STRINGY HODGE NUMBERS AND VIRASORO ALGEBRA

Victor V. Batyrev

Abstract. In this paper we define for singular varieties $X$ a rational number $c_{1,n-1}^s(X)$ which is a stringy version of the product of Chern numbers $c_1$ and $c_{n-1}$. We show that the number $c_{1,n-1}^s(X)$ can be expressed via stringy Hodge numbers of singular $X$ in the same way as $c_1c_{n-1}$ expresses via usual Hodge numbers for smooth manifolds. Our result provides some evidences for the existence of quantum cohomology theory of singular varieties $X$ based on representation of the Virasoro algebra whose central charge is the rational number $e_{st}(X)$ which equals the stringy Euler number of $X$.

1. Introduction

Let $X$ be an arbitrary smooth $n$-dimensional projective variety. It was discovered by Libgober and Wood that the product of the Chern classes $c_1(X)c_{n-1}(X)$ depends only on the Hodge numbers of $X$ [11]. This result has been used by Eguchi, Jinzenji and Xiong in their approach to the quantum cohomology of $X$ via a representation of the Virasoro algebra with the central charge $c_n(X)$ [8, 9].

We recall that the $E$-polynomial of $X$ is defined as

$$E(X; u, v) := \sum_{p, q} (-1)^{p+q} h^{p,q}(X) u^p v^q,$$

where $h^{p,q}(X) = \dim H^q(X, \Omega^p_X)$ are Hodge numbers of $X$. Using the Hirzebruch-Riemann-Roch theorem, Libgober and Wood [11] have proved the following equality (see also results of Borisov [6] and Salamon [12]):

**Theorem 1.1.**

$$\frac{d^2}{du^2} E_{st}(X; u, 1)|_{u=1} = \frac{3n^2 - 5n}{12} c_n(X) + \frac{c_1(X)c_{n-1}(X)}{6}.$$  

By Poincaré duality for $X$, one immediately obtains the following equivalent reformulation of the above equality:

**Theorem 1.2.** Let $X$ be an arbitrary smooth $n$-dimensional projective variety. Then $c_1(X)c_{n-1}(X)$ can be expressed via the Hodge numbers of $X$ using the following equality

$$\sum_{p, q} (-1)^{p+q} h^{p,q}(X) \left( p - \frac{n}{2} \right)^2 = \frac{n}{12} c_n(X) + \frac{1}{6} c_1(X)c_{n-1}(X),$$

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where
\[ c_n(X) = \sum_{p,q} (-1)^{p+q} h^{p,q}(X) \]

is the Euler number of \( X \).

In particular, one has:

**Corollary 1.3.** Let \( X \) be an arbitrary smooth \( n \)-dimensional projective variety with \( c_1(X) = 0 \). Then the Hodge numbers of \( X \) satisfy the following equation
\[ \sum_{p,q} (-1)^{p+q} h^{p,q}(X) \left( p - \frac{n}{2} \right)^2 = \frac{n}{12} \sum_{p,q} (-1)^{p+q} h^{p,q}(X). \]

**Remark 1.4.** We note that if \( X \) is a K3-surface, then the relation 1.3 is equivalent to the equality \( c_2(X) = 24 \). For smooth Calabi-Yau 4-folds \( X \) the equality 1.3 has been observed by Sethi, Vafa, and Witten [13]. It is equivalent to the equality
\[ c_4(X) = 6(8 - h^{1,1}(X) + h^{2,1}(X) - h^{3,1}(X)), \]
if \( h^{1,0}(X) = h^{2,0}(X) = h^{3,0}(X) = 0 \).

There are a lot of examples of Calabi-Yau varieties \( X \) having at worst Gorenstein canonical singularities which are hypersurfaces and complete intersections in Gorenstein toric Fano varieties [1, 3]. It has been shown in [5] that for all these singular Calabi-Yau varieties \( X \) one can define so called *stringy Hodge numbers* \( h_{st}^{p,q}(X) \) [2]. Moreover, the stringy Hodge numbers of Calabi-Yau complete intersections in Gorenstein toric varieties agree with the topological mirror duality test [4]. It is natural to expect that one has the same kind of identity for stringy Hodge numbers of singular Calabi-Yau varieties as for usual Hodge numbers of smooth Calabi-Yau manifolds, i.e.,

\[ (1) \quad \sum_{p,q} (-1)^{p+q} h_{st}^{p,q}(X) \left( p - \frac{n}{2} \right)^2 = \frac{n}{12} \sum_{p,q} (-1)^{p+q} h_{st}^{p,q}(X) = \frac{n}{12} c_{st}(X). \]

