Comparison of Chinese and Japanese Developments in Mathematics during the Late 19th and Early 20th Centuries

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Foreword [1]

After the invention of calculus by I. Newton and G. Leibniz much fundamental progress in science has occurred. There were a large number of outstanding mathematicians in Europe in the period of the eighteenth and the nineteenth centuries. I list here the names of some of the most famous ones according to their birth years in the eighteenth and nineteenth centuries.

D. Bernoulli (1700-1782)
G. Cramer (1704-1752)
L. Euler (1707-1783)
A. Clairaut (1713-1765)
J. d'Alembert (1717-1783)
J. Lambert (1728-1777)
A. Vandermonde (1735-1796)
E. Waring (1736-1798)
J. Lagrange (1736-1814)
G. Monge (1746-1818)
P. Laplace (1749-1827)
A. Legendre (1752-1833)
J. Argand (1768-1822)
J. Fourier (1768-1830)
C. Gauss (1777-1855)
A. Cauchy (1789-1857)
A. Möbius (1790-1868)
N. Lobachevsky (1792-1856)
G. Green (1793-1841)
N. Abel (1802-1829)
J. Bolyai (1802-1860)
C. Jacobi (1804-1851)
J. Dirichlet (1805-1859)
W. Hamilton (1805-1865)
H. Grassmann (1809-1877)
J. Liouville (1809-1892)
E. Kummer (1810-1893)
E. Galois (1811-1832)
G. Boole (1815-1864)
K. Weierstrass (1815-1897)
G. Stokes (1819-1903)
P. Chebyshev (1821-1894)
A. Cayley (1821-1895)
C. Hermite (1822-1901)
G. Eisenstein (1823-1852)
L. Kronecker (1823-1891)
W. Kelvin (1824-1907)
B. Riemann (1826-1866)
J. Maxwell (1831-1879)
L. Fuchs (1833-1902)
E. Beltrami (1835-1900)
S. Lie (1842-1899)
J. Darboux (1842-1917)
H. Schwarz (1843-1921)
G. Cantor (1845-1918)
F. Frobenius (1849-1917)
F. Klein (1849-1925)
G. Ricci (1853-1925)
H. Poincare (1854-1912)
A. Markov (1856-1922)
L. Bianchi (1856-1928)
C. Picard (1856-1941)
A. Lyapunov (1857-1918)
D. Hilbert (1862-1943)
H. Minkowski (1864-1909)
G. Castelnuovo (1865-1952)
J. Hadamard (1865-1963)
G. Voronoi (1868-1908)
F. Hausdorff (1868-1942)
E. Cartan (1869-1951)
F. Enriques (1871-1946)
G. Fano (1871-1952)

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E. Borel (1871-1956)
T. Levi-Civita (1873-1941)
H. Lebesgue (1875-1941)
G. Hardy (1877-1947)
G. Fubini (1879-1943)
F. Severi (1879-1961)
G. Birkhoff (1884-1944)
H. Weyl (1885-1955)
J. Littlewood (1885-1977)
I. Vinogradov (1891-1983)
N. Wiener (1894-1964)
R. Nevanlinna (1895-1980)
C. Siegel (1896-1981)
O. Zariski (1899-1986)

These mathematicians brought in new concepts and disciplines, and created great advance, for mathematics in the past three hundred years. Many profound and powerful tools were devised. On the other hand, the advance of mathematics contributed in an essential way to the evolution of natural science, which forms the pillar of modern culture.

In contrast, during their period, mathematics in Asia was abnormally silent, considering the fact that very few mathematicians, in China, Indian or Japan, were able to make any achievements comparable to the western masters. Since mathematics is the foundation of natural science, poor performance in mathematics thus led to slow development of science in the Asian countries. It is therefore worthwhile to ponder over this issue.

Before the Meiji Restoration, Japan had achieved much less in mathematics compared with China except for the discovery of determinants by Takakazu Seki (關孝和, 1642–1708). However, Japan surpassed China in mathematics during the last part of the nineteenth century. In this article, I intend to explore this phenomenon by reviewing some historical incidents.

Importing Western Mathematics in China and in Japan during the 19th Century

In 1859 Shanlan Li (李善蘭, 1811–1882), a Chinese mathematician, and A. Wylie, a Scotland missionary, collaborated to translate De Morgan's textbook *Algebra* (consisting of thirteen volumes) and E. Loomis' *Geometry, Differential and Integral Calculus* (consisting of 18 volumes) into Chinese. They also completed the translation of Euclid's classic *Elements with Comments* which was half done by Guanqi Xu (徐光啟, 1562–1633) and Matteo Ricci (利瑪竇, 1552–1610) in Ming Dynasty.

The translation of De Morgan's and Loomis's books (*Algebra*, and *Geometry, Differential and Integral Calculus*) has greater significance in regard to the effect on the development of oriental modern mathematics. De Morgan's book brought in modern algebra, while Euclid's *Elements* and Loomis's book introduced the concepts of axiomatic geometry and calculus.

Shanlan Li could comprehend the power series expansions of trigonometric and logarithmic functions and their inverse functions. Indeed, he knew how to calculate the volume of a cone. He reproved Fermat's little theorem, and deduced the so-called "identity of Shanlan Lee" in the area of combinatorial mathematics. Due to the above contributions, Shanlan Li can be viewed as the most prominent mathematician at the end of Qing Dynasty. However, his work was not comparable to the works of the European masters because he was not able to expand mathematics on the basis of the knowledge of calculus.

