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CORRIGENDUM : SOME *IHX*-TYPE RELATIONS ON TRIVALENT GRAPHS AND SYMPLECTIC REPRESENTATION THEORY

MATHEMATICAL RESEARCH LETTERS, **5** (1998), 391 – 402.

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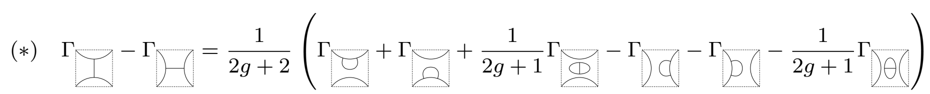
We correct the statement of Theorem 1 of [GN] as follows.

Theorem 1. *There exists a surjective homomorphism of graded algebras*

$$\mathcal{C}(\phi)/(IH_0^{bis}, loop) \longrightarrow (\Lambda U/([2^2]_{\mathfrak{sp}}))^{\mathfrak{sp}}$$

which multiplies degrees by 2. It gives also an isomorpshism in the range of $3m \le g$.

Recall that $\mathcal{C}(\phi)$ is the commutative graded algebra freely generated by the trivalent graphs, and '*loop*' denotes the ideal generated by graphs containing a 1-loop. The ideal IH_0^{bis} of $\mathcal{C}(\phi)$ is the correct substitute for IH_0 in loc. cit. and is, by definition, generated by the relations of type:

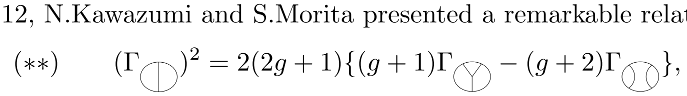


$$(*)\quad \Gamma - \Gamma = \frac{1}{2g+2}\left(\Gamma + \Gamma + \frac{1}{2g+1}\Gamma - \Gamma - \Gamma - \frac{1}{2g+1}\Gamma\right),$$

where $\Gamma_{\square}$ are graphs differing from each other only in parts where certain 4 *distinct* edges are connected as illustrated in the boxes. According to this correction, Corollary 2.2 and Proposition 2.3(c) of loc.cit. should also be modified: in Corollary 2.2 IH_0 should be replaced by IH_0^{bis}, while it turns out that a simple stable structure of $(\Lambda U/([2^2]_{\mathfrak{sp}}))^{\mathfrak{sp}}$ as in Proposition 2.3(c) will not be easy to detect (cf. [KM], see also [A]).

The above correction is obtained from adding an extra argument to the last part of §3.3, where we should have identified precisely the images of $\alpha_{\Gamma_I} - \alpha_{\Gamma_H}$ for IH_0-related pairs under the projection $Im(\bar{f}_{IH}) \wedge (\Lambda U)^{2m-2} \to Im(f') \wedge (\Lambda U)^{2m-2}$. This can be done by examining explicitly the action of the Casimir operator C of $\mathfrak{sp}_{2g}$ on the elements of type $\sum_e \mathrm{sgn}(e)(x_a \wedge x_b \wedge x_e) \wedge (x_c \wedge x_d \wedge x_{-e})$; we interpret the projection $Im(\bar{f}_{IH}) \to [2^2]_{\mathfrak{sp}}$ as the linear map $\frac{1}{2g+1}((g+1)C^2 - gC)$.

In [KM] §12, N.Kawazumi and S.Morita presented a remarkable relation



$$(**)\qquad (\Gamma)^2 = 2(2g+1)\{(g+1)\Gamma - (g+2)\Gamma\},$$

in the cohomology ring $H^*(M_g, \mathbb{Q})$, and Theorem 1.1 of [KM] tells us that this relation holds also at the level of the graded algebra $(\Lambda U/([2^2]_{\mathfrak{sp}}))^{\mathfrak{sp}}$ in the stable range of $3m \le g$. One can check $(**)$ by the relation $(*)$ of Thoerem 1 with the dotted squares of $\Gamma_{\square}$'s respectively applied to the shaded part of the following graph:

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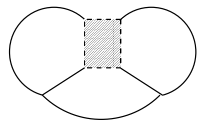

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A detailed proof of Theorem 1 and some generalizations have recently been worked out by Hiroki Akazawa. See [A].

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