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CORRECTION TO "MEROMORPHIC SOLUTIONS OF SOME FUNCTIONAL EQUATIONS" *

WALTER BERGWEILER†, KATSUYA ISHIZAKI‡, AND NIRO YANAGIHARA§

In [1] we treated the functional equation

$$\sum_{j=0}^{n} a_j(z) f(c^j z) = Q(z), \tag{1.1}$$

where $0 < |c| < 1$ is a complex number, and $a_j(z)$, $j = 0, 1, \ldots, n$, and $Q(z)$ are rational functions, $a_0(z) \not\equiv 0$, $a_n(z) \equiv 1$. We mentioned that *if all coefficients of* (1.1) *are constant, then* (1.1) *has no transcendental meromorphic solution.* The argument given in [1], however, is not correct. The purpose of this note is to give a correct proof of this claim.

To do this we assume that (1.1) with constant coefficients a_j and Q possesses a meromorphic solution f. First we note that if f has a pole $z_0 \neq 0$, then, by (1.1), f has infinitely many poles of the form $c^j z_0$, $j \in \mathbb{N}$, so that the poles of f accumulate at 0, a contradiction. Thus the only possible pole of f is at 0.

We now argue analogously to the proof of Theorem 1.1 in [1]. As there we put $s = 1/c$ and we define

$$M_k = \max_{j=0,1,\ldots,k} M(|s|^j, f) + 1$$

and $L_k = \log M_k$ for $k \geq 0$. It follows from (1.1) that

$$|a_0| M(|s|^k, f) \leq \sum_{j=1}^{n} |a_j| M(|s|^{k-j}, f) + |Q| \leq \left(\sum_{j=1}^{n} |a_j| \right) M_{k-1} + |Q|$$

for $k \geq n$, and this implies that there exist constants A, B such that $M_k \leq A M_{k-1} + B$ for all $k \geq 1$. We deduce that $L_k \leq L_{k-1} + C$ for some $C > 0$ and all $k \geq 1$. It follows that $L_k \leq Ck + L_0$ and hence that

$$m(|s|^k, f) \leq L_k \leq Ck + L_0 = \frac{C}{\log|s|} \log\left(|s|^k\right) + L_0$$

for all $k \in \mathbb{N}$. We deduce that

$$T(r, f) = m(r, f) + N(r, f) = O(\log r)$$

for $r = |s|^k$, $k \in \mathbb{N}$, $k \to \infty$, and this implies that $T(r, f) = O(\log r)$ as $r \to \infty$ through any sequence of r-values. Thus f is rational.

REFERENCES

[1] W. BERGWEILER, K. ISHIZAKI, AND N. YANAGIHARA, *Meromorphic solutions of some functional equations*, Method Appl. Anal., 5:3 (1998), pp. 248–258.

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†Mathematisches Seminar, Christian-Albrechts-Universität zu Kiel, Ludewig-Meyn-Str. 4, D-24098 Kiel, Germany (bergweiler@math.uni-kiel.de).
‡Department of Mathematics, NIPPON Institute of Technology, 4-1 Gakuendai Miyashiro, Minamisaitama Saitama, 345 JAPAN (ishi@nit.ac.jp).
§Department of Mathematics, Faculty of Science, Chiba University, 1-33 Yayoi-cho Inage, Chiba 263, JAPAN (yanagi@math.s.chiba-u.ac.jp).