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ON LEGENDRIAN SURGERIES

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Abstract. We use the Ozsváth-Szabó contact invariants to distinguish between tight contact structures obtained by Legendrian surgeries on stabilized Legendrian links in tight contact 3-manifolds. We also discuss the implication of our result on the tight contact structures on the Brieskorn homology spheres $-\Sigma(2,3,6n-1)$.

1. Introduction

A contact structure ξ on an oriented 3-manifold M is an oriented tangent plane distribution such that there is a 1-form α on M satisfying $\xi = \ker\alpha$, $d\alpha|_\xi > 0$, and $\alpha \wedge d\alpha > 0$. Such a 1-form is called a contact form for ξ. A curve in M is said to be Legendrian if it is tangent to ξ everywhere. ξ is said to be overtwisted if there is an embedded disk D in M such that ∂D is Legendrian, but D is transversal to ξ along ∂D. A contact structure that is not overtwisted is called tight.

There are three types of symplectic fillability for contact structures.

(1) ξ is called Stein fillable if there is a Stein surface (W, J) such that $M = \partial W$ and $\xi = TM \cap J(TM)$.
(2) ξ is called strongly fillable if there is a symplectic 4-manifold (W, ω) such that $M = \partial W$, ω is exact near M, and there exists a primitive α of ω near M satisfying $\xi = \ker(\alpha|_M)$ and $\omega|_\xi > 0$.
(3) ξ is called weakly fillable if there is a symplectic 4-manifold (W, ω) such that $M = \partial W$ and $\omega|_\xi > 0$.

From the works of Eliashberg [3], Etnyre and Honda [8], Gromov [16] and Ghiggini [9, 10], we know

$$\begin{aligned} &\{\text{Stein fillable contact structures}\}\\ \subsetneq\ &\{\text{strongly fillable contact structures}\}\\ \subsetneq\ &\{\text{weakly fillable contact structures}\}\\ \subsetneq\ &\{\text{tight contact structures}\}. \end{aligned}$$

The classification problem of overtwisted contact structures was solved by Eliashberg [2]. The classification of tight contact structures up to isotopy is much more complex, and is only known for limited classes of 3-manifolds.

Eliashberg [5] and, independently, Weinstein [26] defined the Legendrian surgery, which turns out to be a very useful method of constructing tight contact structures.

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We will recall Weinstein's construction in details in Section 2. From [4, 8, 26], Legendrian surgery is known to preserve the above three types of symplectic fillability. It has been used to produce many interesting examples of tight contact structures.

In many cases, in order to classify tight contact structures, we need to distinguish between tight contact structures constructed by different Legendrian surgeries. If the Legendrian surgeries are done on the standard contact S^3, which is Stain filled by the standard complex B^4, then the next two theorems provide an easy criterion.

Theorem 1.1. [20, Theorem 1.2] *Let X be a smooth 4-manifold with boundary. Suppose J_1, J_2 are two Stein structures with boundary on X with associated Spin-structures $\mathfrak{s}_1$ and $\mathfrak{s}_2$. If the induced contact structures ξ_1 and ξ_2 on ∂X are isotopic, then $\mathfrak{s}_1$ and $\mathfrak{s}_2$ are isomorphic (and, in particular, have the same first Chern class).*

Theorem 1.2. [14, Proposition 2.3] *If (W, J) is obtained from the standard complex B^4 by Legendrian surgery on a Legendrian link in the standard contact S^3, then the first Chern class $c_1(J)$ of the induced Stein structure J is represented by a cocycle whose value on the 2-dimensional homology class corresponding to a component of L equals the rotation number of that component.*

In particular, we have:

Corollary 1.3. *Let L_1, L_2 be two smoothly isotopic Legendrian links in the standard contact S^3 (which is Stein fillable). Suppose that the Thurston-Bennequin numbers of corresponding components of L_1 and L_2 are equal. Then the Legendrian surgeries on L_1, L_2 give two tight contact structures ξ_1 and ξ_2 on the same ambient 3-manifold. And, if ξ_1 and ξ_2 are isotopic, then the rotation numbers of corresponding components of L_1 and L_2 are equal.*

In practice, we can attain different rotation numbers by stabilizing a Legendrian link in different ways. Then Corollary 1.3 implies that Legendrian surgeries on these stabilized Legendrian links give non-isotopic contact structures. This method can be modified to apply to other Stein fillable contact 3-manifolds. See, e.g., [12, 17, 27] for applications. The goal of the present paper is to generalize Corollary 1.3 to distinguish between tight contact structures obtained by Legendrian surgeries on stabilized Legendrian links in larger classes of tight contact 3-manifolds, including all weakly fillable ones. Our main technical tool is the Ozsváth-Szabó contact invariant.

Theorem 1.4. *Let (M, ξ) be a tight contact 3-manifold, and*

$$L = K^1 \coprod K^2 \coprod \cdots \coprod K^m$$

a Legendrian link in it. For $j = 1, 2, \cdots, m$, $i = 1, 2$, fix integers s^j, p_i^j, so that $0 \leq p_i^j \leq s^j$. Let K_i^j be the Legendrian knot constructed from K^j by p_i^j positive stabilizations and $s^j - p_i^j$ negative stabilizations. Then the Legendrian surgeries on $L_i = K_i^1 \coprod K_i^2 \coprod \cdots \coprod K_i^m$ give two contact structures ξ_1 and ξ_2 on the same ambient 3-manifold M'. Assume that ξ_1 and ξ_2 are isotopic. We have:

(1) *If (M, ξ) is weakly filled by a symplectic 4-manifold (W, ω), then, for each $j = 1, \cdots, m$,*

$$2(p_1^j - p_2^j) \begin{cases} = 0, & \text{if } K^j \text{ represents a torsion element in } H_1(W); \\ \equiv 0 \mod d^j, & \text{otherwise, where } d^j = \gcd\{\langle \zeta, [K^j] \rangle | \zeta \in H^1(W)\}. \end{cases}$$

(2) *If (M,ξ) has non-vanishing Ozsváth-Szabó c^+-invariant, then, for each $j = 1, \cdots, m$,*

$$2(p_1^j - p_2^j)\begin{cases} = 0, & \text{if } K^j \text{ represents a torsion element in } H_1(M); \\ \equiv 0 \mod d^j, & \text{otherwise, where } d^j = \gcd\{\langle \zeta, [K^j]\rangle | \zeta \in H^1(M)\}. \end{cases}$$

The above theorem was proved in the author's attempt to classify tight contact structures on the Brieskorn homology spheres $-\Sigma(2,3,6n-1)$. In Section 4, we will discuss the tight contact structures on these homology spheres using Theorem 1.4. It is known to many contact topologists that there are at most $\frac{n(n-1)}{2}$ tight contact structures on $-\Sigma(2,3,6n-1)$. Using the tight contact structures on $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$, which are all weakly fillable, we can give $\frac{n(n-1)}{2}$ different Legendrian surgery constructions of tight contact structures on $-\Sigma(2,3,6n-1)$. But it is not known whether these surgeries give non-isotopic tight contact structures. We will use Theorem 1.4 to show that, among these surgeries, any two different Legendrian surgeries on the same tight contact structure on $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$ give non-isotopic tight contact structures on $-\Sigma(2,3,6n-1)$, which implies the following theorem.

Theorem 1.5. *There are at least $2n-3$ pairwise non-isotopic tight contact structures on $-\Sigma(2,3,6n-1)$.*

It is still an open problem whether surgeries on different tight contact structures on $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$ give non-isotopic tight contact structures on $-\Sigma(2,3,6n-1)$. The author believes that the answer is yes, and the proof will likely require a better understanding of the Heegaard-Floer homology and the Ozsváth-Szabó contact invariants.

2. Standard symplectic 2-handle and Legendrian surgery

In this section, we recall Weinstein's construction of the standard symplectic 2-handle and the Legendrian surgery in [26].

Let (x_1, y_1, x_2, y_2) be the standard Cartesian coordinates of $\mathbb{R}^4$, and

$$\omega_{st} = dx_1 \wedge dy_1 + dx_2 \wedge dy_2$$

the standard symplectic form on $\mathbb{R}^4$. Define

$$f_2 = x_1^2 - \frac{y_1^2}{2} + x_2^2 - \frac{y_2^2}{2},$$

$$v_2 = \nabla f_2 = 2x_1\frac{\partial}{\partial x_1} - y_1\frac{\partial}{\partial y_1} + 2x_2\frac{\partial}{\partial x_2} - y_2\frac{\partial}{\partial y_2},$$

and

$$\alpha_2 = \iota_{v_2}\omega_{st} = y_1dx_1 + 2x_1dy_1 + y_2dx_2 + 2x_2dy_2.$$

Then v_2 is a symplectic vector field, in the sense that $d(\iota_{v_2}\omega_{st}) = \omega_{st}$. Let

$$X_- = \{(x_1, y_1, x_2, y_2) \in \mathbb{R}^4 \mid f_2(x_1, y_1, x_2, y_2) = -1\}.$$

X_- is positively transverse to v_2, and, hence, $\alpha_2|_{X_-}$ is a contact form. Let

$$S^1_- = \{(0, y_1, 0, y_2) \mid f_2(0, y_1, 0, y_2) = -1\}.$$

This is a Legendrian knot in $(X_-, \alpha_2|_{X_-})$.

