

**SYMPLECTICALLY ASPHERICAL  
MANIFOLDS WITH NONTRIVIAL  $\pi_2$**

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ABSTRACT. We construct closed symplectic manifolds for which spherical classes generate arbitrarily large subspaces in 2-homology, such that the first Chern class and cohomology class of the symplectic form both vanish on all spherical classes. We construct both Kähler and non-Kähler examples, and show independence of the conditions that these two cohomology classes vanish on spherical homology. In particular, we show that the symplectic form can pair trivially with all spherical classes even when the Chern class pairs nontrivially.

Floer’s original approach to the Arnol’d Conjecture [F] worked for closed symplectic manifolds  $(X, \omega)$  satisfying the additional hypotheses that  $[\omega]$  and  $c_1(\omega)$  in  $H^2(X; \mathbb{R})$  should vanish on all spherical homology classes. While more recent work has avoided these hypotheses in this particular context, elsewhere the question has persisted of which symplectic manifolds satisfy the above hypotheses. Clearly the hypotheses are true for aspherical manifolds, but there are apparently no other published examples for which  $[\omega]$  vanishes on all spherical classes, prompting various people to ask whether such “symplectically aspherical” manifolds are always aspherical. In this note, we answer the question in the negative. (The author has been informed that J. Kollár has also reached this conclusion — see Remark 8.) To be more precise, let  $\Pi(X) = \text{span}_{\mathbb{R}}(\text{Im}(\pi_2(X) \rightarrow H_2(X; \mathbb{R})))$  denote the subspace of  $H_2(X; \mathbb{R})$  spanned by spherical classes. We construct families of closed, symplectic 4-manifolds  $(X, \omega)$  for which  $\Pi(X)$  has arbitrarily large dimension, but both  $\omega$  and  $c_1(\omega)$  vanish on  $\Pi(X)$ . We produce such a family for which each  $\omega$  is a Kähler form, and a different family for which no  $X$  is homotopy equivalent to a Kähler manifold. We also show that the conditions that  $\omega$  and  $c_1(\omega)$  vanish on  $\Pi(X)$  are independent — in particular, we can have  $\omega|_{\Pi(X)} = 0$  but  $c_1(\omega)|_{\Pi(X)} \neq 0$ . Note that since  $\Pi(X) = H_2(X; \mathbb{R})$  when  $X$  is simply connected, any closed symplectic manifold with  $\omega|_{\Pi(X)} = 0$  must have infinite fundamental group. The question of which topologies (e.g. fundamental groups) are realized by symplectically aspherical manifolds seems highly nontrivial.

Our construction is based on symplectic branched coverings. As observed by Gromov ([G], 3.4.4(E)), symplectic structures can be lifted under branched

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Received September 24, 1998.

Partially supported by NSF grant DMS-9625654.

coverings with symplectic branch loci. The following lemma pins down the details. We denote the Poincaré duality isomorphism by  $PD$ .

**Lemma 1.** *Let  $(X, \omega)$  be a closed symplectic manifold with a codimension-2 symplectic submanifold  $B$ . Suppose that  $\pi : \tilde{X} \rightarrow X$  is a branched covering map with branch locus  $B$ , and the manifold  $\pi^{-1}(B) \subset \tilde{X}$  has components  $B_1, \dots, B_n$ , oriented via the symplectic structure on  $B$ . Let  $d_i > 0$  denote the multiplicity of  $\pi$  along  $B_i$ . Then there is a symplectic form  $\tilde{\omega}$  on  $\tilde{X}$  with  $[\tilde{\omega}] = \pi^*[\omega] \in H^2(\tilde{X}; \mathbb{R})$ , and  $c_1(\tilde{\omega}) = \pi^*c_1(\omega) + \sum(1 - d_i)PD[B_i] \in H^2(\tilde{X}; \mathbb{Z})$ .*