Afterwards, Hengfang Hua (華蘅芳, 1833–1902), in collaboration with the English missionary J. Fryer (傅蘭雅), translated John Wallis' books *Algebra* and *Origin of Calculus and Trigonometry*. In addition, they translated T. Galloway and R.E. Anderson's book Probability Theory (published in 1896) which was an important work about classic probability theory.

During this period, many widely used mathematical terms were created, including a system of symbols, such as using the Chinese character "禾" to represent integration. Also, they selected 26 Chinese characters to represent the 26 English letters and used the traditional Chinese names of 28 stars to represent the Greek alphabet.

The reason for such "special" symbols was to introduce western culture without deviating too much from the traditional Chinese culture so that Chinese might comprehend western mathematics more easily. It turned out that such symbols became a major obstruction for later generations to understand western mathematics. We might point out that Arabic mathematics had been transmitted to China since the Yuan Dynasty; however, the system of symbols invented by Greek mathematicians was not accepted by ancient Chinese.

Due to the above translations, Chinese scholars bean to have access to relatively modern mathematics, especially to the content of calculus whose pivotal role in modern science cannot be exaggerated. Unfortunately, calculus was not included in the major courses of the Tongwen School (同文館, established in 1861; later it was transformed into Peking University), which played an important role in the Westernization Movement in China, during the later 40 years in the 19th century.

On the other hand, the Fuzhou Shipping School (1866) invited L. Medrod, a French mathematician, to give some advanced courses in calculus. In 1875, the school sent some students to England and France. For examples, Fu Yan (嚴復) was dispatched to England to learn mathematics and natural science; Shoujian Zeng (鄭守箴) and Zhenfeng Lin (林振峰) had opportunities to study at the École Normale Supérieure in Paris and got bachelor degrees. It is a pity that they could not appreciate the beauty of mathematics, which resulted in their lack of enthusiasm for the subject. None of these pursued mathematics after they came home.

Before the Meiji Restoration in 1868, Japanese mathematicians were largely influenced by China and

Netherlands. In 1862, Japanese scholars visited China and brought back Shanlan Li's translations *Algebra* and *Geometry, Differential and Integral Calculus*. The books were widely disseminated in Japan. Then Japanese mathematicians began to make their own translation, using symbols and equations not only from Chinese translations but also from western books.

The Emperor Meiji decreed the study of knowledge from all over the world. He urged Japanese people to learn western mathematics instead of the traditional Japanese mathematics, Wasan. Japan dispatched many students to Europe. What is more, three thousand foreigners were once invited to Japan to provide assistance. Although some Wasan mathematicians such as Morishizu Takaku (高久守靜) strongly resisted western mathematics, the government insisted on its western mathematics policies. Thus western culture was absorbed rapidly in Japan. As a result, Japan surpassed China in a short time.

In 1872 Akitake Tsukamoto (冢本明毅) finished the Japanese translation of Wallis' *Algebra*. Subsequently the Japanese translation of Loomis' book *Geometry, Differential and Integral Calculus* was published in 1865. This Japanese translation not only corrected the errors in the Chinese translation, but also added new ingredients, such as some formulae and calculations in Dutch. How conscientious these Japanese scholars were!

Since then, Japanese mathematicians not only made translations from English or Dutch but also published their own original work. For instance, Riken Fukuda (福田理軒) completed a book named *Introduction to Calculus* (筆算微積入門). Although Japanese mathematics had been mainly influenced by the Netherlands and China, England soon became a more significant influence. During the Meiji Restoration, there were two mathematicians of enlightenment, Dairoku Kikuchi (菊池大麓, 1855–1917) and Rikitaro Fujisawa (藤澤利喜太郎, 1861–1933). Both of them were professors in the Science College of the Imperial University, which was later renamed the University of Tokyo (the Kyoto Imperial University was not established until 1897).

Kikuchi's father was a Dutch scholar in the Edo period. Kikuchi studied geometry at Cambridge. At that time, projective geometry had just been introduced into England from the Continent. According to his biography, Kikuchi always ranked first in his class. Kikuchi and his classmates respected each other; in spite of the fierce competition in their studies. According to Kikuchi's autobiography, the British gentleman style made an unforgettable impression on him. Afterwards Kikuchi was offered prestigious jobs in the academic world and even in the government. He became a model for many Japanese scholars. Kikuchi received both bachelor's and master's degrees from Cambridge. Upon his return to Japan in 1877, he became the first mathematics professor in Japan. Furthermore, Kikuchi went on to lay the foundation of projective geometry in Japan. In fact, decades later when the Chinese mathematician Buqing Su (蘇步青) came to Japan to learn mathematics, it was projective and differential geometry that captured his attention and he intensely studied this renowned Japanese geometric tradition.

As we already alluded to above, Kikuchi came from a scholarly family. Many of his relatives, including his sons, went on to become famous scholars in Japan. Kikuchi earned his position as the Dean of the Science College (1881–1893), as well as the president of the Tokyo Imperial University (1898–1901). He later became the Minister of Education (1901–1903), the president of the Kyoto Imperial University (1908–1912), and subsequently the president of the Imperial Academy of Japan.

