Methods and Applications of Analysis.
Vol. 21, No. 3, pp. 357–364, September 2014

005

SOME REMARKS ON YAU'S CONJECTURE AND COMPLEX PLATEAU PROBLEM*

RONG DU† AND YUN GAO‡

Dedicated to Professor Stephen S. T. Yau on the occasion of his 60th Birthday

Abstract. The invariant $g^{(1,1)}$ was introduced by Du and Yau for solving the complex Plateau problem. In this paper, we prove that this invariant never vanishes for any minimal elliptic surface singularity, which confirms Yau's conjecture for the strictly positivity of $g^{(1,1)}$. We also show that this invariant can be arbitrarily large. As an application of this invariant, we give a new criterion for the regularity problem of the Harvey-Lawson solution to the complex Plateau problem for a strongly pseudoconvex compact CR manifold of dimension 3.

Key words. Complex Plateau problem, CR manifold, Harvey-Lawson solution.

AMS subject classifications. 32V15, 32S05, 32S20.

1. Introduction.

One of the natural fundamental questions of complex geometry is to study the boundaries of complex varieties. For example, the famous classical complex Plateau problem asks which odd dimensional real sub-manifolds of $\mathbb{C}^N$ are boundaries of complex sub-manifolds in $\mathbb{C}^N$. Harvey and Lawson in their paper [Ha-La] proved that for any compact connected CR manifold X in $\mathbb{C}^N$, there is a unique complex variety V in $\mathbb{C}^N$ such that the boundary of V is X.

If X is a strongly pseudoconvex CR manifold of dimension $2n-1$, $n \geq 2$, contained in the boundary of a bounded strongly pseudoconvex domain D in $\mathbb{C}^N$, then V has boundary regularity at every point of X and V has only isolated singularities in $V - X$ ([Lu-Ya1]). In 1981, Yau [Ya1] solved the classical complex Plateau problem for the case $n \geq 3$ by calculation of Kohn-Rossi cohomology groups $H^{p,q}_{KR}(X)$. More precisely, suppose that X is a compact connected strongly pseudoconvex CR manifold of real dimension $2n-1$, $n \geq 3$, in the boundary of a bounded strongly pseudoconvex domain D in $\mathbb{C}^{n+1}$. Then X is a boundary of the complex sub-manifold $V \subset D - X$ if and only if Kohn-Rossi cohomology groups $H^{p,q}_{KR}(X)$ are all zero for $1 \leq q \leq n-2$ (see Theorem 3.1).

For $n = 2$, i.e. X is a 3-dimensional CR manifold, the classical complex Plateau problem remains unsolved for over a quarter of a century. The main difficulty is that the Kohn-Rossi cohomology groups are infinite dimensional in this case. If we assume that V is a complex variety with X as its boundary then the singularities of V are surface singularities. In [Lu-Ya2], the holomorphic De Rham cohomology, which is derived form Kohn-Rossi cohomology, is considered to determine what kind of singularities can exist in V. In [Lu-Ya2], Luk and Yau proved that if $(V, 0)$ is a Gorenstein surface singularity with vanishing s-invariant, then $(V, 0)$ is a quasihomogeneous singularity whose link is rational homology sphere. In [Lu-Ya2] they proved

*Received September 16, 2012; accepted for publication March 26, 2013.

†Department of Mathematics, Shanghai Key Laboratory of PMMP, East China Normal University, Rm. 312, Math. Bldg, No. 500, Dongchuan Road, Shanghai, 200241, P. R. China (rdu@math.ecnu.edu.cn). The Research Sponsored by The National Natural Science Foundation of China, Shanghai Pujiang Program and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry.

‡Department of Mathematics, Shanghai Jiao Tong University, Shanghai 200240, P. R. China (gaoyunmath@sjtu.edu.cn). The Research Sponsored by National Natural Science Foundation of China and The Innovation Program of Shanghai Municipal Education Commission.

that if X is a strongly pseudoconvex compact Calabi-Yau CR manifold of dimension 3 contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^N$ and the holomorphic De Rham cohomology $H_h^2(X)$ vanishes, then X is a boundary of a complex variety V in D with only isolated singularities in the interior and the normalization of these singularities are Gorenstein surface singularities with vanishing s-invariants (see Theorem 3.2). As a corollary of this theorem, they got that if $N = 3$, the variety V bounded by X has only isolated quasi-homogeneous singularities such that the dual graphs of the exceptional sets in the resolution are star shaped and all the curves are rational (see Corollary 3.3). Recently, in [Du-Ya1], the first author and Yau introduced a new CR invariant $g^{(1,1)}(X)$. This new invariant gives a necessary and sufficient condition for the variety V bounded by X with the holomorphic De Rham cohomology $H_h^2(X) = 0$ being smooth.