Lemma 2.1. [26, Lemma 3.1] *For $A > 1$, let*

$$F(x_1, y_1, x_2, y_2) = A(x_1^2 + x_2^2) - \frac{y_1^2 + y_2^2}{2} - 1.$$

Then the hypersurface

$$\Sigma = F^{-1}(0)$$

is positively transverse to v_2, and the region

$$\mathcal{H}_2 = \{(x_1, y_1, x_2, y_2) \in \mathbb{R}^4 \mid f_2(x_1, y_1, x_2, y_2) \geq -1,\ F(x_1, y_1, x_2, y_2) \leq 0\}$$

is diffeomorphic to $D^2 \times D^2$. Moreover, by choosing $A \gg 1$, we can make $\mathcal{H}_2 \cap X_-$ an arbitrarily small neighborhood of S_-^1 in X_-.

Definition 2.2. $(\mathcal{H}_2, \omega_{st}|_{\mathcal{H}_2})$ is called a standard symplectic 2-handle.

Proposition 2.3. [26, Proposition 4.2] *Suppose, for $i = 1, 2$, (W_i, ω_i) is a symplectic 4-manifold, u_i is a symplectic vector field in (W_i, ω_i), M_i is a 3-dimension submanifold of W transverse to u_i, and K_i is a Legendrian knot in M_i with respect to the contact form $\iota_{u_i}\omega_i|_{M_i}$. Then there is an open neighborhood U_i of K_i in W_i, for $i = 1, 2$, and a diffeomorphism $\varphi : U_1 \to U_2$, s.t., $\varphi^*(\omega_2|_{U_2}) = \omega_1|_{U_1}$, $\varphi_*(u_1|_{U_1}) = u_2|_{U_2}$, $\varphi(U_1 \cap M_1) = U_2 \cap M_2$, $\varphi(K_1) = K_2$.*

Let (W, ω) be a symplectic 4-manifold with boundary, M a component of ∂W, and ξ a contact structure on M so that $\omega|_\xi > 0$. Let K be a Legendrian knot in (M, ξ). By [8, Lemma 2.4], we isotope ξ near K so that there exit a neighborhood U of K in W, and a non-vanishing symplectic vector field v defined in U, s.t., v transversally points out of W along $U \cap M$, and $\xi|_{U\cap M} = \ker(\iota_v\omega|_{U\cap M})$. Let $\{\psi_t\}$ be the flow of v. Without loss of generality, we assume there exists $\tau > 0$ such that

$$U = \bigcup_{0\leq t<\tau} \psi_{-t}(U \cap M).$$

Choose a small $\varepsilon \in (0, \tau)$. By Proposition 2.3, there is an open neighborhood V of S_-^1 in $\mathbb{R}^4$, and an embedding $\varphi : V \to U$, s.t., $\varphi^*(\omega) = \omega_{st}$, $\varphi_*(v_2) = v$, $\varphi(V \cap X_-) \subset \psi_{-\varepsilon}(U \cap M)$, and $\varphi(S_-^1) = \psi_{-\varepsilon}(K)$. Choosing $A \gg 1$ in Lemma 2.1, we get a standard symplectic 2-handle $\mathcal{H}_2$, such that $\mathcal{H}_2 \cap X_- \subset V$. We extend the map $\varphi : V \to U$ by mapping the flow of v_2 to the flow of v. Then φ becomes a symplectic diffeomorphism from a neighborhood of $\mathcal{H}_2 \cap X_-$ to a neighborhood of K in W. Now, let

$$W' = W \cup_\varphi \mathcal{H}_2,\ \omega' = \begin{cases} \omega, & \text{on } W; \\ \omega_{st}, & \text{on } \mathcal{H}_2, \end{cases} \text{, and } v' = \begin{cases} v, & \text{on } U; \\ v_2, & \text{on } \mathcal{H}_2. \end{cases}$$

Then (W', ω') is a symplectic 4-manifold, and v' is a symplectic vector field defined in $U \cup_\varphi \mathcal{H}_2$, transversally pointing out of the boundary of W'. Let

$$M' = (M \setminus \mathcal{H}_2) \cup (\mathcal{H}_2 \cap \Sigma), \text{ and } \xi' = \begin{cases} \xi, & \text{on } M \setminus \mathcal{H}_2; \\ \ker \alpha_2, & \text{on } \mathcal{H}_2 \cap \Sigma. \end{cases}$$

Then (M', ξ') is the contact 3-manifold obtained from (M, ξ) by Legendrian surgery on K, and $\omega'|_{\xi'} > 0$.

Remark 2.4. If (M, ξ) is weakly fillable, then the above construction gives (M', ξ') a weak symplectic filling. For a general contact 3-manifold (M, ξ), consider the symplectic 4-manifold $(M \times I, d(e^t\alpha))$, where α is a contact form for ξ, and t is the variable of I. We can carry out the above construction near $M \times \{1\}$, and get a symplectic cobordism from (M, ξ) to (M', ξ').

3. Ozsváth-Szabó invariants and proof of Theorem 1.4

Ozsváth and Szabó [23] introduced the Ozsváth-Szabó invariant $c(\xi)$ of a contact structure ξ on a 3-manifold M. $c(\xi)$ is an element of the quotient $\widehat{HF}(-M)/\{\pm 1\}$ of the Heegaard-Floer homology group of $-M$, and is invariant under isotopy of ξ. $c(\xi)$ vanishes when ξ is overtwisted. For our purpose, it is more convenient to use the following variant of the Ozsváth-Szabó invariant.

Definition 3.1. [10, 25] Let M be a closed, oriented 3-manifold, and

$$\iota : \widehat{HF}(-M) \to HF^+(-M)$$

the canonical map. Define $c^+(\xi) = \iota(c(\xi))$ for any contact structure ξ on M.

Clearly, $c^+(\xi)$ is also invariant under isotopy of ξ, and vanishes when ξ is overtwisted.

The behavior of Ozsváth-Szabó invariants under Legendrian surgeries is described in the following theorem of Ozsváth and Szabó.

Theorem 3.2. [23] *Let (M', ξ') be the contact 3-manifold obtained from (M, ξ) by Legendrian surgery on a Legendrian link, then $F^+_W(c^+(\xi')) = c^+(\xi)$, where W is the cobordism induced by the surgery.*

Specially, this implies that ξ' is tight if $c(\xi) \neq 0$.

Ghiggini [10] refined Theorem 3.2 to the following.

Proposition 3.3. [10, Lemma 2.11] *Suppose that (M', ξ') is obtained from (M, ξ) by Legendrian surgery on a Legendrian link. Then we have $F^+_{W,\mathfrak{t}}(c^+(\xi')) = c^+(\xi)$, where W is the cobordism induced by the surgery and $\mathfrak{t}$ is the canonical $Spin^{\mathbb{C}}$-structure associated to the symplectic structure on W. Moreover, $F^+_{W,\mathfrak{s}}(c^+(\xi')) = 0$ for any $Spin^{\mathbb{C}}$-structure $\mathfrak{s}$ on W with $\mathfrak{s} \neq \mathfrak{t}$.*

In order to prove Theorem 1.4 in the weakly fillable case, we need to use the Ozsváth-Szabó contact invariant twisted by a 2-form as defined in [21]. Let (M, ξ) be a contact 3-manifold with weak symplectic filling (W, ω), and B an embedded 4-ball in the interior of W. Consider the element $\underline{F}^+_{W\setminus B,\mathfrak{s}|_{W\setminus B};[\omega|_{W\setminus B}]}(c^+(\xi; [\omega|_M]))$ of the group $\underline{HF}^+(S^3; [\omega|_{S^3}])$, where $S^3 = -\partial B$, $\mathfrak{s}$ is a $Spin^{\mathbb{C}}$-structure on W, $c^+(\xi; [\omega|_M]) \in \underline{HF}^+(-M; [\omega|_M])$ is the Ozsváth-Szabó contact invariant of ξ twisted by $[\omega|_M]$, and $\underline{F}^+_{W\setminus B,\mathfrak{s}|_{W\setminus B};[\omega|_{W\setminus B}]}$ is the homomorphism between the two twisted Heegaard-Floer homology groups induced by the cobordism $W \setminus B$. Note that both $c^+(\xi; [\omega|_M])$ and $\underline{F}^+_{W\setminus B,\mathfrak{s}|_{W\setminus B};[\omega|_{W\setminus B}]}$ are defined up to an overall multiplication by a factor of the form $\pm T^c$ for some $c \in \mathbb{R}$. To make them absolute, we fix the auxiliary choices in the constructions of them, including a triple Heegaard diagram, a base Whitney triangle to define the homomorphisms, and a representation of $c^+(\xi; [\omega|_M])$. We also fix a minimal grading generator Θ^+ of $HF^+(S^3)$. Note that $\underline{HF}^+(S^3; [\omega|_{S^3}]) = HF^+(S^3) \otimes \mathbb{Z}[\mathbb{R}]$. Let $P_{\xi,\mathfrak{s};[\omega]} \in \mathbb{Z}[\mathbb{R}]$ be the coefficient of $\Theta^+ \otimes 1$ in

$$\underline{F}^+_{W\setminus B,\mathfrak{s}|_{W\setminus B};[\omega|_{W\setminus B}]}(c^+(\xi; [\omega|_M])).$$

Define a degree on $\mathbb{Z}[\mathbb{R}]$ by setting $\deg 0 = +\infty$ and $\deg P = c_1$ for

$$P = \sum_{i=1}^{m} a_i T^{c_i} \ \in \ \mathbb{Z}[\mathbb{R}],$$

where $a_i \neq 0$ and $c_1 < \cdots < c_m$. Denote by $\mathfrak{s}_\omega$ the canonical $Spin^{\mathbb{C}}$-structure of (W, ω).