He made exceptional contributions to the academic development during the Meiji Restoration, and was open-minded; for example, he gave mathematical courses in English for several years.

The year Fujisawa entered the Tokyo Imperial University, 1877, was the same year when Kikuchi became a professor there. Incidentally, Fujisawa's father was also a Dutch scholar. Under Kikuchi's instruction, Fujisawa spent five years in the Tokyo Imperial University. Then he went to London University and transferred to Berlin and Strasbourg after several months. In Berlin, his teachers were E. E. Kummer, L. Kronecker and K. Weierstrass, the masters of Germany at that time.

Fujisawa returned to Japan in 1887, bringing with him the German academic system and practices. Fujisawa was an expert in elliptic function theory on which he wrote fourteen papers. He became a senator in 1925 and was elected to the Imperial Academy of Japan (1932).

Kikuchi and Fujisawa not only made great contributions to higher education, but also devoted much energy to middle school and girl's education. They compiled a lot of textbooks.

Mathematics in China and Japan at the beginning of the 20th century

Mathematics in Japan

Tsuruichi Hayashi (林鶴一, 1873–1935) and Teiji Takagi (高木貞治, 1875–1960) were the most important Japanese mathematicians at the beginning of the 20th century. Hayashi was the founder of the Department of Mathematics in Tohoku Imperial University. Using his own income, he created a mathematical research journal, the *Tohoku Journal of Mathematics*.

But the true founder of modern Japanese mathematics is Takagi. Takagi grew up in the countryside; his father was an accountant. Takagi went to middle school in 1886; the textbook he read was I. Todhunter's, *Algebra for Beginners* and Wilson's *Geometre*. He went to the Third High School (in Kyoto) in 1891 and then moved to the Tokyo Imperial University three years later.

According to Takagi's autobiography, his university textbooks were *Elliptic Function* by Duregi and *Algebraic*

Curves by G. Salmon. At that time, he did not understand the close relation of projective geometry and the subjects in these books. Kikuchi was then the minister of Education; thus he had a few hours per week available for lecturing. The major teaching load fell on Fujisawa's shoulder. Fujisawa adopted the German style of teaching. He emphasized the central role of algebra in mathematics, which traced back to Kronecker. Takagi learned the abelian equations from J. A. Serret's book *Cours d'Algèbre Supérieure*. He also had studied the two volumes of *Algebra* by H. Weber, which were just published.

In 1898 Takagi traveled to Berlin and studied under F. G. Frobenius. At that time L. Fuchs and H. A. Schwarz were still alive. There were no great differences in the subject of course in Berlin and Tokyo. However, he could communicate with the masters and enjoyed the academic atmosphere there.

In 1900, Takagi visited Göttingen and had the opportunity to see C. F. Klein and D. Hilbert. Most of the young mathematicians in Europe gathered in Göttingen to communicate with each other and perform research under the guidance of the great masters. Takagi felt the big gap of mathematics level between Europe and Japan. He believed that Japan was nearly a half century behind Europe. After working hard for one and a half years, Takagi became much more confident about himself, which illustrated the importance of the academic atmosphere in the gathering of scholars on research of individual mathematicians.

Takagi studied algebraic number theory under Hilbert. He worked on complex multiplications of the lemniscate function. He finished his Ph.D. thesis in 1903.His doctoral degree was granted by Tokyo University while he was offered an associate professorship by the Tokyo Imperial University in 1900.

In 1901, Takagi returned to Tokyo, bringing the Göttingen research methods promoted by Hilbert. Takagi claimed that the Colloquia was indispensable for research. The department of mathematics must have its own library and café where the faculty members could communicate ideas of mathematics freely. In 1904, Takagi was promoted to professorship. Takagi devoted much energy not only to research but also to education. He compiled many textbooks which had a great impact on the development of Japan mathematics.

The outbreak of World War I in 1914 led to the isolation of the scientific community in Japan from the West. However, in Takagi's view, short-term academic isolation was actually beneficial to him, affording him enough time for research on the class field theory. During this period, he realized the inadequacies of Hilbert's theory and, in 1920, published his new theory in the International Congress of Mathematicians in Strasbourg. Two years later, C. Siegel recognized the significance of this paper and recommended E. Artin to study it. Hereafter Artin discovered the most general reciprocity law, which was a milestone in the modern class field theory.

Shokichi Iyanaga (彌永昌吉), a student of Takagi, graduated from the Tokyo Imperial University in 1931. He went to Germany and France. There he studied under Artin. He became a professor in the Tokyo Imperial University in 1942. Iyanaga had a lot of students and had a great influence on the development of mathematics in Japan.

Below is a list of famous Japanese mathematicians whose degrees were obtained between 1930's to 1960's:

University of Tokyo: Kosaku Yoshida (吉田耕作, 1931), Tadashi Nakayama (中山正, 1935), Kiyoshi Ito (伊藤清, 1938), Kenkichi Iwasawa (岩澤健吉, 1945), Nagayoshi Iwahori (岩堀長慶, 1948), Kunihiko Kodaira (小平邦彦, 1949), Tosio Kato (加藤敏夫, 1951), Mikio Sato (佐藤幹夫, 1952), Goro Shimura (志村五郎, 1952), Michio Suzuki (鈴木通夫, 1952), Yutaka Taniyama (谷山豐, 1953), Tsuneo Tamagawa (玉河恒夫, 1954), Ichiro Satake (佐武一郎, 1950), Yasutaka Ihara (伊原康隆).