The CR invariant $g^{(1,1)}(X)$ is deduced from the corresponding invariants $g^{(1,1)}$'s of singularities. The invariant $g^{(1,1)}$ of a surface singularity was first defined and studied by the first author and Yau ([Du-Ya1]). Moreover, Yau has the following conjecture.

Conjecture. For each normal surface singularity, the invariant $g^{(1,1)}$ is strictly positive.

The first author and Yau showed that this numerical invariant is strictly positive when a surface singularity has $\mathbb{C}^*$-action. They also gave explicit calculations for $g^{(1,1)}$'s for rational double points and cyclic quotient singularities and proved that they are exact 1. In [Du-Ga], the authors prove that for each rational surface singularity, the invariant $g^{(1,1)}$ also never vanishes and for each rational triple point it is also equal to 1. In this paper, we confirm Yau's conjecture for minimal elliptic singularities and show that $g^{(1,1)}$ can be arbitrarily large.

Theorem A.
- The invariant $g^{(1,1)}$ *can be arbitrarily large.*
- *For minimal elliptic surface singularity,* $g^{(1,1)} \geq 1$.

As an application of the CR invariant $g^{(1,1)}(X)$ which is deduced from the invariants of singularities $g^{(1,1)}$'s, we give a new criterion for regularity problem of the Harvey-Lawson solution to the complex Plateau problem for a strongly pseudoconvex compact CR manifold of dimension 3.

Theorem B. *Let X be a strongly pseudoconvex compact rational CR manifold of dimension* 3. *Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^N$. If there is a non-nilpotent derivation in $\Gamma(\widehat{T}(X))$, then X is a boundary of the complex sub-manifold (up to normalization) $V \subset D - X \Longleftrightarrow$ $g^{(1,1)}(X) = 0$.*

2. Invariants of singularities. Let V be an n-dimensional complex analytic subvariety in $\mathbb{C}^N$ with only isolated singularities. In [Ya2], Yau considered four kinds of sheaves of germs of holomorphic p-forms

1. $\bar{\Omega}_V^p := \pi_* \Omega_M^p$, where $\pi : M \longrightarrow V$ is a resolution of singularities of V.
2. $\bar{\bar{\Omega}}_V^p := \theta_* \Omega_{V \backslash V_{sing}}^p$ where $\theta : V \backslash V_{sing} \longrightarrow V$ is the inclusion map and V_{sing} is the singular set of V.
3. $\Omega_V^p := \Omega_{\mathbb{C}^N}^p / \mathscr{K}^p$, where $\mathscr{K}^p = \{f\alpha + dg \wedge \beta : \alpha \in \Omega_{\mathbb{C}^N}^p; \beta \in \Omega_{\mathbb{C}^N}^{p-1}; f, g \in \mathscr{I}\}$ and $\mathscr{I}$ is the ideal sheaf of V in $\mathbb{C}^N$.
4. $\widetilde{\Omega}_V^p := \Omega_{\mathbb{C}^N}^p / \mathscr{H}^p$, where $\mathscr{H}^p = \{\omega \in \Omega_{\mathbb{C}^N}^p : \omega|_{V \backslash V_{sing}} = 0\}$.

Ω_V^p is Grauert-Grothendieck sheaf of germs of holomorphic p-form on V. In case V is a normal variety, the dualizing sheaf ω_V of Grothendieck is actually the sheaf $\bar{\bar{\Omega}}_V^n$. Clearly Ω_V^p, $\widetilde{\Omega}_V^p$ are coherent. $\bar{\Omega}_V^p$ is a coherent sheaf because π is a proper map. $\bar{\bar{\Omega}}_V^p$ is also a coherent sheaf by a theorem of Siu (cf. Theorem A of [Si]).

In [Du-Ya1] , another sheave $\bar{\bar{\Omega}}_V^{1,1}$ was considered.