Lemma 3.4. [21, Theorem 4.2]

$$\deg P_{\xi,\mathfrak{s}_\omega;[\omega]} < \deg P_{\xi,\mathfrak{s};[\omega]}$$

for any $Spin^{\mathbb{C}}$*-structure* $\mathfrak{s}$ *on* W *with* $\mathfrak{s} \neq \mathfrak{s}_\omega$.

Proof. (Following the proof of [21, Theorem 4.2].) Fix an open book of M adapted to ξ with connected binding and genus greater than 1. Eliashberg [6, Theorem 1.1] showed than ω extends over the the Giroux 2-handle $M \xrightarrow{W_0} M_0$ corresponding to the 0-surgery on the binding of the open book, where M_0 is the surface bundle over S^1 resulted from this surgery. Moreover, [6, Theorem 1.3] implies that there is a 4-manifold V with $\partial V = -M_0$, $b_2^+(V) > 1$, such that the extension of ω over $W \cup_M W_0$ further extends to a symplectic structure $\widetilde{\omega}$ on $X = W \cup_M W_0 \cup_{M_0} V$. Let $\widetilde{\mathfrak{s}}_{\widetilde{\omega}}$ be the canonical $Spin^{\mathbb{C}}$-structure of $(X, \widetilde{\omega})$.

Let $\mathfrak{s}$ be any $Spin^{\mathbb{C}}$-structure on W such that $\mathfrak{s}|_M$ is the canonical $Spin^{\mathbb{C}}$-structure of (M, ξ). Using the Composition Law [24, Theorem 3.9] and the arguments in the proof of [21, Theorem 4.2], one can show that there exists a non-zero element $P \in \mathbb{Z}[\mathbb{R}]$ independent of $\mathfrak{s}$ such that

$$\begin{aligned} & P \cdot \underline{F}^+_{W\setminus B,\mathfrak{s}|_{W\setminus B};[\omega|_{W\setminus B}]}(c^+(\xi;[\omega|_M])) \\ = & \sum_{\widetilde{\mathfrak{s}}\in Spin^{\mathbb{C}}(X),\ \widetilde{\mathfrak{s}}|_W=\mathfrak{s},\ \widetilde{\mathfrak{s}}|_{W_0}=\widetilde{\mathfrak{s}}_{\widetilde{\omega}}|_{W_0},\ \widetilde{\mathfrak{s}}|_V=\widetilde{\mathfrak{s}}_{\widetilde{\omega}}|_V} \Phi_{X,\widetilde{\mathfrak{s}}} \cdot T^{\langle \omega\cup c_1(\widetilde{\mathfrak{s}}),[X]\rangle}, \end{aligned}$$

where $\Phi_{X,\widetilde{\mathfrak{s}}}$ is the closed 4-manifold invariant defined in [24]. By [22, Theorem 1.1], the degree of the right hand side of the above equation is equal to $\langle \omega \cup c_1(\widetilde{\mathfrak{s}}_{\widetilde{\omega}}), [X]\rangle$ if $\mathfrak{s} = \mathfrak{s}_\omega$, and is strictly greater than $\langle \omega \cup c_1(\widetilde{\mathfrak{s}}_{\widetilde{\omega}}), [X]\rangle$ otherwise. This implies the lemma. □

The next two lemmas are technical results needed to prove Theorem 1.4.

Lemma 3.5. *Let* X *be a compact manifold with boundary, and* Y *a closed submanifold of* X*. Suppose that* L_1 *and* L_2 *are two complex line bundles over* X*, and there is an isomorphism* $\Psi : L_1|_Y \to L_2|_Y$*. Let* $j : (X, \emptyset) \to (X, Y)$ *be the natural inclusion. Then there exists* $\beta \in H^2(X, Y)$*, such that* $j^*(\beta) = c_1(L_1) - c_1(L_2)$*, and, for any embedded 2-manifold* Σ *in* X *with* $\partial\Sigma \subset Y$*, and any non-vanishing section* v *of* $L_1|_{\partial\Sigma}$*, we have* $\langle \beta, [\Sigma]\rangle = \langle c_1(L_1, v), [\Sigma]\rangle - \langle c_1(L_2, \Psi(v)), [\Sigma]\rangle$*, where* $[\Sigma]$ *is the relative homology class in* $H_2(X, Y)$ *represented by* Σ*.*

Proof. Denote by J_i the complex structure on L_i. Choose a metric g_2 on $L_2|_Y$ compatible with J_2, and let $g_1 = \Psi^*(g_2)$. Consider the complex line bundle $L = L_1 \otimes \overline{L}_2$, where $\overline{L}_2$ is L_2 with the complex structure $-J_2$. Let $\mathcal{I} : L_2 \to \overline{L}_2$ be the identity map, and $\overline{\Psi} = \mathcal{I} \circ \Psi$. We define a smooth non-vanishing section η of $L|_Y$ as following: at any point p on Y, pick a unit vector $u_p \in L_1|_p$, and define $\eta_p = u_p \otimes \overline{\Psi}(u_p)$. It is clear that η_p does not depend on the choice of u_p since $\overline{\Psi}$ is conjugate linear.

This gives a smooth non-vanishing section η of $L|_Y$. Now, let $\beta = c_1(L,\eta)$. Then $j^*(\beta) = c_1(L) = c_1(L_1) - c_1(L_2)$.

Without loss of generality, we assume that v is of unit length. Choose a section V_1 of $L_1|_\Sigma$ with only isolated singularities that extends v, and a section V_2 of $L_2|_\Sigma$ with only isolated singularities that extends $\Psi(v)$. Then it is easy to see that

$$\begin{aligned}\langle\beta,[\Sigma]\rangle &= \text{Sum of indices of singularities of } (V_1\otimes\mathcal{I}(V_2))\\ &= (\text{Sum of indices of singularities of } V_1)\\ &\quad -(\text{Sum of indices of singularities of } V_2)\\ &= \langle c_1(L_1,v),[\Sigma]\rangle - \langle c_1(L_2,\Psi(v)),[\Sigma]\rangle.\end{aligned}$$

□

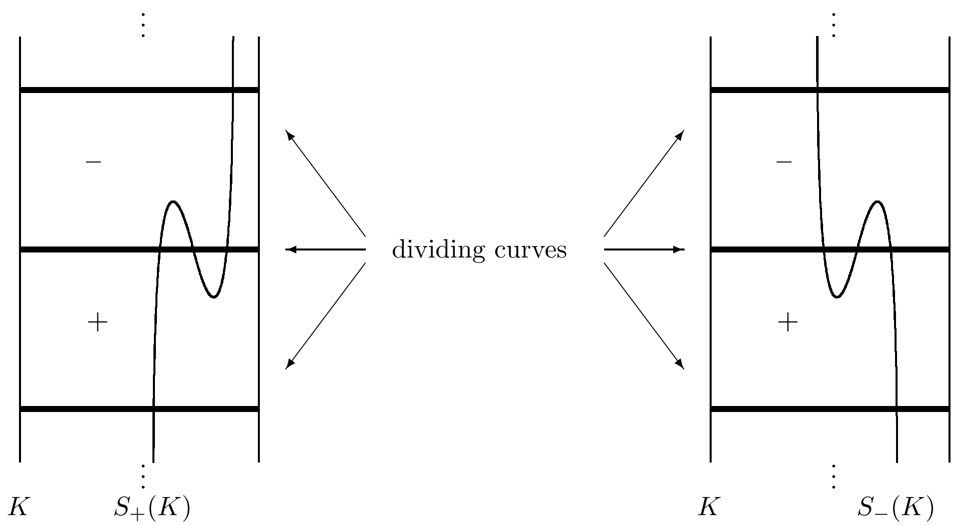

Figure 1. Positive and Negative Stabilizations.

Let K be a Legendrian knot in a contact 3-manifold (M,ξ). Choose an oriented embedded annulus $\widetilde{A}$ which has $-K$ as one of it is boundary components, and such that the index of the contact framing of K relative to the framing given by $\widetilde{A}$ is negative. We can isotope $\widetilde{A}$ relative to K to make it convex, and such that K has a standard annular collar A in $\widetilde{A}$. (See, e.g., [17] for the definition of standard annular collars.) Then, by Legendrian Realization Principle [17, Theorem 3.7], we can isotope A relative to K to make the curved lines in Figure 1 Legendrian without changing the dividing curves. Then these Legendrian curves are (Legendrianly isotopic to) the positive and negative stabilizations of K. By Giroux's Flexibility, we can again assume the stabilization has a standard annular collar neighborhood in A, and repeat the above process to obtain repeated stabilizations of K. This observation and [17, Proposition 4.5] give:

Lemma 3.6. *Let K be a Legendrian knot in a contact 3-manifold (M,ξ). Then there is an embedded convex annulus A in M, such that $\partial A = (-K)\cup K'$, and K' is (Legendrianly isotopic to) the repeated stabilization of K obtained by p positive stabilizations and $s-p$ negative stabilizations. Moreover, if u and u' are the unit tangent vector fields of K and K', then $\langle c_1(\xi,(-u)\sqcup u'),[A,\partial A]\rangle = 2p-s$.*

Proof of Theorem 1.4. For notational simplicity, we assume $L=K$ is a Legendrian knot, and K_i, $i=1,2$, is a Legendrian knot obtained from K by p_i positive stabilizations and $s-p_i$ negative stabilizations. The generalization to Legendrian links is straightforward.

