z) \mathcal{S} | z_1, \dots \rangle = -\frac{e^2}{(4\pi)^2} F_a^{out}(z; z_k) \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle.$$

Straightforward algebra reveals that  $F_a^{out}(z; z_k)$  obeys the differential equation

$$(4.9) \quad \begin{aligned} &\sqrt{\gamma}(D^2 - 2)D^a F_a^{out}(z; z_k) \\ &= -(4\pi)^2 \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \delta^4(z - z_k) - \sum_{k=1}^n \mathcal{Q}_k \delta^4(z - z_k) \right]. \end{aligned}$$

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<sup>2</sup> $P$  is known as the invariant distance on the  $S^4$ , and is related to the cosine of the geodesic distance.

We may also consider Weinberg’s soft theorem for an incoming soft photon, which reads

$$(4.10) \quad \lim_{\omega \rightarrow 0} \omega \langle z_{n+1}, \dots | \mathcal{S} a_\alpha(q)^\dagger | z_1, \dots \rangle \\ = -e\omega \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \frac{p_k \cdot \varepsilon_\alpha^*}{p_k \cdot q} - \sum_{k=1}^n \mathcal{Q}_k \frac{p_k \cdot \varepsilon_\alpha^*}{p_k \cdot q} \right] \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle.$$

We similarly define

$$(4.11) \quad F_a^{in}(z, z_1, \dots, z_{n+n'}) \\ \equiv \partial_a \hat{x}^i(z) \omega \sum_\alpha \varepsilon_i^\alpha \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \frac{p_k \cdot \varepsilon_\alpha^*}{p_k \cdot q} - \sum_{k=1}^n \mathcal{Q}_k \frac{p_k \cdot \varepsilon_\alpha^*}{p_k \cdot q} \right] \\ = - \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \partial_a \log(1 + P(z, z_k)) - \sum_{k=1}^n \mathcal{Q}_k \partial_a \log(1 + P(z, z_k)) \right],$$

which is in turn related to the zero mode insertion

$$(4.12) \quad \langle z_{n+1}, \dots | \mathcal{S} B_a^{0(1)}(z) | z_1, \dots \rangle = \frac{e^2}{(4\pi)^2} F_a^{in}(z; z_k) \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle.$$

Combining equations (3.16), (4.8), (4.9), and (4.12), we obtain the relation

$$(4.13) \quad \frac{1}{2e^2} \int d^4z \sqrt{\gamma} \varepsilon(z) (D^2 - 2) D^a \langle z_{n+1}, \dots | A_a^{0(1)}(z) \mathcal{S} | z_1, \dots \rangle \\ + \frac{1}{2e^2} \int d^4z \sqrt{\gamma} \varepsilon^-(z) (D^2 - 2) D^a \langle z_{n+1}, \dots | \mathcal{S} B_a^{0(1)}(z) | z_1, \dots \rangle \\ = \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \varepsilon(z_k) - \sum_{k=1}^n \mathcal{Q}_k \varepsilon(z_k) \right] \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle.$$

We can rewrite this expression as a Ward identity

$$(4.14) \quad \langle z_{n+1}, \dots | (Q_\varepsilon^+ \mathcal{S} - \mathcal{S} Q_\varepsilon^-) | z_1, \dots \rangle = 0,$$

where  $Q_\varepsilon^\pm$  are charges acting on  $\mathcal{I}^\pm$  states.  $Q_\varepsilon^\pm$  can be decomposed into a hard charge and a soft charge:

$$(4.15) \quad Q_\varepsilon^\pm = Q_H^\pm + Q_S^\pm.$$

The hard charges  $Q_H^\pm$  are defined so that

$$\begin{aligned}
 Q_H^- |z_1, \dots\rangle &= \sum_{k=1}^n \mathcal{Q}_k \varepsilon(z_k) |z_1, \dots\rangle, \\
 \langle z_{n+1}, \dots | Q_H^+ &= \langle z_{n+1}, \dots | \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \varepsilon(z_k).
 \end{aligned}
 \tag{4.16}$$

The soft charges are given by

$$\begin{aligned}
 Q_S^+ &= -\frac{1}{2e^2} \int d^4z \sqrt{\gamma} \varepsilon(z) [D^2 - 2] D^a A_a^{0(1)}(z), \\
 Q_S^- &= \frac{1}{2e^2} \int d^4z \sqrt{\gamma} \varepsilon^-(z) [D^2 - 2] D^a B_a^{0(1)}(z).
 \end{aligned}
 \tag{4.17}$$

### 5. From Ward identity to asymptotic symmetry

We would now like to interpret the Ward identity (4.14) in terms of symmetry transformations on the matter and gauge fields and to identify the asymptotic symmetry group of six dimensional massless QED.