This paper is to show that the formula (1) holds true. Moreover, one can define a rational number \( c_{st}^{4,n-1}(X) \), a stringy version \( c_4(X)c_{n-1}(X) \), such that the stringy analogue of the equation in 1.2
\[ (2) \quad \sum_{p,q} (-1)^{p+q} h_{st}^{p,q}(X) \left( p - \frac{n}{2} \right)^2 = \frac{n}{12} c_{st}(X) + \frac{1}{6} c_{st}^{4,n-1}(X), \]
holds true provided the stringy Hodge numbers of \( X \) exist.
2. Stringy Hodge numbers

Recall our general approach to the notion of stringy Hodge numbers $h_{st}^{p,q}(X)$ for projective algebraic varieties $X$ with canonical singularities (see [2]). Our main definition in [2] can be reformulated as follows:

**Definition 2.1.** Let $X$ be an arbitrary $n$-dimensional projective variety with at worst log-terminal singularities, $\rho : Y \to X$ a resolution of singularities whose exceptional locus $D$ is a divisor with normally crossing components $D_1, \ldots, D_r$. We set $I := \{1, \ldots, r\}$ and $D_J := \bigcap_{j \in J} D_j$ for all $J \subseteq I$, where $D_J = Y$ if $J = \emptyset$. Define the stringy $E$-function of $X$ to be

$$E_{st}(X; u, v) := \sum_{\emptyset \subseteq J \subseteq I} E(D_J; u, v) \prod_{j \in J} \left( \frac{uv - 1}{(uv)^{a_j+1} - 1} \right),$$

where the rational numbers $a_1, \ldots, a_r$ are determined by the equality

$$K_Y = \rho^* K_X + \sum_{i=1}^r a_i D_i.$$

Then the stringy Euler number of $X$ is defined as

$$e_{st}(X) := \lim_{u, v \to 1} E_{st}(X; u, v) = \sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J) \prod_{j \in J} \left( -\frac{a_j}{a_j+1} \right),$$

where $c_{n-|J|}(D_J)$ is the Euler number of $D_J$ (we set $c_{n-|J|}(D_J) = 0$ if $D_J$ is empty).

**Remark 2.2.** It is important that the above definitions do not depend on the choice of a desingularization $\rho : Y \to X$ [2].

**Definition 2.3.** Let $X$ be an arbitrary $n$-dimensional projective variety with at worst Gorenstein canonical singularities. We say that stringy Hodge numbers of $X$ exist, if $E_{st}(X; u, v)$ is a polynomial, i.e.,

$$E_{st}(X; u, v) = \sum_{p+q} a_{p,q}(X) u^p v^q.$$

Under the assumption that $E_{st}(X; u, v)$ is a polynomial, we define the stringy Hodge numbers $h_{st}^{p,q}(X)$ to be $(-1)^{p+q} a_{p,q}(X)$.

**Remark 2.4.** In the above definitions, the condition that $X$ has at worst log-terminal singularities means that $a_i > -1$ for all $i \in I$; the condition that $X$ has at worst Gorenstein canonical singularities is equivalent for $a_i$ to be nonnegative integers for all $i \in I$ (see [10]).

The following statement has been proved in [2]:

**Theorem 2.5.** Let $X$ be an arbitrary $n$-dimensional projective variety with at worst Gorenstein canonical singularities. Assume that stringy Hodge numbers of $X$ exist. Then they have the following properties:

(i) $h_{st}^{0,0}(X) = h_{st}^{n,n}(X) = 1;$
(ii) \( h_{st}^{p,q}(X) = h_{st}^{n-p,n-q}(X) \) and \( h_{st}^{p,q}(X) = h_{st}^{q,p}(X) \) \( \forall p, q; \)

(iii) \( h_{st}^{p,q}(X) = 0 \) \( \forall p, q > n. \)