Kyoto University: Kiyoshi Oka (岡潔, 1924), Yasuo Akizuki (秋月康夫, 1926), Shigeo Nakano (中野茂男), Hiroshi Toda (戸田宏), Sigeru Mizohata (溝畑茂), Fuzihiro Araki (荒木不二洋), Teruhisa Matsusaka (松阪輝久), Heisuke Hironaka (廣中平祐, M.S. 1953), Masayoshi Nagata (永田雅宜, Nagoya Univ., B. Sc.1950; Kyoto Univ., Doc. Sc. 1957), Shinzo Watanabe (渡邊信三, B.S. 1958).

Nagoya University: Shizuo Kakutani (角谷靜夫, 1941), Masatake Kuranishi (倉西正武, 1948), Goro Azumaya (東屋五郎, 1949), Kiiti Morita (森田紀一, 1950).

Tohoku University: Tadahiko Kubota (窪田忠彦, 1915), Shigeo Sasaki (佐佐木重夫, 1935).

Osaka University: Shingo Murakami (村上信吾), Yozo Matsushima (松島与三, 1942).

Scholars in the Tokyo Imperial University and the Kyoto Imperial University followed the tradition of Takagi. Together with their western colleagues, they built up the foundation of mathematics in the mid-20th century. It is fair to say that many of them became giants in the history of mathematics.

Among these scholars, Kunihiko Kodaira (1915–1997) and Heisuke Hironaka (1931–) were Fields Medal winners. Both of them stayed in America for quite a long time. Hironaka received his Ph.D. at Harvard in 1960 where he remained as a professor for a number of years; he returned to Japan in the 1990's. Kodaira returned to Japan in 1967; he had only 4 Ph.D. students in the US while he had as many as 13 Ph.D. students in Japan, such as Shigeru Iitaka (飯高茂), Kenj. Ueno (上野健爾), Eiji Horikawa (堀川穎二, 1947-2006), Iku. Nakamura (中村郁), Fumio. Sakai (酒井文雄), Yoichi. Miyaoka (宮岡洋一), Takao. Fujita (藤田隆夫) and Toshiyuki Katsura (桂利行). The return of Kodaira was crucial to the development of algebraic geometry in Japan.

Mikio Sato (1928–) also had many brilliant students, e.g., Takahiro Kawai (河合隆裕), Masaki Kashiwara (柏原正

樹), Tetsuji Miwa (三輪哲二) and Michio Jimbo (神保道夫); all of them were authorities in algebraic analysis and integrable systems. Among the students of Masayoshi Nagata (1927–2008) we mentioned Masayoshi Miyanishi (宮西正宜), Masaki Maruyama (丸山正樹, 1945–2009), Shigefumi Mori (森重文), and Shigeru Mukai (向井茂); in particular, Mori was a Fields medal winner.

Kiyoshi Ito pioneered the theory of stochastic calculus and was awarded the Gauss prize in ICM 2006 in Spain. Kiyoshi Oka was a founder of several complex variables. Kenkichi Iwasawa, Goro Shimura, Yutaka Taniyama, Jun-Ichi Igusa (井草準一), Michio Kuga (久賀道郎), Ichiro Satake and Tsuneo Tamagawa, along with other western mathematicians, are major architects of modern number theory and arithmetic geometry. Shigeo Sasaki, Kentaro Yano (矢野健太郎), Yozo Matsushima, Shigeo Nakano and Shoshichi Kobayashi (小林昭七) made important contributions to geometry. Tosio Kato is a leader in perturbation theory.

Mathematics in China

As mentioned before, the mathematical development in China started to lag behind that of Japan by late nineteenth century, except during the short period of Shanlan Li's and Wylie's translation of Loomis' book. Although the Fuzhou Shipping School and the Tongwen School provided courses on calculus, the effect was rather meager.

Fu Yan (嚴復, 1854–1921) graduated from the Fuzhou Shipping School and went on to Plymouth and the Royal Naval College at Greenwich. He studied engineering and mathematics there; unfortunately there were no good teachers in mathematics in that school. Hong Rong (容閎, 1828–1912), leading a team of young boys to the US with the aim of learning engineering, made no great contributions to the rebuilding of basic science in China.

After the first Sino-Japanese War (1894), the Chinese government realized the urgent need of modernization and sent many students to Japan. In 1901, two top madarins, Zhidong Zhang (張之洞) and Kunyi Liu (劉坤一), proposed to the Emperor that China should learn from Japan the educational system in various stages to meet the needs of China. They asked the Japanese Ministry of Education and the Ministry of Army to help the Chinese government. In 1906, there were some 8,000 Chinese students studying in Japan. Also, many Japanese teachers were hired to give lessons in China. Zuxun Feng (馮祖荀) was among the first group of students who studied in Japan. Feng was a student at the Kyoto Imperial University. He founded the Department of Mathematics at Peking University in 1913.