*Proof.* The form  $\pi^*\omega$  is symplectic on  $\tilde{X} - \bigcup_{d_i \geq 2} B_i$ . By standard symplectic topology ([MS] Example 5.10 and Theorem 3.29), we can find a closed tubular neighborhood  $N$  of  $B$  in  $X$  with a Hamiltonian circle action, whose fixed set is  $B = H^{-1}(0)$  for the Hamiltonian  $H : N \rightarrow [0, \varepsilon]$ . Symplectic reduction of each  $H^{-1}(t)$  gives a family  $\omega_t$ ,  $t \in [0, \varepsilon]$ , of symplectic forms on  $B$ , with  $\omega_0 = \omega|_B$ . For a suitable family  $\alpha_t$  of 1-forms on the total space of the circle bundle  $p : \partial N \rightarrow B$ , the formula  $p^*\omega_t + \alpha_t \wedge dt$  defines a symplectic form on  $[0, \varepsilon] \times \partial N$ , and symplectic cutting at  $t = 0$  recovers the original form  $\omega$  on  $N$  ([L], cf. also [MS] Example 5.10). Now we use  $\pi$  to lift this structure to a tubular neighborhood  $N_i$  of each component  $B_i$  of  $\pi^{-1}(B)$ . The form and  $S^1$ -action on  $[0, \varepsilon] \times \partial N$  lift to a form  $\omega'$  and Hamiltonian circle action on  $[0, \varepsilon] \times \partial N_i$  with Hamiltonian  $d_i H \circ \pi$ . Since  $\omega'$  agrees with  $\pi^*\omega$  on  $(0, \varepsilon] \times \partial N_i \approx N_i - B_i$ , symplectic cutting at 0 yields a symplectic form  $\tilde{\omega}$  on  $\tilde{X}$  that agrees with  $\pi^*\omega$  outside of  $\bigcup N_i$  (and with  $\tilde{\omega} = \pi^*(\omega|_B)$  on each  $B_i$ ). To check that  $[\tilde{\omega}] = \pi^*[\omega]$ , we evaluate both on an arbitrary homology class, which we can assume is represented by a surface  $F \subset \tilde{X}$  intersecting each  $N_i$  in a union of  $S^1$ -invariant normal disks. For each such disk  $D$ , it is easy to see that  $\int_D \tilde{\omega} = d_i \int_{\pi(D)} \omega$ , so since  $\tilde{\omega} = \pi^*\omega$  on  $\tilde{X} - \bigcup \text{int } N_i$ , we have  $\langle \tilde{\omega}, [F] \rangle = \langle \omega, \pi_*[F] \rangle = \langle \pi^*\omega, [F] \rangle$ , as required.

To compute  $c_1(\tilde{\omega})$ , fix an almost-complex structure  $J$  on  $X$  tamed by  $\omega$ , with  $B$  and the fibers of  $N$   $J$ -holomorphic. Then  $J$  lifts to a  $C^0$  almost-complex structure  $\tilde{J}$  on  $\tilde{X}$  tamed by  $\tilde{\omega}$ . Now  $c_1(\omega) = c_1(J)$  is Poincaré dual to the zero locus of a generic section of  $\Lambda TX$ , where  $\Lambda$  denotes the complex top exterior power. On  $N$ , we have  $TX = TB \oplus \nu B$  and  $\Lambda TX = \Lambda TB \otimes \nu B$ . Let  $\sigma : B \rightarrow \Lambda TB$  be a generic section, and let  $\tau : N \rightarrow \nu B$  be given by the Tubular Neighborhood Theorem (so  $\tau^{-1}(0) = B$ ). Extend the section  $\sigma \otimes \tau$  of  $\Lambda TX|_N$  generically over all of  $X$ . The resulting zero locus exhibits  $c_1(J)$  as  $PD[C] + PD[B]$ , for some codimension-2 submanifold  $C$  of  $X$  intersecting  $B$  transversely. Since  $\tau$  lifts to a continuous map  $N_i \rightarrow \nu B_i$  vanishing with multiplicity 1 on  $B_i$ , the corresponding section of  $\Lambda T\tilde{X}$  exhibits  $c_1(\tilde{\omega}) = c_1(\tilde{J})$ , as  $\pi^*PD[C] + \sum PD[B_i] = \pi^*(c_1(J) - PD[B]) + \sum PD[B_i] = \pi^*c_1(J) + \sum(1 - d_i)PD[B_i]$ .  $\square$