Definition 2.1. Let $(V, 0)$ be a Stein germ of a 2-dimensional analytic space with an isolated singularity at 0. Define a sheaf of germs $\bar{\bar{\Omega}}_V^{1,1}$ by the sheaf associated to the presheaf

$$U \mapsto < \Gamma(U, \bar{\bar{\Omega}}_V^1) \wedge \Gamma(U, \bar{\bar{\Omega}}_V^1) >,$$

where U is an open set of V.

In [Du-Ya1], the first author and Yau also showed that this sheave is coherent and found the relation between $\bar{\bar{\Omega}}_V^{1,1}$ and $\bar{\bar{\Omega}}_V^2$ by short exact sequence stated in the following lemma.

Lemma 2.2. *([Du-Ya1])* Let V be a 2-dimensional Stein space with 0 as its only singular point at 0. Let $\pi : (M, A) \to (V, 0)$ be a resolution of the singularity with A as the exceptional set. Then $\bar{\bar{\Omega}}_V^{1,1}$ is coherent and there is a short exact sequence

$$0 \longrightarrow \bar{\bar{\Omega}}_V^{1,1} \longrightarrow \bar{\bar{\Omega}}_V^2 \longrightarrow \mathscr{G}^{(1,1)} \longrightarrow 0 \tag{2.1}$$

where $\mathscr{G}^{(1,1)}$ is a sheaf supported on the singular point of V. Let

$$G^{(1,1)}(M \backslash A) := \Gamma(M \backslash A, \Omega_M^2) / < \Gamma(M \backslash A, \Omega_M^1) \wedge \Gamma(M \backslash A, \Omega_M^1) >, \tag{2.2}$$

then $g^{(1,1)}(0) := dim \mathscr{G}_0^{(1,1)} = dim G^{(1,1)}(M \backslash A)$.

The invariant is independent on the resolutions of a singularity. We will omit 0 in $g^{(1,1)}(0)$ if there is no confusion from the context.

Yau conjectured that for each normal surface singularity, the invariant $g^{(1,1)}$ is strictly positive. This conjecture was confirmed when the singularities are with $\mathbb{C}^*$-action.

Theorem 2.3. *([Du-Ya1]) Let V be a 2-dimensional Stein space with 0 as its only normal singular point with $\mathbb{C}^*$-action, then $g^{(1,1)} \geq 1$.*

Theorem 2.3 is the crucial part for the solution of the classical complex Plateau problem in [Du-Ya1].

In [Du-Ga], the authors also showed that $g^{(1,1)}$ is strictly positive for each rational singularity. Moreover, for rational double points, rational triple points and quotient singularities $g^{(1,1)}$'s are all equal to 1. (see [Du-Ya2] or [Du-Lu-Ya]).

Theorem 2.4. *([Du-Ga]) Let V be a 2-dimensional Stein space with 0 as its only rational singularity, then $g^{(1,1)} \geq 1$.*

It is well known that the minimal elliptic surface singularities are the simplest singularities except for the rational singularities. The definition of a minimal elliptic surface singularity was first introduced by H. Laufer (see [Lau]).

Definition 2.5. Let $(V, 0)$ be a 2-dimensional Stein space with 0 as its only normal singular point and $\pi : M \to V$ be the minimal resolution of V with A as its exceptional set. Then $(V, 0)$ is called minimal elliptic if $(V, 0)$ is elliptic and every connected proper subvariety of A is the exceptional set for a rational singularity.

Minimal elliptic singularities can be characterized without explicit use the resolution as follows.

THEOREM 2.6. *([Lau]) The singularity $(V, 0)$ is minimal elliptic if and only if it is Gorenstein and the geometry genus $p_g = 1$.*

We will show that the invariant $g^{(1,1)}$ is strictly positive for each minimal elliptic surface singularity.

THEOREM 2.7. *For each minimal elliptic surface singularity, $g^{(1,1)} \geq 1$.*

Proof. Let $(V, 0)$ be a 2-dimensional Stein space with 0 as its only minimal elliptic singularity. So the geometry genus $p_g = 1$ and the singularity is Gorenstein by Theorem 2.6. Let $\pi : (M, A) \to (V, 0)$ be a resolution of the singularity with A as exceptional set. From [St-St], we know that the irregularity q is less than or equal to the geometry genus p_g. Hence $q = 0$ or 1.