3. The number \( c_{st}^{1, n-1}(X) \)

**Definition 3.1.** Let \( X \) be an arbitrary \( n \)-dimensional projective variety \( X \) having at worst log-terminal singularities and \( \rho : Y \to X \) is a desingularization with normally crossing irreducible components \( D_1, \ldots, D_r \) of the exceptional locus. We define the number

\[
c_{st}^{1, n-1}(X) := \sum_{\emptyset \subseteq J \subseteq I} \rho^* c_1(X) c_{n-|J|-1}(D_J) \prod_{j \in J} \left( \frac{-a_j}{a_j + 1} \right),
\]

where \( \rho^* c_1(X) c_{n-|J|-1}(D_J) \) is considered as the intersection number of the 1-cycle \( c_{n-|J|-1}(D_J) \in A_1(D_J) \) with the \( \rho \)-pullback of the class of the anticanonical \( \mathbb{Q} \)-divisor of \( X \).

**Remark 3.2.** It is not clear a priori that the number \( c_{st}^{1, n-1}(X) \) in the above definition does not depend on the choice of a desingularization \( \rho \). Later we shall show this independence.

The proof of the next obvious statement is left to the reader:

**Proposition 3.3.** For any smooth \( n \)-dimensional projective variety \( V \), one has

\[
\frac{d}{du} E(V; u, 1)|_{u=1} = \frac{n}{2} c_n(V).
\]

**Proposition 3.4.** For any \( n \)-dimensional projective variety \( X \) having at worst log-terminal singularities, one has

\[
\frac{d}{du} E_{st}(X; u, 1)|_{u=1} = \frac{n}{2} c_{st}(X).
\]

**Proof.** By definition 2.1, we have

\[
E_{st}(X; u, 1) = \sum_{\emptyset \subseteq J \subseteq I} E(D_J; u, 1) \prod_{j \in J} \left( \frac{u-1}{u^{a_j+1}-1} - 1 \right).
\]

Applying 3.3 to every smooth submanifold \( D_J \subset Y \), we obtain

\[
\frac{d}{du} E_{st}(X; u, 1)|_{u=1} = \sum_{\emptyset \subseteq J \subseteq I} \frac{(n-|J|)}{2} c_{n-|J|}(D_J) \prod_{j \in J} \left( \frac{-a_j}{a_j + 1} \right) + \sum_{\emptyset \subseteq J \subseteq I} \frac{|J|}{2} c_{n-|J|}(D_J) \prod_{j \in J} \left( \frac{-a_j}{a_j + 1} \right) + \frac{n}{2} \sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J) \prod_{j \in J} \left( \frac{-a_j}{a_j + 1} \right) = \frac{n}{2} c_{st}(X).
\]

\( \square \)
Proposition 3.5. Let $V$ be a smooth projective algebraic variety of dimension $n$ and $W \subseteq V$ a smooth irreducible divisor on $V$ or empty divisor (the latter means that $\mathcal{O}_V(W) \cong \mathcal{O}_V$). Then

$$c_1(\mathcal{O}_V(W))c_{n-1}(V) = c_{n-1}(W) + c_1(\mathcal{O}_W(W))c_{n-2}(W),$$

where $c_{n-1}(W)$ is considered to be zero if $W = \emptyset$.

Proof. Consider the short exact sequence

$$0 \to T_W \to T_{V|W} \to \mathcal{O}_W(W) \to 0,$$

where $T_W$ and $T_V$ are tangent sheaves on $W$ and $V$. It gives the following relation between Chern polynomials

$$(1 + c_1(\mathcal{O}_W(W))t)(1 + c_1(W)t + c_2(W)t^2 + \cdots + c_{n-1}(W)t^{n-1}) = 1 + c_1(T_{V|W})t + c_2(T_{V|W})t^2 + c_{n-1}(T_V|W)t^{n-1}.$$

Comparing the coefficients by $t^{n-1}$ and using $c_{n-1}(T_V|W) = c_1(\mathcal{O}_V(W))c_{n-1}(V)$, we come to the required equality. $\square$

Corollary 3.6. Let $Y$ be a smooth projective variety, $D_1, \ldots, D_r$ smooth irreducible divisors with normal crossings, $I := \{1, \ldots, r\}$. Then for all $J \subseteq I$ and for all $j \in J$ one has

$$c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_j))c_{n-|J|}(D_{J\setminus\{j\}}) - c_{n-|J|}(D_j) = c_1(\mathcal{O}_{D_j}(D_j))c_{n-|J|+1}(D_j),$$

where $D_j$ is the complete intersection $\bigcap_{j \in J} D_j$.