**Example 2.** Let  $X = F_1 \times F_2$  be a product of two surfaces of nonzero genus, with a product symplectic form  $\omega$ . Fix integers  $m_1, m_2 \geq 1$ , and  $d \geq 2$ . Choose  $m_1 d$  surfaces in  $X$  of the form  $\{p\} \times F_2$  and  $m_2 d$  surfaces  $F_1 \times \{q\}$ , and let

$B^*$  denote their union. Let  $B \subset X$  be the smooth symplectic surface obtained by smoothing the  $m_1 m_2 d^2$  positive double points of  $B^*$  via the local model  $\{z_1 z_2 = 0\} \mapsto \{z_1 z_2 = \varepsilon\}$ . Let  $\pi : \tilde{X} \rightarrow X$  be the  $d$ -fold cyclic branched covering of  $X$ , branched along  $B$  with multiplicity  $d$  on  $\pi^{-1}(B)$ . (There is only one such covering, up to diffeomorphisms of  $X$ .) The following two theorems immediately show that  $\dim \Pi(\tilde{X}) \geq m_1 m_2 d^2 (d - 1)$ , but the form  $\tilde{\omega}$  given by Lemma 1 satisfies  $\tilde{\omega}|_{\Pi(\tilde{X})} = c_1(\tilde{\omega})|_{\Pi(\tilde{X})} = 0$ .

**Theorem 3.** *For  $(\tilde{X}, \tilde{\omega})$  as in Lemma 1, if  $\omega|_{\Pi(X)} = 0$ , (e.g., if  $\pi_2(X) = 0$ ), then  $\tilde{\omega}|_{\Pi(\tilde{X})} = 0$ . If  $\pi_2(X) = 0$  and  $\pi^{-1}(B)$  is connected, then  $c_1(\tilde{\omega})|_{\Pi(\tilde{X})} = 0$ .*

*Proof.* If  $\alpha \in H_2(\tilde{X}; \mathbb{Z})$  is represented by a map of a sphere, then so is  $\pi_* \alpha \in H_2(X; \mathbb{Z})$ . Thus  $\langle [\tilde{\omega}], \alpha \rangle = \langle [\omega], \pi_* \alpha \rangle = 0$ . If, in addition,  $\pi_2(X) = 0$ , then  $\pi_* \alpha = 0$ , so the intersection number  $\pi_* \alpha \cdot B$  vanishes. Connectedness of  $\pi^{-1}(B)$  now implies that  $\alpha \cdot \pi^{-1}(B) = 0$  and  $\langle c_1(\tilde{\omega}), \alpha \rangle = \langle c_1(\omega), \pi_* \alpha \rangle = 0$ .  $\square$

**Theorem 4.** *Let  $\pi : \tilde{X} \rightarrow X$  be a  $d$ -fold branched covering of orientable 4-manifolds, whose branch locus  $B \subset X$  is obtained from a generically immersed surface  $B^* \subset X$  by smoothing all double points as in Example 2 (up to orientation). If  $B^*$  has  $k \geq 1$  double points, then  $\Pi(\tilde{X}) \neq 0$ . If  $\pi$  is injective on  $\pi^{-1}(B)$ , then  $\dim \Pi(\tilde{X}) \geq k(d - 1)$ .*

*Proof.* At each double point of  $B^*$ , a 4-ball neighborhood  $K$  intersects  $B$  in the complex curve  $z_1 z_2 = \varepsilon$ , which is the Milnor fiber  $M(2, 2)$ . Each component of  $\pi^{-1}(K)$  intersects one  $B_i$ , and is the  $d_i$ -fold cover of  $K$  branched along  $K \cap B$ . This is diffeomorphic to the Milnor fiber  $M(2, 2, d_i)$ , which is the plumbing along a linear graph of  $d_i - 1$  spheres of square  $-2$ . The intersection matrix of these  $d_i - 1$  spheres has nonzero determinant, so each component of each  $\pi^{-1}(K)$  contributes  $d_i - 1$  spheres to a linearly independent subset of  $\Pi(\tilde{X})$ . If  $\pi|_{\pi^{-1}(B)}$  is injective, then each  $d_i = d$ , so we obtain  $\dim \Pi(\tilde{X}) \geq k(d - 1)$  (and in general,  $\dim \Pi(\tilde{X}) \geq kd - \ell$ , where  $\ell$  is the number of components of  $\bigcup \pi^{-1}(K)$ ).  $\square$