If $q = 0$, then any holomorphic 1-form in $\Gamma(M\backslash A, \Omega^1_M)$ can be extended across the exceptional set A, i.e. $\Gamma(M\backslash A, \Omega^1_M) = \Gamma(M, \Omega^1_M)$. So

$$\frac{\Gamma(M\backslash A, \Omega^2_M)}{< \Gamma(M\backslash A, \Omega^1_M) \wedge \Gamma(M\backslash A, \Omega^1_M) >} = \frac{\Gamma(M\backslash A, \Omega^2_M)}{< \Gamma(M, \Omega^1_M) \wedge \Gamma(M, \Omega^1_M) >}.$$

Therefore

$$\begin{aligned} g^{(1,1)} &= \dim \frac{\Gamma(M\backslash A, \Omega^2_M)}{< \Gamma(M\backslash A, \Omega^1_M) \wedge \Gamma(M\backslash A, \Omega^1_M) >} \\ &= \dim \frac{\Gamma(M\backslash A, \Omega^2_M)}{< \Gamma(M, \Omega^1_M) \wedge \Gamma(M, \Omega^1_M) >} \geq \frac{\Gamma(M\backslash A, \Omega^2_M)}{\Gamma(M, \Omega^2_M)} \\ &= p_g = 1. \end{aligned}$$

If $q = 1$, by Theorem 3.2 in [Wa], the singularity has $\mathbb{C}^*$-action since the singularity is Gorenstein. So by Theorem 2.3, $g^{(1,1)} \geq 1$. □

REMARK 2.8. It is well known that cusp singularities are included in the minimal elliptic singularities, so for each cusp singularity $g^{(1,1)} \geq 1$. In fact, the irregularity is equal to 0 for cusp singularities.

PROPOSITION 2.9. *The invariant $g^{(1,1)}$ can be arbitrarily large.*

Proof. Recall that Wahl's example 4.6 in [Wa]: the singularity is defined by taking positive weight deformation of the singularity $z^2 + x^{2a+1} + y^{2a+2} = 0$, where a is an integer and $a \geq 1$. The geometry genus of the singularity is $p_g = a(a-1)/2$ and the irregularity is $q = 0$. As same as the arguments in the above theorem, $g^{(1,1)} \geq a(a-1)/2$. □

3. Complex Plateau problem of 3 dimensional strongly pseudoconvex *CR* manifold. Suppose that X is a strongly pseudoconvex CR manifold. For the definitions and results stated in this section, we recommend the preliminaries in [Du-Ya1]. If X is a strongly pseudoconvex CR manifold of dimension $2n-1$, $n \geq 2$, contained in the boundary of a bounded strongly pseudoconvex domain D in $\mathbb{C}^N$, then

V has boundary regularity at every point of X and V has only isolated singularities in $V - X$ ([Lu-Ya1]). In 1981, Yau ([Ya1]) solved the classical complex Plateau problem of hypersurface type for the case $n \geq 3$.

THEOREM 3.1. *([Ya1]) Let X be a compact connected strongly pseudoconvex CR manifold of real dimension $2n - 1$, $n \geq 3$, in the boundary of a bounded strongly pseudoconvex domain D in $\mathbb{C}^{n+1}$. Then X is a boundary of the complex sub-manifold $V \subset D - X$ if and only if Kohn-Rossi cohomology groups $H^{p,q}_{KR}(X)$ are zeros for $1 \leq q \leq n-2$.*

In 2007, Luk and Yau ([Du-Ya1]) introduced so called s-invariant in order to solve the complex Plateau problem as $n = 2$. But they could not even give a sufficient condition on the boundary such that it determines the smoothness in the interior.

THEOREM 3.2. *([Lu-Ya2]) Let X be a strongly pseudoconvex compact Calabi-Yau CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^N$. If the holomorphic De Rham cohomology $H^2_h(X) = 0$, then X is a boundary of a complex variety V in D with only isolated singularities in the interior and the normalization of these singularities are Gorenstein surface singularities with vanishing s-invariant.*

COROLLARY 3.3. *([Lu-Ya2]) Let X be a strongly pseudoconvex compact CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^3$. If the holomorphic De Rham cohomology $H^2_h(X) = 0$, then X is a boundary of a complex variety V in D with boundary regularity and V has only isolated quasi-homogeneous singularities such that the dual graphs of the exceptional sets in the resolution are star shaped and all the curves are rational.*

Recently, the first author and Yau used new invariants $g^{(1,1)}$'s for singularities to relate to a new CR invariant $g^{(1,1)}(X)$([Du-Ya1]).