In 1902, Da Zhou (周達) went to Japan. Then he met Kiyoshi Ueno (上野清) and Kamenosuke Nagasawa (長澤龜之助). He studied the Japanese curriculum and the mathematics education there. According to Zhou, the core courses were the following:

The first year: Differentiation, integration, analytical geometry of the plane and the space, basic number theory, basic astronomy, and the least square method, elementary theoretical physics, exercises in physics, exercises in mathematics;

The second year: Elementary complex function theory, algebra, mechanics, exercises in number theory, physics experiments;

The third year: General complex function theory, elliptic function theory, higher geometry, algebra, advanced differential equations, topics in analysis, mechanics, calculus of variations, number theory. The courses were so comprehensive that they were similar to the courses of today's top universities, except for topology, which did not appear until the 20th century. In contrast, Chinese universities were still in their infancy and did not cover the above comprehensive subjects.

Ueno told Zhou that, in the translation of Wallis's book *Algebra*, Hengfang Hua should not have omitted the exercises in the original book. Zhou's third son, Wei-Liang Chow (Weiliang Zhou 周煒良, 1911–1995), was to become a prominent algebraic geometer in the 20th century.

Hindsight shows that it was not a wise idea to rely entirely on Japan in the westernization process, because there was still some gap between Japan and Europe regardless of the tremendous progress Japan had made in science during the end of the 19th century and the early 20th century. China devoted much time to learn from Japan, but not directly from Europe, solely because Japan was nearby, which kept traveling and living costs under control. It is understandable that the Chinese government and the Chinese students chose a neighboring country as their mentor for economical reasons. But, if you don't learn from the master, how can you make it to the top?

Shortly after, China began to send students to the USA. Dunfu Hu (胡敦復) and Zhifan Zheng (鄭之蕃) were among these students. Both Hu and Zheng studied at Cornell; Zheng then visited Harvard for one year. They taught at Tsinghua University in succession (1911 and 1920). The Department of Mathematics in Tsinghua University was established in 1927; Zheng was the first department chairman. Fen Qin (秦汾) was another student at Harvard. He became a professor in Peking University. The above three were among the founding members of the Chinese Mathematical Society in 1935. Dunfu Hu was in charge of sending three batches of students (180 in total) to the Unites States.

The Chinese Education and Culture Foundation was established in 1909 because of the USA Boxer Indemnity refund. Then other countries, including Britain, also followed. The effect was that the process of sending students to Europe and the USA became better organized and more efficient. The quality of students increased. Harvard was the most popular university for Chinese students. Mingfu Hu (胡明復, 1891–1927) was the first Chinese student to earn a Ph.D.; He studied integral equations under W. F. Osgood and M. Bocher. Lifu Jiang

(姜立夫, 1890–1978), the second Chinese mathematics Ph.D., was a geometer who studied under J. L. Coolidge.

Dawei Yu (俞大維, 1897–1993) studied mathematical logic with H. M. Sheffer and C. I. Lewis; he got a doctorate from the department of Philosophy of Harvard University in 1922. Jinnian Liu (劉晉年, 1904–1968) followed G. D. Birkhoff and obtained a doctorate in 1929. Zehan Jiang (江澤涵, 1902–1994) followed H. M. Morse to learn topology and obtained a doctorate in 1930. Youcheng Shen (申又棖, 1901–1978) followed J. L. Walsh to learn analysis and obtained a doctorate in 1934.

The University of Chicago was another destination of Chinese students. Wuzhi Yang (楊武之, 1896–1973) studied number theory with L. E. Dickson (Ph.D. in 1928). Guangyuan Sun (孫光遠) studied projective differential geometry with E. P. Lane and obtained his Ph.D. in the same year. Kunsheng Hu (胡坤升) studied analysis and calculus of variation with Bliss and obtained his Ph. D. in 1932. In addition, Yuanrong Zeng (曾遠榮) and Ruqi Huang (黃汝琪) also studied in Chicago and obtained their doctorates in 1933 and 1937 respectively.

There were also Chinese students at other U. S. universities. For examples, Rong Sun (孫榮, 1921, Syracuse University), Zhaoan Zeng (曾昭安, 1925, Columbia University), Jinchang Hu (胡金昌, 1932, University of California), Shuting Liu (劉叔廷, 1930, University of Michigan), Hongji Zhang (張鴻基, 1933, University of Michigan), Piji Yuan (袁丕濟, 1933, University of Michigan), Xiping Zhou (周西屏, 1933, University of Michigan), and Qinglai Shen (沈青來, 1935, University of Michigan).

Also, some Chinese students studied complex function theory in France. Jinyi Zhao (趙進義) and Juxian Liu (劉俊賢) at Lyon (1930), and Huiguo Fan (范會國) at Paris. Qinglai Xiong (熊慶來, 1893–1969) was the most influential one among those Chinese students in France. Xiong taught in Tsinghua University in 1926 and served as the chairman of the department in 1928. In 1932, Xiong went to France and completed a national science doctorate a year later. In 1934, Xiong returned to Tsinghua University, resuming the position of the chairmanship. Xiong laid the foundation for complex function theory in China. Lo Yang (楊樂, 1939–) and Guanghou Zhang (張廣厚, 1937–1987) were his students.

At that time German and French mathematicians were the leaders of mathematics in the world. R. Courant in Göttingen had many Chinese students, such as Shizhen Wei (魏時珍, 1925), Gongjin Zhu (朱公謹, 1927), and Shuomin Jiang (蔣碩民, 1934). All of them studied the theory of differential equations.

