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Problems

E. Lieb

Department of Mathematics
Princeton University
Princeton, NJ 08544

1. Derive a rigorous version of Quantum electrodynamics for low energies (atoms and molecules). It will involve a cutoff, but all renormalization effects should be logarithmic in the cutoff.

2. Prove long range order in the Hubbard model in three or more dimensions. (Also in two dimensions for the ground state.) (See E.H. Lieb, 'The Hubbard model – Some Rigorous Results and Open Problems', in *Proceedings of the XIth International Congress of Mathematical Physics*, Paris, 1994, D. Iagolnitzer ed., pp. 392-412, International Press (1995).

3. Forty years ago Yang and others worked out the first two terms in the ground state energy of a hard sphere Bose gas as a function of density for low density. It was non rigorous, based on Bogoliubov's theory (his seminal 1947 paper, whose existence seems to be little known nowadays). Very recently Yngvason and I have given a rigorous derivation of the first term. However, a rigorous derivation of the second term is still to be found. (If you are more ambitious, you can try to derive the third term, for which T.T. Wu gave an expression).

(see E.H. Lieb and J. Yngvason, Ground State Energy of the Low Density Bose Gas, Phys. Rev. Lett. **80**, 2504-2507 (1998).

4. Prove the existence of Bose-Einstein condensation in a continuum gas. (It is already known for a lattice model. (See T. Kennedy, E.H. Lieb and S. Shastry, "The XY Model has Long-Range Order for all Spins and all Dimensions Greater than One", Phys. Rev. Lett. **61**, 2582-2584 (1988).)