#### 5.1. Action on matter fields

Equation (4.16) indicates that the charges  $Q_H^\pm$  generate a gauge transformation on the matter fields. We can express  $Q_H^\pm$  in terms of the gauge current:

$$\begin{aligned}
 Q_H^+ &= \lim_{r \rightarrow \infty} \int_{\mathcal{I}^+} r^4 \sqrt{\gamma} du d^4z \varepsilon(z) J_u^M(u, r, z), \\
 Q_H^- &= \lim_{r \rightarrow \infty} \int_{\mathcal{I}^-} r^4 \sqrt{\gamma} dv d^4z \varepsilon^-(z) J_v^M(v, r, z).
 \end{aligned}
 \tag{5.1}$$

For a matter field  $\Psi_{\mathcal{Q}}$  of charge  $\mathcal{Q}$  we have

$$\begin{aligned}
 [Q_H^+, \Psi_{\mathcal{Q}}(u, r, z)] &= \left[ \lim_{r \rightarrow \infty} \int_{\mathcal{I}^+} r^4 \sqrt{\gamma} \varepsilon J_u^M, \Psi_{\mathcal{Q}}(u, r, z) \right] \\
 &= -\varepsilon(z) \mathcal{Q} \Psi_{\mathcal{Q}}(u, r, z).
 \end{aligned}
 \tag{5.2}$$

The soft charges  $Q_S^\pm$  commute with  $\Psi_{\mathcal{Q}}$ , so we see that the total charges  $Q_\varepsilon^\pm$  generate gauge transformations on the matter fields with gauge parameter  $\varepsilon(z)$ .

## 5.2. Action on gauge fields

Since the full theory is invariant only under combined gauge transformations of the matter and gauge fields, it is intuitively obvious that  $Q_S^\pm$  must generate a large gauge transformation for the gauge fields  $A_a$  and  $B_a$ . In order to make this relationship precise, we can use the constraint equation (2.8) along with the boundary conditions from Section 3.1 to rewrite the total charge as a boundary integral

$$(5.3) \quad \begin{aligned} Q_\varepsilon^+ &= \frac{1}{e^2} \lim_{r \rightarrow \infty} \int_{\mathcal{I}_+^+} r^4 \sqrt{\gamma} d^4 z \varepsilon(z) F_{ru}(u, r, z) \\ &= \frac{1}{e^2} \int_{\mathcal{I}_+^+} \sqrt{\gamma} d^4 z \varepsilon(z) E_r(z), \end{aligned}$$

$$(5.4) \quad \begin{aligned} Q_\varepsilon^- &= \frac{1}{e^2} \lim_{r \rightarrow \infty} \int_{\mathcal{I}_+^-} r^4 \sqrt{\gamma} d^4 z \varepsilon^-(z) G_{rv}(v, r, z) \\ &= \frac{1}{e^2} \int_{\mathcal{I}_+^-} \sqrt{\gamma} d^4 z \varepsilon^-(z) E_r^-(z). \end{aligned}$$

At this point several comments are in order. For  $\varepsilon(z) = 1$  these expressions reduce to the familiar expressions for total electric charge at  $\mathcal{I}_+^+$  and  $\mathcal{I}_+^-$ . For non-constant  $\varepsilon(z)$  they are the natural generalization of the asymptotic symmetry generators in the four dimensional case [6]. Both charges are written as pure boundary integrals of the free data  $E_r$  and  $E_r^-$ , allowing for a canonical identification of asymptotic symmetry transformations at  $\mathcal{I}^+$  and  $\mathcal{I}^-$ . In the next subsection we demonstrate that it is possible to define a symplectic structure on the phase space of the theory so that the charges do in fact generate large gauge transformations on the gauge fields.