Part (1). We assume that (M,ξ) is weakly filled by (W,ω).

First, by [8, Lemma 2.4], we isotope ξ in near K so that there is an open neighborhood U of K in W and a non-vanishing symplectic vector field v defined in U, s.t., $\xi|_{U\cap M} = \ker(\iota_v\omega|_{U\cap M})$, and v transversally points out of W along $U\cap M$. Let $\{\psi_t\}$ be the flow of v. Without loss of generality, we assume that $K_i \subset U\cap M$, and

$$U = \bigcup_{0\leq t<\tau} \psi_{-t}(U\cap M).$$

Let W' be the smooth 4-manifold obtained from W by attaching a 2-handle to W along K with the framing given by the contact framing of K plus $s+1$ left twists, and $M'=\partial W'$. Then the Legendrian surgeries along K_1 and K_2 give two contact structures ξ_1 and ξ_2 on M', and two corresponding symplectic structures ω_1 and ω_2 on W', such that (W',ω_i) is a weak symplectic filling of (M',ξ_i).

Lemma 3.7. *We can arrange that $[\omega_1]=[\omega_2]\in H^2(W';\mathbb{R})$.*

Proof. Choose a small $\varepsilon\in(0,\tau)$. Let $N=\psi_{-\varepsilon}(U\cap M)$, and $\hat{K}_i=\psi_{-\varepsilon}(K_i)$. Then We find a standard 2-handle $\mathcal{H}_2$, a neighborhood V of $\mathcal{H}_2\cap X_-$ in $\mathbb{R}^4$, and an embedding $\varphi_i : V\to U$, s.t., $\varphi^*(\omega)=\omega_{st}$, $\varphi_*(v_2)=v$, $\varphi(V\cap X_-)\subset N$, and $\varphi(S^1_-)=\hat{K}_i$. Since K_1 and K_2 are isotopic as framed knots, $\varphi_1|_{\mathcal{H}_2\cap X_-}$ and $\varphi_2|_{\mathcal{H}_2\cap X_-}$ are isotopic as smooth embeddings. So there is a smooth isotopy $\hat{\varphi}_s : \mathcal{H}_2\cap X_-\to N$, $1\leq s\leq 2$, s.t., $\hat{\varphi}_i=\varphi_i|_{\mathcal{H}_2\cap X_-}$. After a change of variable in s, we assume that

$$\hat{\varphi}_s = \begin{cases} \varphi_1|_{\mathcal{H}_2\cap X_-}, & \text{if } 1\leq s\leq 1.1;\\ \varphi_2|_{\mathcal{H}_2\cap X_-}, & \text{if } 1.9\leq s\leq 2.\end{cases}$$

Let $\widetilde{W}=W\times[1,2]$, and $\widetilde{\mathcal{H}}_2=\mathcal{H}_2\times[1,2]$. Define $\widetilde{\omega}$ and $\widetilde{\omega}_{st}$ to be the pull backs of ω and ω_{st} onto $\widetilde{W}$ and $\widetilde{\mathcal{H}}_2$. And define $\widetilde{v}$ and $\widetilde{v}_2$ to be the lifts of v and v_2 to $U\times[1,2]$ and $\widetilde{\mathcal{H}}_2$ that are tangent to the horizontal slices $U\times\{s\}$ and $\mathcal{H}_2\times\{s\}$, $1\leq s\leq 2$. Then $\iota_{\widetilde{v}}\widetilde{\omega}$ and $\iota_{\widetilde{v}_2}\widetilde{\omega}_{st}$ are the pull backs of $\iota_v\omega$ and $\iota_{v_2}\omega_{st}$.

Define $\hat{\Phi} : (\mathcal{H}_2\cap X_-)\times[1,2]\to N\times[1,2]$ by $\hat{\Phi}(p,s)=(\hat{\varphi}_s(p),s)$. By mapping the flow of $\widetilde{v}_2$ to the flow of $\widetilde{v}$, we extend $\hat{\Phi}$ to a diffeomorphism Φ from a neighborhood of $(\mathcal{H}_2\cap X_-)\times[1,2]$ in $\widetilde{\mathcal{H}}_2$ to a neighborhood of $\{\psi_\varepsilon\circ\hat{\varphi}_s(S^1_-)\mid 1\leq s\leq 2\}$ ($\subset M\times[1,2]$) in $\widetilde{W}$. Clearly, we have $\Phi_*(\widetilde{v}_2)=\widetilde{v}$, and, near $(\mathcal{H}_2\cap X_-)\times\{1,2\}$, we have $\Phi^*(\widetilde{\omega})=\widetilde{\omega}_{st}$.

Consider the 1-form $\Phi^*(\iota_{\widetilde{v}}\widetilde{\omega})$ defined in a neighborhood of $(\mathcal{H}_2 \cap X_-) \times [1,2]$ in $\widetilde{\mathcal{H}}_2$. It equals $\iota_{\widetilde{v}_2}\widetilde{\omega}_{st}$ near $(\mathcal{H}_2 \cap X_-) \times \{1,2\}$. So, there is a 1-form $\widetilde{\alpha}$ on $\widetilde{\mathcal{H}}_2$, s.t.,

$$\widetilde{\alpha} = \begin{cases} \Phi^*(\iota_{\widetilde{v}}\widetilde{\omega}), & \text{near } (\mathcal{H}_2 \cap X_-) \times [1,2]; \\ \iota_{\widetilde{v}_2}\widetilde{\omega}_{st}, & \text{near } \mathcal{H}_2 \times \{1,2\}. \end{cases}$$

Define

$$\widetilde{W}' = \widetilde{W} \cup_\Phi \widetilde{\mathcal{H}}_2, \text{ and } \widetilde{\omega}' = \begin{cases} \widetilde{\omega}, & \text{on } \widetilde{W}; \\ d\widetilde{\alpha}, & \text{on } \widetilde{\mathcal{H}}_2. \end{cases}$$

Then $\widetilde{\omega}'$ is a well defined closed 2-form on $\widetilde{W}'$.

For $s \in [1,2]$, let

$$g_s : W' \to \widetilde{W}'$$

be the embedding given by $g_s(p) = (p,s)$ for any point p in W or $\mathcal{H}_2$. Then $\{g_s\}$ is an isotopy of embeddings of W' into $\widetilde{W}'$. For $i = 1,2$, let $\omega_i = g_i^*(\widetilde{\omega}')$. Then (W', ω_i) is a weak symplectic filling of (M', ξ_i), and $[\omega_1] = [\omega_2] \in H^2(W'; \mathbb{R})$. □

Now we are in the situation that the contact 3-manifold (M', ξ_i) is weakly symplectically filled by (W', ω_i) for $i = 1,2$, ξ_1 and ξ_2 are isotopic, and $[\omega_1] = [\omega_2]$. Let $\mathfrak{s}_i$ be the canonical $Spin^{\mathbb{C}}$-structure of (W', ω_i). Suppose that $\mathfrak{s}_1 \neq \mathfrak{s}_2$. Then, by Lemma 3.4, we have

$$\deg P_{\xi_1, \mathfrak{s}_1; [\omega_1]} = \deg P_{\xi_2, \mathfrak{s}_1; [\omega_2]} > \deg P_{\xi_2, \mathfrak{s}_2; [\omega_2]},$$

and, similarly,

$$\deg P_{\xi_2, \mathfrak{s}_2; [\omega_2]} = \deg P_{\xi_1, \mathfrak{s}_2; [\omega_1]} > \deg P_{\xi_1, \mathfrak{s}_1; [\omega_1]}.$$

This is a contradiction. Thus, $\mathfrak{s}_1 = \mathfrak{s}_2$.

Next we construct a symplectic decomposition of (TW', ω_i) in a neighborhood of the 2-handle $\mathcal{H}_2$. First, define a 2-plane distribution $\widetilde{\xi}$ on U by $\widetilde{\xi}|_{\psi_t(p)} = \psi_{t*}(\xi_p)$ for $p \in M$. And let $\widetilde{\eta} = \widetilde{\xi}^{\perp_\omega}$, the ω-normal bundle of $\widetilde{\xi}$. Clearly, v is a non-vanishing section of $\widetilde{\eta}$.