Chiungtze Tsen (曾炯之, 1898–1940) was a student of Emmy Noether in Göttingen. Tsen's thesis was an important contribution to mathematics. Yuhuai Cheng (程毓淮) studied analysis and got his degree in Göttingen too. In the summer of 1935, Daren Wu (吳大任) went to Hamburg to study with Blaschke. Here at Hamburg he met with Shiing-Shen Chern (陳省身, 1911–2004); this was the third time that they were classmates. Wu returned to China in 1937.

Some students studied in Japan. Jiangong Chen (Kien-Kwong Chen 陳建功, 1893–1971) studied trigonometric series with Matsusaburo Fujiwara (藤原松三郎) at the Tohoku Imperial University; he got a Ph.D. in 1929. In the same university Buqing Su (Buchin Su 蘇步青, 1902–2003) studied projective differential geometry with Tadahiko Kubota (窪田忠彥) and got a Ph.D. in 1931. After returning to China, both Jiangong Chen and Buqing Su served as chairmen of the Department of Mathematics of Zhejiang University in succession. Chuan-Chih Hsiung (熊全治), Chaohao Gu (谷超豪, 1926–2012), and Hesheng Hu (胡和生), were the famous students of Buqing Su. Other students who also studied in Japan were Guoping Li (李國平), Yongfang Yang (楊永芳), Qianxiu Yu (余潛修), and Wenqing Li (李文清).

In summary, most of the first generation of Chinese students who studied abroad and were awarded Ph. D.'s returned to China. They played very crucial roles in the modernization of China. Among them, Chiungtze Tsen was in algebra; Wuzhi Yang was in number theory; Qinglai Xiong, Jiangong Chen, Mingfu Hu, and Gongjin Zhu were in analysis; Lifu Jiang, Guangyuan Sun and Buqing Su were in geometry; and Zehan Jiang was in topology.

Zehan Jiang was the chairman of the Department of Mathematics in Peking University. Lifu Jiang founded the Department of Mathematics in Nankai University in 1920; Guangyuan Sun was the chairman of the Department of Mathematics in Central University (the later Nanjing University); Jiangong Chen was the chairman of the Department of Mathematics in Zhejiang University; Zhaoan Zeng was the chairman of the Department of Mathematics in Wuhan University.

A lot of the masters in the western world, such as J. Hadamard, N. Wiener, W.J.E. Blaschke, E. Sperner, G.D. Birkhoff, and Osgood, were invited to visit China because of their connection to their former Chinese overseas students. These great mathematicians sped up the development of Chinese mathematics tremendously. Both Chern and Hua were influenced by their visits. In addition, the French mathematicians E. Borel and P. Painleve, as well as the British mathematician B.A.W. Russell (1920–1921) visited China; however, their visits were less influential.

The next generation of Chinese mathematician were the great masters: Shiing-Shen Chern, Loo-Keng Hua (Luogeng Hua 華羅庚, 1910–1985), and Wei-Liang Chow, who are of course well-known to everybody. The ascension of these shining stars signified that Chinese mathematicians were beginning to play a more crucial role on the world stage. A contemporary, Pao-Lu Hsu (Baolu Xu 許寶騄, 1910–1970), a world renowned statistician, was a pioneer of probability and statistics in China. He graduated from Tsinghua University in 1935.

During the period of the Western-Southern United University (a united university of Peking University, Tsinghua University and Nankai University during the second Sino-Japanese War in 1937–1945), Chern, Hua and Hsu trained a team of brilliant young mathematicians, such as Hsio-fu Tuan (Xuefu Duan 段學復, 1914–2005), Shianghaw Wang (Xianghao Wang 王湘浩, 1915–1993), Hsien-chung Wang (王憲鍾, 1918–1978), Zhida Yan (Chih-Ta Yen 嚴志達, 1917–1999), Kai-lai Chung (鍾開萊, 1917–1999), Hao Wang (王浩, 1921–1995), Zhexian Wan (萬哲先, 1927–); Kang Feng (馮康, 1920–1993) was another student in this period. He graduated from the Central University and became one of the founders of finite element methods.

In the mean time, several young students trained in Zhejiang University also made serious contributions to mathematics. This included Ching-Tsun Loo (Qingjun Lu 盧慶駿, 1913–1995), Minde Cheng (程民德, 1917–1998), Chung-Tao Yang (楊忠道, 1923–2005), Chaohao Gu (谷超豪), Daoxing Xia (夏道行), Hesheng Hu (胡和生), Yuan Wang (王元), and Zhongci Shi (石鐘慈) and Wenjun Wu (Wen-Tsun Wu,吳文俊). Some of them become core members of the Academia Sinica after the war.

Shiing-Shen Chern, Loo-Keng Hua, and Pao-Lu Hsu were educated in Tsinghua University before the War. All of them are the Chinese scholars to whom I have great respect. Chern's Chinese students in the US included Shantao Liao (San-Dao Liao 廖山濤, 1920–1997), Shiu-Yuen Cheng (鄭紹遠), Peter Li (李偉光) and myself. Loo-Keng Hua was in the US during 1946–1950; he was a professor in University of Illinois at Urbana from 1948 to 1950. But he decided to return to China in 1950. Back home, he created a school of distinguished scholars, such as Qi-Keng Lu (陸啟鏗), Jingrun Chen (陳景潤, 1933–1996), Yuan Wang, Zhexian Wan, etc. Together with other young mathematicians, they became the founders of the new Chinese mathematics community.