## 5.3. Bracket for the free data

In order to claim that the  $Q_\varepsilon^\pm$  generate gauge transformations we need to define the symplectic structure on the phase space of the theory. The bracket for the radiative modes is unambiguous [40, 41] and can be deduced from the mode expansion:

$$(5.5) \quad [A_a^{(1)}(u, z), \partial_{u'} A_b^{(1)}(u', z')] = i \frac{e^2}{2} \gamma_{ab} \delta(u - u') \frac{\delta^4(z - z')}{\sqrt{\gamma}}.$$

The bracket for the zero modes can then be defined so that the charge  $Q_\varepsilon^\pm$  generates the correct gauge transformation. The correct bracket is given by

$$(5.6) \quad [E_r(z), \phi(z')] = ie^2 \frac{\delta^4(z - z')}{\sqrt{\gamma}}.$$

The bracket (5.6) resembles that of the constant modes in [6]. Similar expressions hold for  $\mathcal{I}^-$  quantities. It follows that

$$(5.7) \quad [Q_\varepsilon^+, A_a(z)] = i\partial_a \varepsilon(z),$$

and we conclude that the charges  $Q_\varepsilon^\pm$  generate large gauge transformations on the matter fields and gauge fields of the theory.

As we have seen,  $Q_S^\pm$  does not annihilate the conventional vacuum of the theory. In fact, when  $Q_S^\pm$  acts on the vacuum it creates a soft photon, indicating that the large gauge symmetries are spontaneously broken. Under a large gauge transformation with parameter  $\varepsilon(z)$ , the free data  $\phi(z)$  transforms as a Goldstone boson:

$$(5.8) \quad \phi(z) \rightarrow \phi(z) + \varepsilon(z).$$

## 6. Generalization to arbitrary even dimensional spacetime

The results of the preceding sections can be straightforwardly generalized to arbitrary even dimensional flat spacetimes. In this section, we sketch the derivation of the Ward identity for  $d = 2m + 2$  dimensional spacetime, omitting a detailed discussion of the boundary conditions and symplectic form.

The plane wave expansion of the gauge field in  $d = 2m + 2$  dimensions is given by

$$(6.1) \quad A_\mu(u, r, z) = e \sum_\alpha \int \frac{d^{2m+1}q}{(2\pi)^{2m+1}} \frac{1}{2\omega_q} \left[ \varepsilon_\mu^{*\alpha}(\vec{q}) a_\alpha(\vec{q}) e^{iqx} + \varepsilon_\mu^\alpha(\vec{q}) a_\alpha(\vec{q})^\dagger e^{-iqx} \right].$$

Here  $\omega_q = |\vec{q}|$  and  $\alpha$  labels the  $2m$  polarizations of the photon with corresponding polarization vectors  $\varepsilon_\mu^\alpha(\vec{q})$ . The operator  $a_\alpha(\vec{q})^\dagger$  is a photon creation operator normalized so that

$$(6.2) \quad [a_\alpha(\vec{p}), a_\beta(\vec{q})^\dagger] = 2\omega_q \delta_{\alpha\beta} (2\pi)^{2m+1} \delta^{2m+1}(\vec{p} - \vec{q}).$$

We can evaluate the leading term in the large  $r$  expansion of (6.1) using the saddle point approximation, yielding an expression for the radiative degrees

of freedom of the Maxwell field in  $d = 2m + 2$  dimensions near  $\mathcal{I}^+$ . The expression for the Fourier image is

$$(6.3) \quad A_a^{\omega(m-1)}(z) = \frac{(-i)^m \omega^{m-1} e}{2(2\pi)^m} \partial_a \hat{x}^j(z) \sum_{\alpha} \varepsilon_j^{*\alpha} a_{\alpha}(\omega \hat{x}(z)),$$

$$(6.4) \quad A_a^{-\omega(m-1)}(z) = \frac{i^m \omega^{m-1} e}{2(2\pi)^m} \partial_a \hat{x}^j(z) \sum_{\alpha} \varepsilon_j^{\alpha} a_{\alpha}(\omega \hat{x}(z))^{\dagger},$$

where  $\hat{x}^i(z)$  is an embedding of  $S^{2m}$  into  $\mathbb{R}^{2m+1}$ . We can define a generalized zero mode operator

$$(6.5) \quad A_a^{0(m-1)} = \frac{1}{2} \lim_{\omega \rightarrow 0} (i\omega)^{2-m} \left[ A_a^{\omega(m-1)} + (-1)^m A_a^{-\omega(m-1)} \right].$$