**Example 5 – Kähler manifolds.** We find a family as in Example 2 with  $\Pi(\tilde{X})$  arbitrarily large, for which each  $\tilde{\omega}$  is a Kähler form. Suppose that each  $F_i$  is given as a complex curve of degree  $m_i$  in  $\mathbb{C}\mathbb{P}^{N_i}$ . Let  $H_i \subset \mathbb{C}\mathbb{P}^{N_i}$  be a generic degree- $d$  hypersurface, and choose the  $m_i d$  points  $p$  in Example 2 to be the set  $F_1 \cap H_1$ . Similarly, choose the points  $q$  to be  $F_2 \cap H_2$ . Thus, each of these finite subsets of  $F_i$  is exhibited as the zero locus of a holomorphic section of  $L_i^{\otimes d}$ , where  $L_i \rightarrow F_i$  is the restriction of the hyperplane bundle of  $\mathbb{C}\mathbb{P}^{N_i}$ . The corresponding zero locus of  $L_1^{\otimes d} \otimes L_2^{\otimes d} \rightarrow X = F_1 \times F_2$  is the singular surface  $B^*$ . By varying our choices of  $H_i$ , it is easy to find three holomorphic sections of  $L_1^{\otimes d} \otimes L_2^{\otimes d}$  whose common zero locus is empty — thus, the complete linear system of holomorphic sections of  $L_1^{\otimes d} \otimes L_2^{\otimes d}$  has empty base locus. By Bertini’s Theorem, a generic holomorphic section  $\rho$  of  $L_1^{\otimes d} \otimes L_2^{\otimes d}$  has smooth zero locus, so the required perturbation of  $B^*$  to a smooth surface  $B = \rho^{-1}(0)$  can be done holomorphically. To find a Kähler form  $\tilde{\omega}$  on the resulting  $\tilde{X}$

satisfying the conclusion of Lemma 1, note that the  $d$ -fold tensor power map  $p : L_1 \otimes L_2 \rightarrow (L_1 \otimes L_2)^d = L_1^{\otimes d} \otimes L_2^{\otimes d}$  is a  $d$ -fold covering map branched along the zero section. Since the section  $\rho : X \rightarrow L_1^{\otimes d} \otimes L_2^{\otimes d}$  is a diffeomorphism onto its image, we can identify  $\tilde{X}$  with  $p^{-1}(\text{Im } \rho) \subset L_1 \otimes L_2$  and  $\pi : \tilde{X} \rightarrow X$  with the restriction of projection  $L_1 \otimes L_2 \rightarrow X$ . If we identify  $X$  with the zero section of  $L_1 \otimes L_2$ , then any Kähler form  $\Omega$  on the total space of  $L_1 \otimes L_2$  restricts to Kähler forms  $\tilde{\omega}$  on  $\tilde{X}$  and  $\omega$  on  $X$ . For any  $\alpha \in H_2(\tilde{X}; \mathbb{R})$ , the classes  $\alpha$  and  $\pi_*\alpha$  have the same image in  $H_2(L_1 \otimes L_2; \mathbb{R})$ , so  $\langle \tilde{\omega}, \alpha \rangle = \langle \Omega, \pi_*\alpha \rangle = \langle \omega, \pi_*\alpha \rangle$ , implying that  $[\tilde{\omega}] = \pi^*[\omega]$  as required. The Chern class formula follows as before (and is well-known since  $\pi$  is holomorphic), so we again have  $\tilde{\omega}|\Pi(\tilde{X}) = c_1(\tilde{\omega})|\Pi(\tilde{X}) = 0$ .