DEFINITION 3.4. Suppose that X is a compact connected strongly pseudoconvex CR manifold of real dimension 3.

$$G^{(1,1)}(X) := \mathscr{S}^2(X)/ < \mathscr{S}^1(X) \wedge \mathscr{S}^1(X) > \tag{3.1}$$

where $\mathscr{S}^p$ are modules of holomorphic sections of $\wedge^p(\widehat{T}(X)^*)$ and $\widehat{T}(X)^*$ is the holomorphic cotangent bundle of X. Then we set

$$g^{(1,1)}(X) := dim G^{(1,1)}(X). \tag{3.2}$$

LEMMA 3.5. *([Du-Ya1]) Let X be a compact connected strongly pseudoconvex CR manifold of real dimension 3, which bounds a strongly pseudoconvex variety V with only isolated singularities $\{0_1, \cdots, 0_k\}$ in $\mathbb{C}^N$. Then $g^{(1,1)}(X) = \sum_i g^{(1,1)}(0_i)$.*

Note that this invariant $g^{(1,1)}(X)$ can be calculated on X directly. In [Du-Ya1], we use this CR invariant to give the sufficient and necessary condition for the variety bounded by a Calabi-Yau CR manifold X being smooth if $H^2_h(X) = 0$.

THEOREM 3.6. *([Du-Ya1]) Let X be a strongly pseudoconvex compact Calabi-Yau CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^N$ with $H^2_h(X) = 0$. Then X is a* *boundary of the complex sub-manifold up to normalization $V \subset D - X$ with boundary regularity if and only if $g^{(1,1)}(X) = 0$.*

THEOREM 3.7. *([Du-Ya1]) Let X be a strongly pseudoconvex compact CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^3$ with $H_h^2(X) = 0$. Then X is a boundary of the complex sub-manifold $V \subset D - X$ if and only if $g^{(1,1)}(X) = 0$.*

In this paper, we give a new criterion for the regularity problem of the Harvey-Lawson solution to the complex Plateau problem for a strongly pseudoconvex compact CR manifold of dimension 3.

THEOREM 3.8. *Let X be a strongly pseudoconvex compact CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^N$. If there is a non-nilpotent derivation in $\Gamma(\widehat{T}(X))$ then X is a boundary of the complex sub-manifold (up to normalization) $V \subset D - X \iff g^{(1,1)}(X) = 0$.*

Proof. $(\Rightarrow)$: If V is smooth, the result is trivial.
$(\Leftarrow)$:Suppose that V is in $\mathbb{C}^N$ such that the boundary of V is X. Let

$$\pi : (M, A_1, \cdots, A_k) \to (V, 0_1, \cdots, 0_k)$$

be the minimal good resolutions of the singularities with $A_i = \pi^{-1}(0_i)$, $1 \le i \le k$, as exceptional sets and $A = \cup_{i=1}^k A_i$. Denote V_i to be a little Stein neighborhood of 0_i, with 0_i as its only singularity and $M_i = \pi^{-1}(V_i)$. Take a one-convex exhausting function ϕ on M such that $\phi \ge 0$ on M and $\phi(y) = 0$ if and only if $y \in A$. Set $N_r = \{y \in M, \phi(y) \ge r\}$. Since $X = \partial M$ is strictly pseudoconvex, any holomorphic tangent field $\theta \in \Gamma(\widehat{T}(X))$ can be extended to a one side neighborhood of X in M, where $\Gamma(\widehat{T}(X))$ is the set of holomorphic cross sections of $\widehat{T}(X)$. Hence, θ can be viewed as a holomorphic tangent field on N_r, i.e. an element in $\Gamma(N_r, T_{N_r})$, where T_{N_r} is a holomorphic tangent bundle on N_r. By Andreotti and Grauert's result in [An-Gr], $\Gamma(N_r, T_{N_r})$ is isomorphic to $\Gamma(M\backslash A, T_M)$, where T_M is a holomorphic tangent bundle on M. From the condition that there is a non-nilpotent derivation in $\Gamma(\widehat{T}(X))$, we know that there also exists a non-nilpotent derivation D in $\Gamma(M\backslash A, T_M)$ and D restricts to each M_i is also non-nilpotent. Recall that any minimal good resolution is equivariant, i.e., $\Gamma(M_i\backslash A_i, T_{M_i}) = \Gamma(M_i, T_{M_i}) = T_{V_i, 0_i}$. Hence there exists a non-nilpotent derivation in each $T_{V_i, 0_i}$. By a Theorem of G. Scheja and H. Wiebe (see [Sc-Wi]) each the singularity $(V_i, 0_i)$ has $\mathbb{C}^*$-action. The proof this theorem is now a consequence of Lemma 3.5 and Theorem 2.3. □