Define $\Theta : \mathbb{R}^4 \setminus \{0\} \to Sp(4)$ by

$$\Theta(x_1,y_1,x_2,y_2) = \frac{1}{\sqrt{4x_1^2 + y_1^2 + 4x_2^2 + y_2^2}} \begin{pmatrix} 2x_1 & y_1 & -2x_2 & y_2 \\ -y_1 & 2x_1 & -y_2 & -2x_2 \\ 2x_2 & y_2 & 2x_1 & -y_1 \\ -y_2 & 2x_2 & y_1 & 2x_1 \end{pmatrix}.$$

Note that Θ factors through the natural inclusion of $SU(2)$ into $Sp(4)$ induced by

$$a + bi \mapsto \begin{pmatrix} a & -b \\ b & a \end{pmatrix}.$$

Since $SU(2)$ is simply connected, we can modify $\Theta|_{\mathcal{H}_2}$ in a small neighborhood of the intersection $\mathcal{H}_2 \cap \{y_1 = y_2 = 0\}$, and then extend it into a smooth map $\hat{\Theta} : \mathcal{H}_2 \to Sp(4)$ (c.f. [14, Proposition 2.3]). Now let $\{e_1, e_2, e_3, e_4\}$ be the symplectic frame of $T\mathbb{R}^4|_{\mathcal{H}_2}$ defined by

$$(e_1, e_2, e_3, e_4) = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial y_2}) \cdot \hat{\Theta}.$$

Let φ_i be the symplectic attaching map used above to construct (W', ω_i), which is a symplectic diffeomorphism from a neighborhood of $\mathcal{H}_2 \cap X_-$ to a neighborhood of

K_i in U. Note that φ_i maps v_2 ($=\sqrt{4x_1^2+y_1^2+4x_2^2+y_2^2}\cdot e_1$ in the attaching region) to v. So, in the attaching region, φ_i identifies $\widetilde{\xi}$ with the 2-plane distribution on $\mathcal{H}_2$ spanned by $\{e_3,e_4\}$, and identifies $\widetilde{\eta}$ with the 2-plane distribution on $\mathcal{H}_2$ spanned by $\{e_1,e_2\}$. Let

$$\widetilde{\xi}_i=\begin{cases}\widetilde{\xi}, & \text{on } U;\\ \langle e_3,e_4\rangle, & \text{on } \mathcal{H}_2.\end{cases} \quad \text{and} \quad \widetilde{\eta}_i=\begin{cases}\widetilde{\eta}, & \text{on } U;\\ \langle e_1,e_2\rangle, & \text{on } \mathcal{H}_2.\end{cases}$$

Then

$$TW'|_{U\cup_{\varphi_i}\mathcal{H}_2}=\widetilde{\xi}_i\oplus\widetilde{\eta}_i.$$

And $\widetilde{\xi}_i$ and $\widetilde{\eta}_i$ are ω_i-orthogonal to each other. Also, it easy to see that $\widetilde{\eta}_i$ has a non-vanishing section since we can modify v_2 near the intersection $\mathcal{H}_2\cap\{y_1=y_2=0\}$, and then extend it to a non-vanishing multiple of e_1.

Choose an almost complex structure J_i on $U\cup_{\varphi_i}\mathcal{H}_2$ compatible with $\omega_i|_{U\cup_{\varphi_i}\mathcal{H}_2}$ so that $\widetilde{\xi}_i$ and $\widetilde{\eta}_i$ are complex sub-bundles of $(TW'|_{U\cup_{\varphi_i}\mathcal{H}_2},J_i)$. Then $\widetilde{\eta}_i$ becomes a trivial complex line bundle. Note that $\mathfrak{s}_i|_{U\cup_{\varphi_i}\mathcal{H}_2}$ is the $Spin^{\mathbb{C}}$-structure associated to J_i. There are natural isomorphisms of complex line bundles

$$\det(\mathfrak{s}_i)|_{U\cup_{\varphi_i}\mathcal{H}_2}\ \cong\ \det(TW'|_{U\cup_{\varphi_i}\mathcal{H}_2},J_i)\ \cong\ \widetilde{\xi}_i.$$

Moreover, there is a natural isomorphism

$$\det(\mathfrak{s}_i)|_W\cong\det(\mathfrak{s}),$$

where $\mathfrak{s}$ is the $Spin^{\mathbb{C}}$-structure on W associated to ω.

Let $A_i\subset M$ be the annulus bounded by $(-K)\cup K_i$ given in Lemma 3.6, and

$$\Sigma_i=A_i\cup(\text{the core of the 2-handle attached to } K_i),$$

oriented so that $\partial\Sigma_i=-K$. Then $[\Sigma_1]=[\Sigma_2]\in H_2(W',W)$. And, by Lemma 3.5, there exists $\beta\in H^2(W',W)$, such that $j^*(\beta)=c_1(\det(\mathfrak{s}_1))-c_1(\det(\mathfrak{s}_2))=0$, and

$$\begin{aligned}\langle\beta,[\Sigma_1]\rangle &= \langle c_1(\det(\mathfrak{s}_1),-\mu_1),[\Sigma_1]\rangle-\langle c_1(\det(\mathfrak{s}_2),-\mu_2),[\Sigma_1]\rangle\\ &= \langle c_1(\widetilde{\xi}_1,-u),[\Sigma_1]\rangle-\langle c_1(\widetilde{\xi}_2,-u),[\Sigma_1]\rangle\\ &= \langle c_1(\widetilde{\xi}_1,-u),[\Sigma_1]\rangle-\langle c_1(\widetilde{\xi}_2,-u),[\Sigma_2]\rangle,\end{aligned}$$

where u is the unit tangent vector field of K, and μ_i is the section of $\det(\mathfrak{s}_i)|_K$ identified with u through the above isomorphisms.

Denote by u_i the unit tangent vector field of K_i. Then u_i extends over the core of the 2-handle as a non-vanishing multiple of e_3. So, by Lemma 3.6, we have $\langle c_1(\widetilde{\xi}_i,-u),[\Sigma_i]\rangle=\langle c_1(\xi,(-u)\sqcup u_i),[A_i,\partial A_i]\rangle=2p_i-s$. Thus, $\langle\beta,[\Sigma_1]\rangle=2(p_1-p_2)$. But, since $j^*(\beta)=0$, there exists $\varsigma\in H^1(W)$, s.t., $\delta(\varsigma)=\beta$, where δ is the connecting map in the long exact sequence of the pair (W',W). So $2(p_1-p_2)=\langle\delta(\varsigma),[\Sigma_1]\rangle=\langle\varsigma,-[K]\rangle$. This implies $p_1=p_2$ when $[K]$ is torsion, and $2p_1\equiv 2p_2 \mod d$ when $[K]$ is non-torsion, where $d=\gcd\{\langle\zeta,[K]\rangle|\zeta\in H^1(W)\}$.

Part (2). We assume that $c^+(\xi)\neq 0$.

Consider the symplectic 4-manifold $(M\times I,d(e^t\alpha))$, where α is a contact form for ξ, and t is the variable of I. Note that $\frac{\partial}{\partial t}$ is a symplectic vector field in this setting, and it transversally points out of $M\times I$ along $M\times\{1\}$. The flow of $\frac{\partial}{\partial t}$ is the translation in the I-direction. Let $\widetilde{\xi}$ be the 2-plane distribution on $M\times I$ generated by translating ξ in the I-direction, and $\widetilde{\eta} = \widetilde{\xi}^{\perp_{d(e^t\alpha)}}$, the $d(e^t\alpha)$-normal bundle of $\widetilde{\xi}$. Note that $\frac{\partial}{\partial t}$ is a section of $\widetilde{\eta}$.

We perform Legendrian surgery along $K_i \times \{1\}$. Let φ_i be the symplectic attaching map, which is a symplectic diffeomorphism from a neighborhood of S^1_- in $\mathcal{H}_2$ to a neighborhood of $K_i \times \{1\}$ in $M \times I$. Let

$$W = (M \times I) \cup_{\varphi_1} \mathcal{H}_2 \cong (M \times I) \cup_{\varphi_2} \mathcal{H}_2.$$

Then the two Legendrian surgeries give two symplectic structures ω_1 and ω_2 on W, so that (W, ω_i) is a symplectic cobordism from (M, ξ) to (M', ξ_i). Similar to the construction used in Part (1), we construct an ω_i-orthogonal decomposition

$$TW = \widetilde{\xi}_i \oplus \widetilde{\eta}_i,$$

where $\widetilde{\xi}_i|_{M\times I} = \widetilde{\xi}$, $\widetilde{\eta}_i|_{M\times I} = \widetilde{\eta}$, and, moreover, $\frac{\partial}{\partial t}$ extends to a non-vanishing section of $\widetilde{\eta}_i$. Let J_i be an almost complex structure on W compatible with ω_i such that both $\widetilde{\xi}_i$ and $\widetilde{\eta}_i$ are complex sub-bundles of (TW, J_i). Then $\widetilde{\eta}_i$ becomes a trivial complex line bundle over W, and, hence, $c_1(J_i) = c_1(\widetilde{\xi}_i)$.

Let $\mathfrak{s}_i$ be the canonical $Spin^{\mathbb{C}}$-structure associated to J_i. Then it is also the canonical $Spin^{\mathbb{C}}$-structure associated to ω_i. If $\mathfrak{s}_1$ and $\mathfrak{s}_2$ are non-isomorphic, according to Proposition 3.3, we have

$$\begin{aligned} F^+_{W,\mathfrak{s}_1}(c^+(\xi_1)) &= F^+_{W,\mathfrak{s}_2}(c^+(\xi_2)) \;=\; c^+(\xi) \neq 0 \\ F^+_{W,\mathfrak{s}_1}(c^+(\xi_2)) &= F^+_{W,\mathfrak{s}_2}(c^+(\xi_1)) \;=\; 0. \end{aligned}$$

But ξ_1 and ξ_2 are isotopic, this is impossible. So $\mathfrak{s}_1$ and $\mathfrak{s}_2$ are isomorphic, and, hence, $c_1(\widetilde{\xi}_1) = c_1(\widetilde{\xi}_2)$.