Proof. One sets in 3.5 $V := D_{J\setminus\{j\}}$ and $W := D_j$. $\square$

Proposition 3.7. Let $\rho : Y \to X$ be a desingularization as in 3.1. Then

$$\sum_{0 \subseteq J \subseteq I} c_1(D_j)c_{n-|J|-1}(D_J) \prod_{j \in J} \left(\frac{-a_j}{a_j + 1}\right) = c_{n-1}^{1, n}(X) + \sum_{0 \subseteq J \subseteq I} \left(\sum_{j \in J} (a_j + 1)c_{n-|J|}(D_j)\right) \prod_{j \in J} \left(\frac{-a_j}{a_j + 1}\right).$$

Proof. Using the formula

$$c_1(Y) = \rho^*c_1(X) + \sum_{i \in I} -a_ic_1(\mathcal{O}_Y(D_i)),$$

and the adjunction formula for every complete intersection $D_j$ ($J \subseteq I$), we obtain

$$c_1(D_J) = \rho^*c_1(X)|_{D_J} + \sum_{j \in J} (-a_j - 1)c_1(\mathcal{O}_{D_j}(D_j)) + \sum_{j \in I \setminus J} (-a_j)c_1(\mathcal{O}_{D_j}(D_j)).$$
Therefore

$$
\sum_{\emptyset \subseteq J \subseteq I} c_1(D_J) c_{n-|J|-1}(D_J) \prod_{j \in J} \left( \frac{-a_j}{a_j+1} \right) = c_{st}^{1,n-1}(X) + \\
\sum_{\emptyset \subseteq J \subseteq I} \left( \sum_{j \in J} (-a_j-1)c_1(\mathcal{O}_{D_j}(D_J))c_{n-|J|-1}(D_J) \right) \prod_{j \in J} \left( \frac{-a_j}{a_j+1} \right) + \\
\sum_{\emptyset \subseteq J \subseteq I} \left( \sum_{j \in I \setminus J} (-a_j)c_1(\mathcal{O}_{D_J}(D_J))c_{n-|J|-1}(D_J) \right) \prod_{j \in J} \left( \frac{-a_j}{a_j+1} \right).
$$

Using 3.6, we obtain

$$
\sum_{j \in J} (-a_j-1)c_1(\mathcal{O}_{D_j}(D_J))c_{n-|J|-1}(D_J) = \\
\sum_{j \in J} (-a_j-1) \left( c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_J))c_{n-|J|}(D_{J\setminus\{j\}}) - c_{n-|J|}(D_J) \right).
$$

By substitution (4) to (3), we come to the required equality, because

$$
\sum_{\emptyset \subseteq J \subseteq I} \left( \sum_{j \in J} (-a_j-1)c_1(\mathcal{O}_{D_{J\setminus\{j\}}}(D_J))c_{n-|J|}(D_{J\setminus\{j\}}) \right) \prod_{j \in J} \left( \frac{-a_j}{a_j+1} \right) + \\
\sum_{\emptyset \subseteq J \subseteq I} \left( \sum_{j \in I \setminus J} (-a_j)c_1(\mathcal{O}_{D_J}(D_J))c_{n-|J|-1}(D_J) \right) \prod_{j \in J} \left( \frac{-a_j}{a_j+1} \right) = 0.
$$

\[\square\]

**Theorem 3.8.** Let \( X \) be an arbitrary \( n \)-dimensional projective variety with at worst log-terminal singularities. Then

$$
\frac{d^2}{du^2} E_{st}(X; u, 1)_{|u=1} = \frac{3n^2 - 5n}{12} c_{st}(X) + \frac{1}{6} c_{st}^{1,n}(X).
$$

**Proof.** Using the equalities

$$
\frac{d}{du} \left( \frac{u-1}{u^{a+1} - 1} - 1 \right)_{u=1} = \frac{-a}{2(a+1)}, \quad \frac{d^2}{du^2} \left( \frac{u-1}{u^{a+1} - 1} - 1 \right)_{u=1} = \frac{a(a+2)}{6(a+1)},
$$

together with the identities in 1.1 and 3.3 for every submanifold \( D_J \subset Y \), we obtain
\[
\frac{d^2}{du^2} E_{st}(X; u, 1)|_{u=1} = \sum_{\emptyset \subseteq J \subseteq I} c_1(D_J)c_{n-|J|-1}(D_J) \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right) + \\
c_{n-|J|}(D_J) \frac{3(n-|J|)^2 - 5(n-|J|)}{12} \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right) + \\
\sum_{\emptyset \subseteq J \subseteq I} \frac{(n-|J|)|J|c_{n-|J|}(D_J)}{2} \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right) + \\
\sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J)(|J|-1)|J| \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right) + \\
\sum_{\emptyset \subseteq J \subseteq I} c_{n-|J|}(D_J)(-\sum_{j \in J}(a_j+2)) \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right).
\]