**Example 6 – Non-Kähler manifolds.** Let  $(X, \omega)$  denote the Kodaira-Thurston manifold, obtained as the quotient of  $(\mathbb{R}^4, dx_1 \wedge dx_2 + dx_3 \wedge dx_4)$  by the discrete group of symplectomorphisms generated by unit translations in the  $x_1, x_2$  and  $x_3$  directions together with  $(x_1, x_2, x_3, x_4) \mapsto (x_1 + x_2, x_2, x_3, x_4 + 1)$ . The projection  $(x_1, x_2, x_3, x_4) \mapsto (x_3, x_4)$  induces a bundle structure  $p : X \rightarrow T^2$  with torus fibers and a section  $x_1 = x_2 = 0$  with trivial normal bundle. As in Example 2, choose  $m_1 d$  fibers and  $m_2 d$  parallel copies of the section, smooth the intersections of the resulting  $B^*$  to obtain a smooth symplectic surface  $B \subset X$ , and let  $\pi : \tilde{X} \rightarrow X$  be the resulting  $d$ -fold branched cover. As before, we obtain examples with  $\Pi(\tilde{X})$  arbitrarily large, but with the form  $\tilde{\omega}$  of Lemma 1 satisfying  $\tilde{\omega}|\Pi(\tilde{X}) = c_1(\tilde{\omega})|\Pi(\tilde{X}) = 0$ . This time, however, no  $\tilde{X}$  has the homotopy type of a Kähler manifold, as we verify by showing that  $b_1(\tilde{X}) = 3$ . Recall that  $b_1(X) = 3$ , since  $H_1(X; \mathbb{Z})$  has 4 canonical generators  $\alpha_1, \dots, \alpha_4$  with  $\alpha_i$  descending from the  $x_i$ -axis, and the monodromy introduces the relation  $\alpha_2 = \alpha_1 + \alpha_2$ . Since  $\pi_* : H_1(\tilde{X}; \mathbb{Z}) \rightarrow H_1(X; \mathbb{Z})$  is surjective, we have  $b_1(\tilde{X}) \geq 3$ . The Milnor fibers  $M(2, 2, d) \subset \tilde{X}$  arising as in the proof of Theorem 4 are simply connected, so we can collapse these to points, obtaining a singular space  $\tilde{X}^*$  with  $b_1(\tilde{X}^*) = b_1(\tilde{X})$  and a map  $\pi^* : \tilde{X}^* \rightarrow X$  that is a  $d$ -fold covering branched along  $B^*$ . The composite  $p \circ \pi^* : \tilde{X}^* \rightarrow T^2$  is a singular fibration whose  $m_1 d$  singular fibers are tori. By transversality,  $H_1(\tilde{X}^*; \mathbb{R})$  is generated by cycles disjoint from the singular fibers, hence, by cycles in a regular fiber together with a pair of classes lifted from the base  $T^2$ . (Meridians to the critical values in  $T^2$  lift to nullhomologous  $\mathbb{R}$ -cycles in  $\tilde{X}^*$ .) Since any cycle in a regular fiber is homologous to one in a preassigned singular fiber,  $H_1(\tilde{X}^*; \mathbb{R})$  is generated by 4 classes  $\tilde{\alpha}_1, \dots, \tilde{\alpha}_4$  lifted from cycles in  $B^*$  representing  $\alpha_1, \dots, \alpha_4$ . Since the relation  $\alpha_2 = \alpha_1 + \alpha_2$  is represented by a surface in  $X$  disjoint from  $B^*$  and lifting trivially to  $\tilde{X}^*$ , we obtain the relation  $\tilde{\alpha}_2 = \tilde{\alpha}_1 + \tilde{\alpha}_2$  in  $H_1(\tilde{X}^*; \mathbb{R})$ , so  $b_1(\tilde{X}) = b_1(\tilde{X}^*) = 3$  as required.

**Theorem 7.** *The conditions  $\omega|\Pi(X) = 0$  and  $c_1(\omega)|\Pi(X) = 0$  are independent.*

*Proof.* We have shown that both  $\omega$  and  $c_1(\omega)$  can vanish on a nontrivial  $\Pi(X)$ . A small perturbation of  $\omega$  gives examples with  $\omega|\Pi(X) \neq 0$ ,  $c_1(\omega)|\Pi(X) = 0$ . (The K3-surface is another such example.) Any simply-connected Kähler surface