Using the conventions (4.2)–(4.4), we can rewrite Weinberg’s soft theorem (4.1) in the form

$$(6.6) \quad \begin{aligned} &\langle z_{n+1}, \dots | A_a^{0(m-1)}(z) \mathcal{S} | z_1, \dots \rangle \\ &= -\frac{(-1)^m e^2}{4(2\pi)^m} F_a^{out}(z; z_k) \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle. \end{aligned}$$

Here the soft factor

$$(6.7) \quad \begin{aligned} &F_a^{out}(z, z_1, \dots, z_{n+n'}) \\ &\equiv \partial_a \hat{x}^i(z) \omega \sum_{\alpha} \varepsilon_i^{*\alpha} \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} - \sum_{k=1}^n \mathcal{Q}_k \frac{p_k \cdot \varepsilon_{\alpha}}{p_k \cdot q} \right] \\ &= \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \partial_a \log(1 - P(z, z_k)) - \sum_{k=1}^n \mathcal{Q}_k \partial_a \log(1 - P(z, z_k)) \end{aligned}$$

satisfies the differential equation

$$(6.8) \quad \begin{aligned} &(-1)^{m+1} \sqrt{\gamma} \prod_{l=m+1}^{2m-1} [D^2 - (2m-l)(l-1)] D^a F_a^{out} \\ &= \Gamma(m) 2^m (2\pi)^m \left[ \sum_{k=1}^n \mathcal{Q}_k \delta^{2m}(z - z_k) - \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \delta^{2m}(z - z_k) \right]. \end{aligned}$$

We can similarly introduce the in-modes

$$(6.9) \quad B_a^{\omega(m-1)}(z) = \frac{i^m \omega^{m-1} e}{2(2\pi)^m} \partial_a \hat{x}^j(z) \sum_{\alpha} \varepsilon_j^{*\alpha} a_{\alpha}(-\omega \hat{x}(z)),$$

$$(6.10) \quad B_a^{-\omega(m-1)}(z) = \frac{(-i)^m \omega^{m-1} e}{2(2\pi)^m} \partial_a \hat{x}^j(z) \sum_{\alpha} \varepsilon_j^{\alpha} a_{\alpha}(-\omega \hat{x}(z))^{\dagger},$$

and the associated zero mode operator

$$(6.11) \quad B_a^{0(m-1)} = \frac{1}{2} \lim_{\omega \rightarrow 0} (i\omega)^{2-m} \left[ B_a^{\omega(m-1)} + (-1)^m B_a^{-\omega(m-1)} \right].$$

We then have

$$(6.12) \quad \begin{aligned} & \langle z_{n+1}, \dots | \mathcal{S} B_a^{0(m-1)}(z) | z_1, \dots \rangle \\ &= \frac{e^2}{4(2\pi)^m} F_a^{in}(z; z_k) \langle z_{n+1}, \dots | \mathcal{S} | z_1, \dots \rangle, \end{aligned}$$

where

$$(6.13) \quad \begin{aligned} & F_a^{in}(z, z_1, \dots, z_{n+n'}) \\ &= - \left[ \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \partial_a \log(1 + P(z, z_k)) - \sum_{k=1}^n \mathcal{Q}_k \partial_a \log(1 + P(z, z_k)) \right]. \end{aligned}$$

Combining equations (3.16), (6.6), (6.8), and (6.12), we can rewrite the soft theorem as the Ward identity

$$(6.14) \quad \langle z_{n+1}, \dots | (Q_{\varepsilon}^+ \mathcal{S} - \mathcal{S} Q_{\varepsilon}^-) | z_1, \dots \rangle = 0.$$