By 3.7, the first term of the above equals

\[
\frac{1}{6} c_{st}^{1,n-1}(X) + \frac{1}{6} \sum_{\emptyset \subseteq J \subseteq I} \left(\sum_{j \in J}(a_j+1)c_{n-|J|}(D_J)\right) \prod_{j \in J} \left(-\frac{a_j}{a_j+1}\right).
\]

Now the required statement follows from the equality

\[
\sum_{j \in J}(a_j+1) = \frac{3(n-|J|)^2 - 5(n-|J|)}{12} + \frac{(n-|J|)|J|}{4} - \frac{\sum_{j \in J}(a_j+2)}{6} = \frac{3n^2 - 5n}{12}.
\]

\[\square\]

**Corollary 3.9.** The number \(c_{st}^{1,n}(X)\) does not depend on the choice of the desingularization \(\rho : Y \to X\).

**Proof.** By 3.4 and 3.8, \(c_{st}^{1,n}(X)\) can be computed in terms of derivatives of the stringy \(E\)-function of \(X\). But the stringy \(E\)-function does not depend on the choice of a desingularization [2]. \[\square\]

**Corollary 3.10.** Let \(X\) be a projective variety with at worst Gorenstein canonical singularities. Assume that the stringy Hodge numbers of \(X\) exist. Then

\[
\sum_{p,q} (-1)^{p+q} p^{p,q}(X) \left(p - \frac{n}{2}\right)^2 = \frac{n}{12} e_{st}(X) + \frac{1}{6} c_{st}^{1,n-1}(X).
\]

**Proof.** The equality follows immediately from 3.8 using the properties of the stringy Hodge numbers 2.5. \[\square\]
Corollary 3.11. Assume that the canonical class of $X$ is numerically trivial. Then $c_{1,n-1}^{1, n-1}(X) = 0$. In particular, for Calabi-Yau varieties with at worst Gorenstein canonical singularities we have

$$\frac{d^2}{du^2}E_{\text{st}}(X; u, 1)|_{u=1} = \frac{3n^2 - 5n}{12} e_{\text{st}}(X),$$

and therefore stringy Hodge numbers of $X$ satisfy the identity (1) provided that stringy numbers exist.

Example 3.12. Let $\Delta$ be an $n$-dimensional reflexive polyhedron and $\Delta^*$ its polar polyhedron [1]. We denote by $P_{\Delta}$ the Gorenstein toric Fano variety associated with $\Delta$. If $\Theta$ a convex lattice polyhedron of dimension $k$ then we denote by $v(\Theta)$ the integer $k! \ vol_k(\Theta)$ where $\ vol_k(\Theta)$ is the $k$-dimensional volume of $\Theta$ with respect to the lattice. It is known that $e_{\text{st}}(P_{\Delta}) = v(\Delta^*)$ (see Cor. 7.7 [5]).

One can ask about the meaning of the new invariant $c_{1, n}^{1, n}$ in terms of $\Delta$. Assume that $P_{\Delta}$ has a smooth crepant toric desingularization $\rho : \hat{P}_{\Delta} \to P_{\Delta}$ which is defined by a triangulation $T$ of the boundary $\partial \Delta^*$ by regular simplices. By 3.1 we obtain that