COROLLARY 3.9. *Let X be a strongly pseudoconvex compact CR manifold of dimension 3. Suppose that X is contained in the boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^3$. If there is a non-nilpotent derivation in $\Gamma(\widehat{T}(X))$ then X is a boundary of the complex sub-manifold $V \subset D - X \iff g^{(1,1)}(X) = 0$.*

Proof. Because every hypersurface singularity is normal, the result follows from Theorem 3.8 directly. □

The invariant $g^{(1,1)}(X)$ can also control the number of singularities in the variety which X bounds in some sense.

COROLLARY 3.10. *Let X be a strongly pseudoconvex compact CR manifold of dimension 3 which is a boundary of a variety V. Suppose that X is contained in the* *boundary of a strongly pseudoconvex bounded domain D in $\mathbb{C}^3$. If $H_h^2(X) = 0$ or there is a non-nilpotent derivation in $\Gamma(\widehat{T}(X))$. Then the number of singularities of variety V is less than or equal to $g^{(1,1)}(X)$.*

Proof. The result follows from Lemma 3.5, Theorem 3.7 and Theorem 3.8 directly. □

REFERENCES

[Du-Ga] Du, R. and Gao, Y., *Strict positivity of new invariants for rational singularities*, Pacific Journal of Mathematics, 269:1 (2014), pp. 73–97.

[Du-Lu-Ya] R. Du and S. S.-T. Yau, *New invariants for complex manifolds and isolated singularities*, Communications in Analysis and Geometry, 19:5 (2011), pp. 991–1021.

[Du-Ya1] R. Du and S. S.-T. Yau, *Kohn-Rossi cohomology and its application to the complex Plateau problem, III*, Journal of Differential Geometry, 90:2 (2012), pp. 251–266.

[Du-Ya2] R. Du and S. S.-T. Yau, *New invariants for complex manifolds, singularities, and CR manifolds with applications*, Fifth International Congress of Chinese Mathematicians. Part 1, 2, pp. 431–445, AMS/IP Stud. Adv. Math., 51, pt. 1, 2, Amer. Math. Soc., Providence, RI, 2012.

[Ha-La] R. Harvey and B. Lawson, *On boundaries of complex analytic varieties* I, Ann. of Math., 102 (1975), pp. 233–290.

[Lau] H. Laufer, *On Minimally Elliptic Singularities*, American Journal of Mathematics, 99:5 (1977), pp. 1257–1295.

[Lu-Ya1] H. S. Luk and S. S.-T. Yau, *Counterexample to boundary regularity of a strongly pseudoconvex CR manifold: An addendum to the paper of Harvey-Lawson*, Annals of Mathematics, 148 (1998), pp. 1153–1154.

[Lu-Ya2] H. S. Luk and S. S.-T. Yau,*Kohn-Rossi cohomology and its application to the complex Plateau problem, II*, J. Differential Geometry, 77 (2007), pp. 135–148.

[Sc-Wi] G. Scheja and H. Wiebe, *Sur Chevalley-Zerlegung von Derivationen*, Manuscripta Math., 33 (1980), pp. 159–176.

[Si] Y.-T. Siu, *Analytic sheaves of local cohomology*, Trans. AMS, 148 (1970), pp. 347–366.

[St-St] D. V. Straten and J. Steenbrink, *Extendability of holomorphic differential forms near isolated hypersurface singularities*, Abh. Math. Sem. Univ. Hamburg, 55 (1985), pp. 97–110.

[Wa] J. M. Wahl, *A characterization of quasi-homogeneous Gorenstein surface singularities*, Compositio Math., 55 (1985), pp. 269–288.

[Ya1] S. S.-T. Yau, *Kohn-Rossi cohomology and its application to the complex Plateau problem, I*, Ann. of Math., 113 (1981), pp. 67–110.

[Ya2] S. S.-T. Yau, *Various numerical invariants for isolated singularities*, Amer. J. Math., 104:5 (1982), pp. 1063–1110.