other than  $K3$  satisfies  $\omega, c_1(\omega) \neq 0$  on  $\Pi(X) = H_2(X; \mathbb{R})$ . For the remaining case  $\omega|_{\Pi(X)} = 0, c_1(\omega)|_{\Pi(X)} \neq 0$ , begin with the torus  $T^4 = \mathbb{C}^2/\mathbb{Z}^4$  with the standard Kähler form  $\omega$ . Let  $B^* \subset T^4$  be the union of tori descending from the 4 lines  $z_i = \pm \frac{1}{4}$  in  $\mathbb{C}^2$ , and let  $B$  be the symplectic surface obtained by smoothing the 4 intersections  $(\pm \frac{1}{4}, \pm \frac{1}{4})$  of  $B^*$  using the local model  $z_1 z_2 = -\varepsilon, 0 < \varepsilon \ll 1$  (up to translation and perturbation away from the double points). The torus  $T \subset T^4$  determined by the equation  $z_2 = \bar{z}_1$  intersects  $B^*$  in the 2 points  $\pm(\frac{1}{4}, \frac{1}{4})$  and is disjoint from  $B$ . Since  $T$  is Lagrangian, it is easy to perturb  $\omega$  so that  $T$  becomes symplectic. Let  $(\tilde{X}, \tilde{\omega})$  be a 2-fold cover of  $T^4$  branched along  $B$  (and chosen to be trivial over  $T$ ), and let  $\tilde{T}$  be one of the 2 lifts of  $T$  to  $\tilde{X}$ . The vanishing cycle of  $B$  created by smoothing  $B^*$  at  $(\frac{1}{4}, \frac{1}{4})$  bounds a disk  $D$  in  $T^4$  intersecting  $T$  transversely in one point. (The coordinate change  $z_1 = u_1 + iu_2, z_2 = -u_1 + iu_2$  (following the appropriate translation) exhibits  $B$  as  $u_1^2 + u_2^2 = \varepsilon, D$  as the  $\sqrt{\varepsilon}$ -disk in  $\mathbb{R}^2$ , and  $T$  as  $i\mathbb{R}^2$ .) The 2 lifts of  $D$  to  $\tilde{X}$  fit together to form a sphere  $S \subset \tilde{X}$  intersecting  $\tilde{T}$  transversely in a single point. Let  $(\hat{X}, \hat{\omega})$  be any branched cover of  $\tilde{X}$  with branch locus given by 2 parallel copies of  $\tilde{T}$ . (Such a branched cover exists since  $\tilde{T}$  is symplectic with trivial normal bundle — e.g., send the two meridians to opposite generators of  $\mathbb{Z}/d$ .) The sphere  $S \subset \tilde{X}$  lifts to a sphere  $\hat{S} \subset \hat{X}$  intersecting two components  $B_i$  of the lifted branch locus of  $\hat{X} \rightarrow \tilde{X}$  each in one point. The two components  $B_i$  have the same multiplicity  $d \geq 2$ , and the signs  $B_i \cdot \hat{S}$  agree if each  $B_i$  is symplectically oriented. Now Theorem 3 implies that  $\hat{\omega}|_{\Pi(\hat{X})} = 0$  and  $c_1(\hat{\omega})|_{\Pi(\hat{X})} = 0$ , so by Lemma 1 applied to  $\hat{X} \rightarrow \tilde{X}$ , we have  $\langle c_1(\hat{\omega}), \hat{S} \rangle = \sum (1 - d_i) B_i \cdot \hat{S} = 2(1 - d) \neq 0$ .  $\square$

**Remark 8.** J. Kollár produced a different construction of symplectically aspherical Kähler manifolds with  $\Pi(X) \neq 0$ , which preceded the author's work by several months. He now observes that both constructions are based on a common idea: If  $(X, \omega)$  is symplectic,  $f : X \rightarrow Y$  is a continuous map to a topological space with  $\pi_2(Y) = 0$ , and  $[\omega] = f^* \Omega$  for some  $\Omega \in H^2(Y; \mathbb{R})$ , then  $\omega|_{\Pi(X)} = 0$ . (The proof is the same as for Theorem 3.) In our case,  $f$  is a branched covering, whereas Kollár took  $f$  to be an inclusion of algebraic varieties with  $Y$  a  $K(\pi, 1)$  (e.g., an Abelian variety) with  $\dim_{\mathbb{C}} Y \geq 3$ .

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