Let A_i be the annulus in $M \times \{0\}$ bounded by $(-K) \times \{0\} \cup K_i \times \{0\}$ given by Lemma 3.6, and

$$\Sigma_i = A_i \cup (K_i \times I) \cup (\text{the core of the 2-handle attached to } K_i \times \{1\}),$$

oriented so that $\partial\Sigma = -K \times \{0\}$. Then Σ_1 and Σ_2 are isotopic relative to boundary. And, by Lemma 3.5, there exists $\beta \in H^2(W, M)$, such that $j^*(\beta) = c_1(\widetilde{\xi}_1) - c_1(\widetilde{\xi}_2) = 0$, and

$$\begin{aligned} \langle \beta, [\Sigma_1] \rangle &= \langle c_1(\widetilde{\xi}_1, -u), [\Sigma_1] \rangle - \langle c_1(\widetilde{\xi}_2, -u), [\Sigma_1] \rangle \\ &= \langle c_1(\widetilde{\xi}_1, -u), [\Sigma_1] \rangle - \langle c_1(\widetilde{\xi}_2, -u), [\Sigma_2] \rangle, \end{aligned}$$

where u is the unit tangent vector field of $K \times \{0\}$. Denote by u_i the unit tangent vector field of $K_i \times \{0\}$. Then, as in Part (1), u_i extends over $K_i \times I$ and the core of the 2-handle without singularities. So, by Lemma 3.6, we have $\langle c_1(\widetilde{\xi}_i, -u), [\Sigma_i] \rangle = \langle c_1(\xi, (-u) \sqcup u_i), [A_i, \partial A_i] \rangle = 2p_i - s$. Thus, $\langle \beta, [\Sigma_1] \rangle = 2(p_1 - p_2)$. But, since $j^*(\beta) = 0$, there exists $\varsigma \in H^1(M)$, s.t., $\delta(\varsigma) = \beta$, where δ is the connecting map in the long exact sequence of the pair $(W, M \times \{0\})$. So $2(p_1 - p_2) = \langle \delta(\varsigma), [\Sigma_1] \rangle = \langle \varsigma, -[K] \rangle$. This implies $p_1 = p_2$ when $[K]$ is torsion, and $2p_1 \equiv 2p_2 \mod d$ when $[K]$ is non-torsion, where $d = \gcd\{\langle \zeta, [K] \rangle | \zeta \in H^1(M)\}$. □

Remark 3.8. The weakly fillable case of Theorem 1.4 can also be proved using the monopole invariant defined by Kronheimer and Mrowka [19]. Indeed, in Part (1) of the proof, after proving Lemma 3.7, we are in the situation where ξ_1 and ξ_2 are isotopic, and $[\omega_1] = [\omega_2] \in H^2(W'; \mathbb{R})$. After a possible isotopy supported near M', we assume that $\xi_1 = \xi_2 = \xi'$. Let $\mathfrak{s}_i \in Spin^{\mathbb{C}}(W', \xi')$ be the element associated to ω_i. Then, by [19, Theorems 1.1 and 1.2], we have $[\omega_1] \cup (\mathfrak{s}_2 - \mathfrak{s}_1) \geq 0$, and $[\omega_2] \cup (\mathfrak{s}_1 - \mathfrak{s}_2) \geq 0$. But $[\omega_1] = [\omega_2]$. Thus, $[\omega_1] \cup (\mathfrak{s}_2 - \mathfrak{s}_1) = 0$. And, according to [19, Theorem 1.2], this implies that $\mathfrak{s}_1 = \mathfrak{s}_2$ as elements of $Spin^{\mathbb{C}}(W', \xi')$, and, specially, that $c_1(\mathfrak{s}_1) = c_1(\mathfrak{s}_2)$. Then we can repeat the rest of Part (1) of the proof, and prove the weakly fillable case of the theorem.

4. Tight contact structures on Brieskorn homology spheres $-\Sigma(2, 3, 6n - 1)$

A small Seifert fibered manifold is a 3-manifold Seifert fibered over S^2 with 3 singular fibers. We denote by $M(r_1, r_2, r_3)$ the small Seifert fibered manifold with 3 singular fibers with coefficients r_1, r_2 and r_3.

The classification of tight contact structures on a small Seifert fibered manifold is a hard problem. When the Euler number of the small Seifert fibered manifold is not -1 or -2, these tight contact structures are all Stein fillable, and are classified in [12, 27]. Note all these manifolds are L-spaces, i.e. have Heegaard-Floer homology like that of a lens space. There are also partial results when the Euler number is -1 or -2, and the manifold is an L-space (see e.g. [11]). In solving these examples, the use of untwisted Ozsváth-Szabó contact invariant is essential. It appears that the classification is much harder to achieve when the the small Seifert fibered manifold is not an L-space.

Brieskorn homology sphere $-\Sigma(2, 3, 6n - 1)$ is the small Seifert fibered manifold $M(-\frac{1}{2}, \frac{1}{3}, \frac{n}{6n-1})$, which is not an L-space when $n \geq 2$. These appear to be good examples of non-L-space small Seifert fibered manifolds to start with. In [13], Ghiggini and Schönenberger showed that there is a unique tight contact structure on $-\Sigma(2, 3, 11)$. This method was extended to classify contact structures on $-\Sigma(2, 3, 17)$ in [10]. Next we discuss the generalization of their method.

Let Σ be an oriented three-hole sphere with boundary components C_1, C_2 and C_3. Then $-\partial\Sigma \times S^1 = T_1 + T_2 + T_3$, where the "$-$" sign means reversing the orientation and $T_i = -C_i \times S^1$. We identify T_i to $\mathbb{R}^2/\mathbb{Z}^2$ by identifying $-C_i \times \{\text{pt}\}$ to $(1, 0)^T$, and $\{\text{pt}\} \times S^1$ to $(0, 1)^T$. Also, for $i = 1, 2, 3$, let $V_i = D^2 \times S^1$, and identify ∂V_i with $\mathbb{R}^2/\mathbb{Z}^2$ by identifying a meridian $\partial D^2 \times \{\text{pt}\}$ with $(1, 0)^T$ and a longitude $\{\text{pt}\} \times S^1$ with $(0, 1)^T$.

Define diffeomorphism $\varphi_i : \partial V_i \to T_i$ by the following matrices.

$$\varphi_1 = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \varphi_2 = \begin{pmatrix} 3 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \varphi_3 = \begin{pmatrix} 6n-1 & 6 \\ -n & -1 \end{pmatrix}.$$

Then

$$-\Sigma(2, 3, 6n - 1) \cong M(-\frac{1}{2}, \frac{1}{3}, \frac{n}{6n-1}) \cong (\Sigma \times S^1) \cup_{(\varphi_1 \cup \varphi_2 \cup \varphi_3)} (V_1 \cup V_2 \cup V_3).$$

Note that each S^1-fiber in the product $\Sigma \times S^1$ becomes a regular fiber of the Seifert fibration, and the framing of the S^1-fiber from the product is the same as the standard framing of a regular fiber of the Seifert fibration. Also, the core curve of each V_i becomes a singular fiber of the Seifert fibration, and our choice of the longitude of ∂V_i gives each singular fiber a framing. If ξ is a contact structure on $-\Sigma(2, 3, 6n - 1)$, and K is a Legendrian regular fiber (resp. Legendrian singular fiber) of the Seifert fibration, then the twisting number $t(K)$ of K is defined to be the index of the contact framing of K with respect to the standard framing (resp. the framing we chose). We define

$$t(\xi) = \max\{t(K) \mid K \text{ is a Legendrian regular fiber.}\}$$

Etnyre and Honda [7] showed that $-\Sigma(2,3,5)$ does not admit tight contact structures. So we assume that $n \geq 2$ in the discussions below. Next two lemmas are proved following the arguments in [13, Subsection 4.2]. Similar methods were also used in e.g. [27].

Lemma 4.1. *If ξ is a tight contact structure on $-\Sigma(2,3,6n-1)$, then $t(\xi) \leq -2$.*

Proof. We prove the lemma in two steps: first prove that $t(\xi) < 0$, and then prove that $t(\xi) \neq -1$.

Assume $t(\xi) \geq 0$. Then we can find a Legendrian regular fiber F with twisting number 0. After possibly an isotopy, assume F is contained in the piece $\Sigma \times S^1$. Let F_i be a Legendrian knot C^0-close to the core curve of V_i. After repeated stabilization of F_i, we assume that $t(F_i) = n_i << 0$. After isotopy, assume that V_i is a standard neighborhood of F_i. Then ∂V_i is convex and has two parallel dividing curves of slope $\frac{1}{n_i}$. Now use the coordinates of T_i. Then the slopes of dividing curves of T_1, T_2 and T_3 are $s_1 = \frac{n_1}{2n_1-1}$, $s_2 = -\frac{n_2}{3n_2+1}$ and $s_3 = -\frac{nn_3+1}{(6n-1)n_3+6}$, respectively. Since $n_i << 0$, we have $s_1 > 0$, $s_2 > -\frac{1}{2}$ and $s_3 > -\frac{1}{5}$. Then we can isotope T_i as in [13, Subsection 4.2.2] and get a decomposition

$$\Sigma \times S^1 = (\Sigma' \times S^1) \cup (T_1 \times [0,1]) \cup (T_2 \times [0,1]) \cup (T_3 \times [0,1]),$$

such that

- Σ' is a three-sphere in Σ with
$$\partial\Sigma' \times S^1 = (-T_1 \times \{1\}) \cup (-T_2 \times \{1\}) \cup (-T_3 \times \{1\});$$
- $\xi|_{T_i \times [0,1]}$ is a minimal twisting tight contact structure with minimal convex boundary;
- The slopes of dividing curves on $T_1 \times \{0\}, T_2 \times \{0\}, T_3 \times \{0\}$ are $0, -\frac{1}{2}, -\frac{1}{5}$, respectively, and the slopes of dividing curves on $T_1 \times \{1\}, T_2 \times \{1\}, T_3 \times \{1\}$ are ∞.