Epilogue

Compared with Japan, the development of modern mathematics in China has been rather slow. Although calculus was introduced in China earlier than in Japan, Japan quickly surpassed China. One of the reasons may be that the Japanese government decreed that students should learn from the western countries during the Meiji Restoration (1868). Most Chinese intellectuals then stuck to the belief that Chinese learning involved the essence of all true knowledge, while western learning was just utilitarian. Such an attitude kept them from pursuing western knowledge without reservation.

Besides studying geometry and analysis in England, Dairoku Kikuchi was deeply influenced by the English gentleman style education which he brought back to Japanese academia. Similarly, Teiji Takagi studied with the Göttingen masters, and he also succeeded in importing their mathematical research methods into Japan. After returning to Japan, Takagi concentrated on research for at least 15 years, becoming a world-class mathematician. His zeal and passion for mathematics and knowledge were on a higher level than those of the Chinese elites at the same time. As a matter of fact, the research works of those Chinese overseas students before 1935 that would be remembered in the history of mathematics are rather scarce; perhaps Tsen's theorem is the only one that has been lasting. Ironically, Chiungtze Tsen was not taken seriously by the authorities after returning to China and he passed away quite early.

Since the time of Dairoku Kikuchi, Japanese government paid great respect to its overseas students once they returned to Japan. These students devoted much of their energy to basic mathematics in middle schools, high schools, and in university education. For example, Takagi wrote as many as fourteen high school textbooks. Japanese mathematicians produced many pioneering works in 1940s; their works were comparable with those in the leading western countries. It is worth noting that in those days, before being promoted to an associate professor, almost every scholar in Japan had visited Europe or the US for some time so that they were exposed to the research frontier.

I have had the privilege of making the acquaintance of several Japanese mathematicians, e.g., Kiyoshi Ito, Kenkichi Iwasawa, Kunihiko Kodaira, Tosio Kato, Goro Shimura, Ichiro Satake, and Heisuke Hironaka. All of them are fine gentlemen who enjoy the pursuit of knowledge, and concentrate completely on academics. They have always earned my greatest admiration.

In contrast to the early period of modernization, most Chinese focused on applied technology without a deep passion for mathematics. In the 1920s, most overseas Chinese students were still ignorant of the frontier of mathematics.The foreign missionaries to China in the 19th century offered great help to translate text books for Chinese to learn. But they were not masters in mathematics and did not have the thorough training in mathematics necessary to hasten the maturation of Chinese mathematics. The books that they translated were basically textbooks for training mid-level scientists, and were not good enough for students who had serious interest in researching mathematics. That led to the unfortunate situation in which Chinese people didn't really know what the frontiers of science were. As to its education policies and systems, China was far behind Japan. After the first Sino-Japan War, most Chinese students went to Japan to study. In the early period when support from the Chinese Education and Culture Foundation was available, many Chinese students went to the US and some others to France and German.

It is evident that, in the early 20th century, the achievements in mathematics of Japan and the US lagged far behind those of Germany and France. However, most Chinese students chose to study in Japan and the USA instead of France and Germany, which showed the mistake of their educational policy. Fortunately, most of these students returned to China after graduation and took part in the construction of mathematics in the 1930s and afterwards.

It should be emphasized that the rapid growth of mathematics in Japan and the USA was closely linked to their education and research system. They succeeded in combining the English system and the German tradition of research universities. In a society that highly respects knowledge, most scholars will devote their life to academics.

The following example shows the different fates of students at Harvard. When the overseas Chinese students were at Harvard, their classmates included H. Whitney, and Morse (in topology), C. Morrey and J. L. Doob (researching in differential equations and probability theory). All of these American classmates became masters eventually, but the achievements of their Chinese classmates were far inferior.

After the rise of the People's Republic of China, Loo-Keng Hua built up a research team that was close to the international level in some areas of mathematics. But these efforts were ruined during the Cultural Revolution. In the past 30 years, the level of Chinese mathematics is still not comparable to the level of western mathematics. In fact, the present achievements of Chinese mathematicians have not regained the heights of the early masters such as Shiing-Shen Chern, Loo-Keng Hua, and Wei-Liang Chow. Alas, they choose to remain outside of China and most good Chinese graduate students still has the strong desire to go abroad. This is the present plight of mathematics in China.

I believe that recruitment of foreign masters of mathematics to China should be well-planned so that these foreign authorities might help us to train our elite graduate students. The crucial factor behind the U.S. success in academics is that U.S. universities stress both teaching and research for undergraduate and graduate students, which confirms the views of Confucius that teaching and research benefit each other. It is impossible to make advance research only with masters but without outstanding young students. Another issue is that, if the young people have no enthusiasm for knowledge, have no noble ideals, they will never make outstanding achievements. It is impossible to buy the zeal for knowledge with money alone.

We can learn a lot from this history. If one wants to make progress in research he or she must go to the most advanced institutions and devote all of his or her energy and time to the academic.

There has been a great amount of academic fraud in China over the past decade. What's worse, plagiarism was committed even by presidents of some universities or some members of the Chinese Academy of Science. Such behavior heavily destroyed the academic atmosphere. With corruption of leaders in the academic world, very few students will devote their energies to academics. Many students studying abroad and academicians are opinionated and preoccupied with personal revenue rather than academics. In essence, many of them lack interest in mathematics and despite their high profile within China, they have not reached the mathematical frontier. Due to different priorities, as well as power struggles, young scholars often suffer. Some managed to delve into knowledge within a small circle; but some gave up academic careers. With a corrupt academic atmosphere, it's not surprising that few Chinese mathematicians reach the world level.