$$c_{1,n}^{1,n}(P_{\Delta}) = c_1(\hat{P}_{\Delta})c_{n-1}(\hat{P}_{\Delta}).$$

Now we use that fact that in the nonsingular case the Chern class $c_{n-1}(\hat{P}_{\Delta})$ is the sum of all 1-dimensional strata in $\hat{P}_{\Delta}$ [7]. These 1-dimensional strata correspond to $(d-2)$-dimensional simplices in the triangulation $T$ of $\partial \Delta^*$. Let $\tau$ be such a $(d-2)$-dimensional simplex which is a common boundary of two $(d-1)$-dimensional ones $\sigma_1$ and $\sigma_2$. Denote by $Z_\tau$ the 1-cycle on $\hat{P}_{\Delta}$ corresponding to $\tau$. It follows from the description of $c_1$ in terms of a canonical piecewise linear function having value 1 on $\partial \Delta^*$ that the intersection number of $Z_\tau$ with $c_1(\hat{P}_{\Delta})$ is zero unless $\sigma_1$ and $\sigma_2$ belong to different faces $\gamma_1^*$ and $\gamma_2^*$ of codimension 1 of $\Delta^*$. The faces $\gamma_1^*$ and $\gamma_2^*$ are dual to two vertices $\gamma_1, \gamma_2 \in \Delta$ and the intersection number $c_1(\hat{P}_{\Delta})Z_\tau$ equals just the integral length $v([\gamma_1, \gamma_2])$ of the segment $[\gamma_1, \gamma_2]$. Let $[\gamma_1, \gamma_2] \subset \Delta^*$ be the dual to $[\gamma_1, \gamma_2] (d-2)$-dimensional face. Then $[\gamma_1, \gamma_2]^*$ contains exactly $v([\gamma_1, \gamma_2]^*)$ simplices from the triangulation $T$, because for all such simplices one has $v(\tau) = 1$. The above arguments show that the number

$$c_{1,n}^{1,n}(P_{\Delta}) = c_1(\hat{P}_{\Delta}) \sum_{\tau \in T} Z_\tau$$

equals

$$\sum_{[\gamma, \gamma'] \subset \Delta} v([\gamma, \gamma']) \cdot v([\gamma, \gamma']^*).$$

So we have

$$c_{1,n-1}^{1,n-1}(P_{\Delta}) = \sum_{\theta \subset \Delta, \dim \theta = 1} v(\theta) \cdot v(\theta^*).$$
4. Virasoro algebra

Recall that the Virasoro algebra with the central charge $c$ consists of operators $L_n \ (m \in \mathbb{Z})$ satisfying the relations

$$[L_n, L_m] = (n - m)L_{n+m} + c \frac{n^3 - n}{12} \delta_{n+m,0} \quad n, m \in \mathbb{Z}.$$

For arbitrary compact Kähler manifold $X$, Eguchi et. al have proposed in [8, 9] a new approach to its quantum cohomology and to its Gromov-Witten invariants for all genera $g$ using so called the Virasoro condition:

$$L_n Z = 0, \forall n \geq -1,$$

where

$$Z = \exp F = \exp \left( \sum_{g \geq 0} \lambda^{2g-2} F_g \right)$$

is the partition function of the topological $\sigma$-model with the target space $X$ and $F_g$ the free energy function corresponding to the genus $g$. In this approach, the central charge $c$ acts as the multiplication by $c_n(X)$. Moreover, all Virasoro operators $L_n$ can be explicitly written in terms of elements of a basis of the cohomology of $X$, their gravitational descendants and the action of $c_1(X)$ on the cohomology by the multiplication. In particular the commutator relation

$$[L_1, L_{-1}] = 2L_0$$

implies precisely the identity of Libgober and Wood in the form

$$\sum_{p+q} (-1)^{p+q} h^{p,q}(X) \left( \frac{n + 1}{2} - p \right) \left( p - \frac{n - 1}{2} \right) =$$

$$\frac{1}{6} \left( \frac{3 - n}{2} c_n(X) - c_1(X)c_{n-1}(X) \right).$$

Now let $X$ be a projective algebraic variety with at worst log-terminal singularities. We conjecture that there exists an analogous approach to the quantum cohomology as well as to the Gromov-Witten invariants of $X$ for all genera using the Virasoro algebra in such a way that for any resolution of singularities $\rho : Y \to X$ the corresponding Virasoro operators can be explicitly computed via the numbers $a_i$ appearing in the formula

$$K_X = \rho^*K_X + \sum_{i=1}^r a_i D_i,$$

and bases in cohomology of all complete intersections $D_J$ together with the multiplicative actions of $c_1(D_J)$ in them. We consider our main result 3.8 as an evidence in favor of this conjecture.
References


Mathematisches Institut, Universität Tübingen, Auf der Morgenstelle 10, 72076 Tübingen, GERMANY

E-mail address: batyrev@bastau.mathematik.uni-tuebingen.de