Then we can follow the arguments in the proof of [13, Theorem 4.14] to show that ξ must be overtwisted. This contradiction shows that $t(\xi) < 0$.

Now assume that $t(\xi) = -1$. Let $F \subset \Sigma \times S^1$ be a Legendrian regular fiber with $t(F) = -1$, and V_i a standard neighborhood of a Legendrian singular fiber F_i with $t(F_i) = n_i << 0$. For $i = 1, 2$, connect F to ∂V_i by a vertical convex annulus A_i that intersects the dividing curves of ∂V_i efficiently. By Imbalance Principle [17, Proposition 3.17], there is a ∂-parallel dividing curve on A_i along $A_i \cap (\partial V_i)$. Using the bypass from this ∂-parallel dividing curve, by the Twisting Number Lemma [17, Lemma 4.4], we can increase n_i by 1. Repeat this procedure, we can increase n_1, n_2 up to $n_1 = 0$, $n_2 = -1$. When measured in the coordinates of T_i, the dividing curves on T_1 and T_2 have slopes 0 and $-\frac{1}{2}$. Connecting T_1 to T_2 by a vertical convex annulus A in $\Sigma \times S^1$ with ∂A intersecting the dividing curves of T_1, T_2 efficiently. Then, by Imbalance Principle, there is a ∂-parallel dividing curve on A along $A \cap T_2$. Adding the bypass from this dividing curve to T_2, we change the slope of dividing curves of T_2 to -1. Connect T_1 to this new T_2 by a vertical convex annulus A' in $\Sigma \times S^1$ with $\partial A'$

intersecting the dividing curves of T_1, T_2 efficiently. If there are ∂-parallel dividing curves on A', then, by Legendrian Realization Principle [17, Theorem 3.7], we can find a Legendrian regular fiber with twisting number 0. This is a contradiction. If there are no ∂-parallel dividing curves on A', cut $\Sigma \times S^1$ along A' and smooth the edges. This gives us torus T isotopic to T_3 with dividing curves of slope 0. Note that the slope of dividing curves of T_3 is negative since $n_3 << 0$. By [17, Proposition 4.16], there is a torus isotopic to T_3 with vertical dividing curves (isotopic to a regular fiber.) By the Legendrian Realization Principle, we can again find a Legendrian regular fiber with twisting number 0, which is a contradiction. This implies that $t(\xi) \neq -1$. □

Lemma 4.2. *There are at most $\frac{n(n-1)}{2}$ pairwise non-isotopic tight contact structures on the Brieskorn homology sphere $-\Sigma(2, 3, 6n - 1)$.*

Proof. Let ξ be a tight contact structure on $-\Sigma(2, 3, 6n - 1)$ with $t(\xi) = t$, where $t \leq -2$ by Lemma 4.1. Let $F \subset \Sigma \times S^1$ be a Legendrian regular fiber with $t(F) = t$. Isotope V_i into a standard neighborhood of a Legendrian singular fiber F_i with $t(F_i) = n_i << 0$. For $i = 1, 2$, connect F to ∂V_i by a vertical convex annulus A_i that intersects the dividing curves of ∂V_i efficiently.

First consider the annulus A_1. Using the Imbalance Principle and the Twisting Number Lemma, we can increase n_1 by 1, and repeat this process till either $n_1 = 0$ or $|2n_1 - 1| \leq |t|$, whichever comes first. If $n_1 = 0$ comes first, then we have $t(F) = -1$, which is a contradiction. This means that the procedure stops at an integer $n_1 \leq -1$ with $|2n_1 - 1| \leq |t|$. If $|2n_1 - 1| < |t|$, then we can use the Imbalance Principle to increase the twisting number of F, which contradicts our choice of F. So $|2n_1 - 1| = |t|$, which implies that $t = 2n_1 - 1 \leq -3$.

Next consider the annulus A_2. Using the Imbalance Principle and the Twisting Number Lemma, we can increase n_2 by 1, and repeat this process till either $n_2 = -1$ or $|3n_2 + 1| \leq |t|$, whichever comes first. If $n_1 = -1$ comes first, then we have $t(F) \geq -2$, which is a contradiction. This means that the procedure stops at an integer $n_2 \leq -2$ with $|3n_2 + 1| \leq |t|$. If $|3n_2 + 1| < |t|$, then we can use the Imbalance Principle to increase the twisting number of F, which contradicts our choice of F. So $|3n_2 + 1| = |t|$, which implies that $t = 3n_2 + 1 \leq -5$.

Clearly, there is a positive integer m satisfying $t = 1 - 6m$, $n_1 = 1 - 3m$ and $n_2 = -2m$. Now connect T_1 and T_2 by a vertical convex annulus A with Legendrian boundary intersecting the dividing curves of T_1, T_2 efficiently. If A has ∂-parallel dividing curves, then we can use the Legendrian Realization Principle to find a Legendrian regular fiber with twisting number greater than t, which contradicts our choice of t. So every dividing curve of A connects one boundary component of A to the other. Cut $\Sigma \times S^1$ along A and smooth the edges. We get a torus T isotopic to T_3 with dividing curves of slope $-\frac{m}{6m-1}$. If $m \geq n$, then

$$-\frac{m}{6m-1} \geq -\frac{n}{6n-1} > s_3 = -\frac{nn_3 + 1}{(6n-1)n_3 + 6},$$

where $s_3 = -\frac{nn_3+1}{(6n-1)n_3+6}$ is the slope of dividing curves of T_3. By [17, Proposition 4.16], there is a torus isotopic to T_3 with vertical dividing curves (isotopic to a regular fiber.) By the Legendrian Realization Principle, we can again find a Legendrian regular fiber with twisting number 0, which is a contradiction. This shows that $m < n$.

The torus T separates $-\Sigma(2,3,6n-1)$ into two sides. One side is a solid torus V isotopic to V_3. The other side $(-\Sigma(2,3,6n-1)) \setminus V$ is the union of V_1, V_2 and a neighborhood of the annulus A. The dividing curves of A are unique up to an isotopy of A fixing one boundary component since none of the dividing curves is ∂-parallel. Fix the dividing curves on A, since V_1 and V_2 are standard neighborhoods of Legendrian knots. it is easy to see that $\xi|_{(-\Sigma(2,3,6n-1))\setminus V}$ is uniquely determined up to isotopy relative to T. When measured in the coordinates of V_3, the slope of dividing curves of T is $m-n$. So, by [17, Theorem 2.3], up to isotopy relative to T, there are $n-m$ tight contact structures on V satisfying the given boundary condition. Note that, for each pair of possible dividing sets of A, there is an isotopy of $-\Sigma(2,3,6n-1)$ that maps one of them to the other. Thus, up to isotopy of $-\Sigma(2,3,6n-1)$, there are at most $n-m$ tight contact structures on $-\Sigma(2,3,6n-1)$ with twisting number $1-6m$. So the number of tight contact structures up to isotopy on $-\Sigma(2,3,6n-1)$ is at most

$$\frac{n(n-1)}{2} = \sum_{m=1}^{n-1}(n-m).$$

□

It seems that the number of tight contact structures on $-\Sigma(2,3,6n-1)$ is exactly $\frac{n(n-1)}{2}$ since there are actually $\frac{n(n-1)}{2}$ different Legendrian surgery constructions of tight contact structures on $-\Sigma(2,3,6n-1)$. Before constructing these surgeries, we need some preliminaries about tight contact structures on the small Seifert fibered manifold $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$, which is also the torus bundle over S^1 given by the monodromy map $\psi : T^2 \to T^2$ induced by

$$\Psi = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} : \mathbb{R}^2 \to \mathbb{R}^2.$$

Proposition 4.3. [18, Theorem 0.1] *There is a sequence of pairwise non-isotopic tight contact structures* $\{\xi_m\}_{m=1}^{\infty}$ *on* $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$. *Any tight contact structure on* $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$ *is isotopic to one of the* ξ_m*'s.*

Proposition 4.4. [1, Propositions 15 and 16] *There is a simply connected symplectic manifold* (W,ω) *that weakly fills* $(M(-\frac{1}{2},\frac{1}{3},\frac{1}{6}),\xi_m)$ *for* $\forall\ m \geq 1$.

Proof. Such a symplectic manifold (W,ω) is constructed in [1, Propositions 15 and 16]. We only need to show that W is simply connected. Note that

$$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

By the construction of W, there is a Lefschetz fibration $W \to D^2$ which has exactly two singular points. The vanishing circles of these two singular points induce a $\mathbb{Z}$-basis for $\pi_1(T^2) \cong H_1(T^2) \cong \mathbb{Z}^2$. By [15, Proposition 8.1.9], there is an exact sequence

$$\pi_1(T^2) \to \pi_1(W) \to \pi_1(D^2)(=0),$$

where the first map is induced by the inclusion of T^2 into W as a regular fiber, and the second is induced by the projection. It follows that $\pi_1(W) = 0$. □

The point $(0,0)^T \in \mathbb{R}^2$ induces the unique fixed point of ψ, and gives a closed orbit K_0 in $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$, which is isotopic to the $\frac{1}{6}$-singular fiber of the Seifert fibration. The torus bundle structure gives K_0 a standard framing (c.f. [10]). For any Legendrian knot K in a tight contact manifold $(M(-\frac{1}{2},\frac{1}{3},\frac{1}{6}),\xi)$ that is smoothly isotopic to K_0, define its twisting number $t(K)$ to be the index of its contact framing relative to this standard framing. Denote by $t(\xi)$ the maximum of all such twisting numbers.