The good news is that the country plans to put more funding into the cultivation of talent with the hope that by 2020 China will become a country full of talent. I am personally convinced that, with sufficient funds and an emphasis on the cultivation of talents, the academic atmosphere will change for the better; and China will catch up with the advanced countries. However, the Chinese proverb says that the education of talents takes a hundred years to bear fruit. Our leaders should not only continue to increase funds but also develop patience for young evaluable mathematician to grow. Only in this way can China develop academic program at the highest level.

In recent years, Korea and Vietnam have begun to increase funding on basic scientific research. It is estimated that young mathematicians from these countries will win Fields Medal this year in the International Congress of Mathematicians, 2010. China has many similarities in culture with these countries. When can China be able to cultivate their own excellent domestic mathematicians? That is what the government and the public care about the most.

Whether a Chinese mathematician can win a major international award is an important matter. However that is not the reason we do fundamental research, we should see a bit farther to make big achievements in basic scientific research. Confucianism often stresses the harmonious relationship between heaven's way and human affairs. I hope Chinese scholars can follow the teachings of our ancestors and reach this harmonious state.

Yet I also feel sad as a mathematician. Because many of my Chinese peers, some of whom are competent and talented, compete each other for petty profit but ignore the broader academic goal. Some Chinese mathematicians with high positions are very arrogant and despise the Third World scholars. A Chinese proverb says, a boat sailing against the current must forge ahead or it will be driven back. Academics have objective criteria so that we can distinguish the strengths and weaknesses of a scholar easily.

In the Han and Tang Dynasties, China was a superpower not only in economics and military, but also in culture. Asian countries viewed China as their parent country. Nowadays China becomes an economic power and is very influential in the world. However, Chinese mathematics, in turn, has regressed from the brilliant times led by Shiing-Shen Chern and Loo-Keng Hua in 1940s and 1960s.

The cultivation of young scholars will determine the fate of Chinese mathematics; and the cultivation of graduate students relies heavily on the education during secondary school. The government should note that most famous mathematicians in history had published important work before thirty. We should pay serious attention to the young generations.

Today we kindle our enthusiasm again in Tsinghua University. Let us forget the pursuit of fame and wealth, forget disputes between people, forget the competition among universities, and forget conflicts between nations. Let us establish a mathematical center dedicated to the exploration of knowledge and the pursuit of truth and beauty. I hope the renewed fire will not extinguish forever under the assistance of governments, universities, and friends from all over the world. Let us leave our own footprints in the boundless history of mathematics!

Appendix. Letters

from David Mumford, Brown University:

Dear Yau,

I just read your article on the lag of Chinese math with respect to Western math. I was astonished by your passionate attack on the contemporary scene of math In China and the final cry for new efforts, new directions. I had thought that Jiang Zemin had significantly modernized math, science and engineering in China and put their importance front and center.

With my limited understanding of Chinese culture, I would give a somewhat different spin on things, especially the contrast of China with Japan and the West. My sense is that Chinese scholars even today are very conscious of the fact that they have 3000 years of tradition behind them, that they are the Middle Kingdom. This seems to me to make it truly hard to change course. It seems to me to be a fact that, in China, math was never a valued or respected profession. It was occasionally a subject some officials happened to love and they advanced it. But mainly it was viewed as purely utilitarian. The one exception was its use in astronomy which was of great concern to all emperors because they needed accurate astronomical predictions, e.g. of eclipses. Although sometimes included in the imperial examinations, math was never as important as memorizing Confucius, writing beautiful classical essays and poems and knowing Chinese history. As I understand it, China has now and had from Han times at least, the most highly developed bureaucratic system in the world and advancing in this hierarchy was the only way for wealth and power. How can you expect this system to change rapidly and give prestige to Western scholarly fields? But surely it is changing now, especially because of the influence of the Hong Kong community.

By the way, a small comment: I was most interested in your description of the introduction of Chinese characters for symbols, mathematical, Greek and Roman letters. I find most striking how Chinese algebra advanced as far as it did without the introduction of any symbol for an unknown. In the Song dynasty, mathematicians represented polynomials exactly the way they are represented in a computer today: by an array of their coefficients. The array was made with sticks on the ground and there were no written formulas (as found for instance in Diophantus or the Bakhshali manuscript). The algorithms of Chinese algebra were centuries ahead of those in the West perhaps because of this method of representation. This seems to me a very striking difference between the Chinese and Indian/Middle Eastern/ Western approach to algebra. I'd like to pursue this further if I have some help translating.

David

From John Coates, Cambridge University:

Dear Yau,

I wonder if one key factor is the fact that there was internal peace and political stability in Japan from the Meiji period onwards, and this made it much easier for universities to develop and flourish there. In China, until 1949 there was great political instability and often fighting within China itself, and I suspect this made it difficult for Chinese universities to even function properly, let alone develop along western lines. Even after 1949, Mao's hostility and suspicions of intellectuals, culminating in the cultural revolution, meant it was very difficult for Chinese universities to develop. Despite China's ancient intellectual traditions, universities need peace and sympathy from the government to flourish.

Let me also send my very best wishes for the Chinese New Year today.

John