The charges  $Q_{\varepsilon}^{\pm} = Q_H^{\pm} + Q_S^{\pm}$  act on  $\mathcal{I}^{\pm}$  states, with the action of  $Q_H^{\pm}$  defined so that

$$(6.15) \quad \begin{aligned} Q_H^- | z_1, \dots \rangle &= \sum_{k=1}^n \mathcal{Q}_k \varepsilon(z_k) | z_1, \dots \rangle, \\ \langle z_{n+1}, \dots | Q_H^+ &= \langle z_{n+1}, \dots | \sum_{k=n+1}^{n+n'} \mathcal{Q}_k \varepsilon(z_k). \end{aligned}$$

The soft charges take the form

$$(6.16) \quad Q_S^+ = -\frac{1}{2e^2} \frac{2^{2-m}}{\Gamma(m)} \int d^{2m} z \sqrt{\gamma} \varepsilon(z) \prod_{l=m+1}^{2m-1} (D^2 - (2m-l)(l-1)) D^a A_a^{0(m-1)},$$

$$(6.17) \quad Q_S^- = \frac{(-1)^m}{2e^2} \frac{2^{2-m}}{\Gamma(m)} \int d^{2m} z \sqrt{\gamma} \varepsilon^-(z) \prod_{l=m+1}^{2m-1} (D^2 - (2m-l)(l-1)) D^a B_a^{0(m-1)}.$$

Note that the careful limiting procedure of section four may still be applied to  $A_a^\omega$  near  $\omega = 0$ . The charges  $Q_H^\pm$  can be written in terms of the gauge current

$$(6.18) \quad \begin{aligned} Q_H^+ &= \lim_{r \rightarrow \infty} r^{2m} \int_{\mathcal{I}^+} \sqrt{\gamma} \varepsilon(z) J_u^M(u, r, z), \\ Q_H^- &= \lim_{r \rightarrow \infty} r^{2m} \int_{\mathcal{I}^-} \sqrt{\gamma} \varepsilon^-(z) J_v^M(v, r, z). \end{aligned}$$

This operator generates a gauge transformation with parameter  $\varepsilon(z)$  when acting on the matter fields. Assuming a natural generalization of the boundary conditions from Section 3 and using Maxwell's equations (2.7), we can write the total charge  $Q_\varepsilon^\pm = Q_H^\pm + Q_S^\pm$  as a boundary integral

$$(6.19) \quad \begin{aligned} Q_\varepsilon^+ &= \frac{1}{e^2} \lim_{r \rightarrow \infty} r^{2m} \int_{\mathcal{I}^+} d^{2m}z \sqrt{\gamma} \varepsilon(z) F_{ru}(u, r, z), \\ Q_\varepsilon^- &= \frac{1}{e^2} \lim_{r \rightarrow \infty} r^{2m} \int_{\mathcal{I}^-} d^{2m}z \sqrt{\gamma} \varepsilon^-(z) G_{rv}(v, r, z). \end{aligned}$$

We can introduce an extended phase space for the modes on  $\mathcal{I}^+$  and  $\mathcal{I}^-$  to include  $\phi(z)$  and  $E_r(z)$ . The symplectic form of Section 5.3 can then be used to demonstrate that (6.19) generates large gauge transformations on the matter fields and gauge fields. These large gauge transformations are the asymptotic symmetries for even dimensional massless QED.

## 7. Open questions and relations to subsequent work

Our analysis has been restricted to the case of massless matter fields. It would be interesting to extend the analysis to include massive particles, along the lines of [42, 43]. Since this work first appeared, there have been a number of papers which analyze the action of large gauge symmetries in four-dimensional QED in a basis of infrared safe states [44–46]. Since the QED  $\mathcal{S}$ -matrix is infrared finite for  $D > 4$ , it is unclear whether or not such analyses are relevant to the questions considered in this paper, but the issue deserves further consideration. In four dimensions, the soft theorems and asymptotic symmetries of QED have been related to so-called electromagnetic memory effects [47–49]. The soft theorems, asymptotic symmetries [50] and memory effects [51, 52] have been separately analyzed in the higher dimensional case. These are outstanding problems for future investigations.



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