Proposition 4.5. [10, Lemma 3.5] $t(\xi_m) = -m$.

Performing a $(-n)$-surgery along the $\frac{1}{6}$-singular fiber of $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$ with respect to the standard framing, we get $-\Sigma(2,3,6n-1) \cong M(-\frac{1}{2},\frac{1}{3},\frac{n}{6n-1})$. For each $m \in \{1,\cdots,n-1\}$, let $K_0^{(m)}$ be a Legendrian knot in $(M(-\frac{1}{2},\frac{1}{3},\frac{1}{6}),\xi_m)$ smoothly isotopic to K_0 with $t(K_0^{(m)}) = -m$. If we stabilize $K_0^{(m)}$ $n-m-1$ times, and then perform a Legendrian surgery on resulted Legendrian knot, we get a weakly fillable tight contact structure on $-\Sigma(2,3,6n-1)$. (It is actually strongly fillable since $-\Sigma(2,3,6n-1)$ is an integral homology sphere, c.f. [6].) There are $n-m$ ways to perform such an iterated stabilization of $K_0^{(m)}$ depending on the number of positive stabilizations used in the process. Denote by $\xi_{m,p}$ the tight contact structure on $-\Sigma(2,3,6n-1)$ from the iterated stabilization of $K_0^{(m)}$ with p positive stabilization and $n-m-p-1$ negative stabilizations. This gives us a tight contact structure on $-\Sigma(2,3,6n-1)$ for each pair (m,p), where $1 \leq m \leq n-1$, and $0 \leq p \leq n-m-1$. Altogether, we get $\frac{n(n-1)}{2}$ tight contact structures on $-\Sigma(2,3,6n-1)$. The hard part is to show these tight contact structures are pairwise non-isotopic. Using Theorem 1.4, we have following partial result.

Proposition 4.6. *If $0 \leq p_1, p_2 \leq n-m-1$ and $p_1 \neq p_2$, then ξ_{m,p_1} and ξ_{m,p_2} are not isotopic.*

Proof. This is a straightforward consequence of part (1) of Theorem 1.4 and the fact that the symplectic filling (W,ω) of ξ_m is simply connected. □

Proof of Theorem 1.5. Let V be the cobordism from $M(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$ to $-\Sigma(2,3,6n-1)$ induced by the $(-n)$-surgery along the $\frac{1}{6}$-singular fiber. Then, from Theorem 3.2, we know that $F_V^+(c^+(\xi_{m,p})) = c^+(\xi_m)$. Ghiggini [10] showed that $c^+(\xi_1) \neq c^+(\xi_2)$. So ξ_{1,p_1} and ξ_{2,p_2} are non-isotopic for $0 \leq p_1 \leq n-2$ and $0 \leq p_2 \leq n-3$. Combine this with Proposition 4.6, we know that $\xi_{1,0},\cdots,\ \xi_{1,n-2},\ \xi_{2,0},\cdots,\ \xi_{2,n-3}$ are $2n-3$ pairwise non-isotopic tight contact structures on $-\Sigma(2,3,6n-1)$. □

The author hopes that, by a more careful computation of the Ozsváth-Szabó contact invariants, we can strengthen Theorem 1.5 and show that ξ_{m_1,p_1} and ξ_{m_2,p_2} are not isotopic when $m_1 \neq m_2$, which would complete the classification of tight contact structures on $-\Sigma(2,3,6n-1)$.

5. An example where our method does not apply

The author was informed of Example 5.1 by Ghiggini, which was proposed by Stipsicz.

Example 5.1. Consider the Stein fillable contact structure on $S^2 \times S^1$. Let K be any Legendrian knot that is smoothly isotopic to an S^1-fiber. Perform a Legendrian surgery on K, we get a Stein fillable contact 3-manifold, where the underlying smooth

3-manifold is S^3. To see this, note that $S^2 \times S^1$ can be constructed by performing a 0-surgery on an unknot in S^3, and an S^1-fiber comes from another unknot that links once with the surgery unknot. So, topologically, the result of performing a Legendrian surgery along K is the same as performing a surgery along a Hopf link in S^3, where one of its components has coefficient 0, and the other has an integer coefficient. This clearly gives S^3. But there is only one tight contact structure on S^3. This means the result of the Legendrian surgery here does not depend on the choice of the Legendrian knot.

Ghiggini further remarked that, in the setting of Theorem 1.4, if $[K]$ is a primitive element of $H_1(M)$, then $H^2((M \times I) \cup_{\varphi_i} \mathcal{H}_2) = H^2(M)$, and there is a unique $Spin^{\mathbb{C}}$-structure on $(M \times I) \cup_{\varphi_i} \mathcal{H}_2$ that extends the $Spin^{\mathbb{C}}$-structure on M given by the contact structure. So it is not possible to use $Spin^{\mathbb{C}}$-structures on the cobordism to distinguish between contact structures resulted from the Legendrian surgeries on stabilizations of K. Clearly, in the weakly fillable case of Theorem 1.4, if $[K]$ is a primitive element of $H_1(W)$, a similar remark applies. (These examples correspond to the situation when $d = 1$ in Theorem 1.4. And Theorem 1.4 does not give any information about the result contact structures when $d = 1, 2$.)

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References

[1] F. Ding and H. Geiges, *Symplectic fillability of tight contact structures on torus bundles*, Algebr. Geom. Topol. **1** (2001) 153–172.

[2] Y. Eliashberg, *Classification of overtwisted contact structures on* 3*-manifolds*, Invent. Math. **98** (1989), no. 3, 623–637.

[3] ———, *Filling by holomorphic discs and its applications*, in Geometry of low-dimensional manifolds, 2 (Durham, 1989), Vol. 151 of *London Math. Soc. Lecture Note Ser.*, 45–67, Cambridge Univ. Press (1990).

[4] ———, *Topological characterization of Stein manifolds of dimension* > 2, Internat. J. Math. **1** (1990), no. 1, 29–46.

[5] ———, *Legendrian and transversal knots in tight contact* 3*-manifolds*, in Topological methods in modern mathematics (Stony Brook, NY, 1991), 171–193, Publish or Perish (1993).

[6] ———, *A few remarks about symplectic filling*, Geom. Topol. **8** (2004), no. 3, 277–293.

[7] J. Etnyre and K. Honda, *On the nonexistence of tight contact structures*, Ann. of Math. (2) **153** (2001), no. 3, 749–766.

[8] ———, *Tight contact structures with no symplectic fillings*, Invent. Math. **148** (2002), no. 3, 609–626.

[9] P. Ghiggini, *Strongly fillable contact 3manifolds without Stein fillings*, Geom. Topol. **9** (2005) 16771687.

[10] ———, *Ozsváth-Szabó invariants and fillability of contact structures*, Math. Z. **253** (2006), no. 1, 159–175.

[11] P. Ghiggini, P. Lisca, and A. Stipsicz, *Tight contact structures on some small Seifert fibered 3-manifolds*. ArXiv:math/0509714v1.

[12] ———, *Classification of tight contact structures on small Seifert 3-manifolds with* $e_0 \geq 0$, Proc. Amer. Math. Soc. **134** (2006), no. 3, 909–916.

[13] P. Ghiggini and S. Schönenberger, *On the classification of tight contact structures*, in Topology and geometry of manifolds (Athens, GA, 2001), Vol. 71 of *Proc. Sympos. Pure Math.*, 121–151, Amer. Math. Soc. (2003).

[14] R. Gompf, *Handlebody construction of Stein surfaces*, Ann. of Math. (2) **148** (1998), no. 2, 619–693.

[15] R. Gompf and A. Stipsicz, 4-manifolds and Kirby calculus, Amer. Math. Soc. (1999), ISBN 0-8218-0994-6.

[16] M. Gromov, *Pseudo holomorphic curves in symplectic minifolds*, Invent. Math. **82** (1985), no. 2, 307–347.

[17] K. Honda, *On the classification of tight contact structures I*, Geom. Topol. **4** (2000) 309–368.

[18] ———, *On the classification of tight contact structures II*, J. Differential Geom. **55** (2000), no. 1, 83–143.

[19] P. Kronheimer and T. Mrowka, *Monopoles and contact structures*, Invent. Math. **130** (1997), no. 2, 209–255.

[20] P. Lisca and G. Matić, *Tight contact structures and Seiberg-Witten invariants*, Invent. Math. **129** (1997), no. 3, 509–525.

[21] P. Ozsváth and Z. Szabó, *Holomorphic disks and genus bounds*, Geom. Topol. **8** (2004) 311–334.

[22] ———, *Holomorphic triangle invariants and the topology of symplectic four-manifolds*, Duke Math. J. **121** (2004), no. 1, 1–34.

[23] ———, *Heegaard Floer homology and contact structures*, Duke Math. J. **129** (2005), no. 1, 39–61.

[24] ———, *Holomorphic triangles and invariants for smooth four-manifolds*, Adv. Math. **202** (2006), no. 2, 326–400.

[25] O. Plamenevskaya, *Contact structures with distinct Heegaard Floer invariants*, Math. Res. Lett. **11** (2004), no. 4, 547–561.

[26] A. Weinstein, *Contact surgery and sympletic handlebodies*, Hokkaido Math. J. **20** (1991), no. 2, 241–251.

[27] H. Wu, *Legendrian Vertical Circles in Small Seifert Spaces*, Commun. Contemp. Math. **8** (2006), no. 2, 219–246.

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