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GROSS-KOHNEN-ZAGIER THEOREM FOR HIGHER WEIGHT FORMS

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ABSTRACT. We calculate the height pairing between higher weight Heegner cycles associated to distinct imaginary quadratic fields. Under the positive definiteness hypothesis of the height pairing we prove a Gross-Kohnen-Zagier theorem for higher weight modular forms.

1. Introduction

Let $X_0(N)$ be the usual modular curve over $\mathbb{Z}$. Let $\mathcal{O}_D$ be the order of discriminant $D<0$ in an imaginary quadratic field $K_D=\mathbb{Q}(\sqrt{D})\subset\mathbb{C}$. Assume that $(N,D)=1$ and all prime factors of N split in K_D. The theory of complex multiplication defines certain points $x\in X_0(N)(H_D)$. These special points are called Heegner points of discriminant D, and H_D is the ring class field corresponding to the order $\mathcal{O}_D$, see [7] for details.

Following the notation of [8], such a Heegner point x is determined by the residue class of r modulo $2N$, where r satisfies

$$r^2\equiv D\,(\mathrm{mod}\,4N).$$

More precisely, let τ be the root in the upper half plane $\mathbb{H}$ of the equations

$$a\tau^2+b\tau+c=0,\quad a>0,\quad N|a,\quad b^2-4ac=D,\quad \text{and } b\equiv r\,(\mathrm{mod}\,2N),$$

then the image of τ in $X_0(N)(\mathbb{C})$ is a Heegner point defined over H_D. There are exactly $h_D=[H_D:K_D]$ such images, permuted transitively by $\mathrm{Gal}(H_D/K_D)$, and their formal sum is denoted by $P_{D,r}$. The Zariski closure of $P_{D,r}$ in $X_0(N)_{\mathbb{Z}}$ is denoted by $\underline{P}_{D,r}$.

Each Heegner point in $P_{D,r}$ can also be interpreted as an isogeny of elliptic curves $(E\xrightarrow{\phi}E')$ both of which have CM by the same order $\mathcal{O}_D$, and the kernel of ϕ is annihilated by the primitive ideal $\left(N,\frac{r+\sqrt{D}}{2}\right)$ of norm N in $\mathcal{O}_D$.

Let

$$S_{2k}^-(\Gamma_0(N))=\left\{g\in S_{2k}(\Gamma_0(N))\,|\,(-Nz^2)^{-k}g\left(\frac{-1}{Nz}\right)=-g(z)\right\}$$

be the subspace of cuspforms of weight $2k$ which have eigenvalue (-1) under the Fricke involution. If $f\in S_{2k}^-(\Gamma_0(N))$ is a normalized newform, then $L(f,s)$ has minus sign in its functional equation under $s\mapsto 2k-s$. By the result of [10] there is a nonzero Jacobi form $\phi_f\in J_{k+1,N}^{\mathrm{cusp}}$, unique up to a scalar multiple, which has the same eigenvalues as f for Hecke operators T_m with $(m,N)=1$. The Jacobi form ϕ_f can be

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chosen to have Fourier coefficients in the same totally real coefficient field of f. We write the Fourier expansion of ϕ_f as

$$\phi_f(\tau, z) = \sum_{\substack{n,r\in\mathbb{Z} \\ r^2 \leq 4nN}} c(n,r) q^n \zeta^r \tag{1.1}$$

where $\tau \in \mathbb{H}$, $z \in \mathbb{C}$, $q = e^{2\pi i\tau}$ and $\zeta = e^{2\pi i z}$.

When $k = 1$, a well-known result of Gross-Kohnen-Zagier [8] shows that for each fundamental discriminant D, $(y^*_{D,r})_f$ is essentially equal to $c(n, r)$ times a fixed y_f in the space $J_0(N)(\mathbb{Q}) \otimes \mathbb{R}$, where $(y^*_{D,r})_f$ is the f-isotypical component of the divisor class

$$y^*_{D,r} = (P_{D,r} - h_D\infty) + \overline{(P_{D,r} - h_D\infty)},$$

and $n = \frac{r^2-D}{4N}$. Our main goal is to generalize this result to f of general weight $2k$.

For this purpose we first review the construction of Kuga-Sato varieties and Heegner cycles on them associated to weight $2k$ modular forms. After that we calculate the height pairing between the Heegner cycles and compare them with the Fourier coefficients of a modular form F, whose inner product with f gives (essentially) the central derivative $L'(f, k)$. In the end, if we assume further that the height pairing between Heegner cycles is positive definite, we are able to prove a higher weight analogue of the Gross-Kohnen-Zagier theorem. It would be interesting to find an unconditional proof, for instance, by generalizing the approach taken in Borcherds [3].

2. Kuga-Sato varieties and CM cycles

Let N' with $N|N'$ be an auxiliary number which is a product of two coprime integers greater than 2. There is a natural morphism $\pi : \mathcal{E}(N') \to X(N')$ of regular, flat and projective schemes, such that π makes $\mathcal{E}(N')$ a universal semistable elliptic curves with full level N' structures over $X(N')$. The universal elliptic curve with level N' structures is denoted by $\mathcal{E}_0(N')$ and the corresponding moduli scheme is denoted by $Y(N')$.

The $(2k-2)$-tuple fiber product $\mathcal{E}(N')^{2k-2}$ over $X(N')$ has a canonical resolution, denoted by $Y_k(N')$, which is described in Zhang [11] (similar to that in Deligne [4] over $\mathbb{Q}$). The scheme $Y_k(N')$ is the Kuga-Sato variety that will be used for the construction of CM cycles or Heegner cycles in this paper.

The Hecke correspondences T_m for $(m, N') = 1$ are defined as follows. Let $Y(N', m)$ denote the moduli scheme classifying elliptic curves E with level N structure and a cyclic isogeny $E \to E'$ of degree m. Let $\mathcal{E}_0(N', m)$ be the universal elliptic curve over $Y(N', m)$ and $\mathcal{E}_0(N', m) \xrightarrow{\psi} \mathcal{E}'_0(N', m)$ be the universal cyclic m-isogeny. We have the following diagram

$$\begin{array}{ccccccc}
\mathcal{E}_0(N')^{2k-2} & \xleftarrow{\phi_1} & \mathcal{E}_0(N',m)^{2k-2} & \xrightarrow{\psi} & \mathcal{E}'_0(N',m)^{2k-2} & \xrightarrow{\phi_2} & \mathcal{E}_0(N')^{2k-2} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
Y(N') & \longleftarrow & Y(N',m) & = & Y(N',m) & \longrightarrow & Y(N')
\end{array}$$

The Hecke correspondence T_m is defined to be the Zariski closure in $\mathcal{E}(N')^{2k-2} \times \mathcal{E}(N')^{2k-2}$ (or in $Y_k(N')$ to be more precise) of the correspondence $\phi_{1*}\psi^*\phi_2^*$ on $\mathcal{E}_0(N')^{2k-2}$.

Next we want to define the notion of CM Chow cycles of weight $2k$, full level N' and discriminant D. For this purpose let R be an integral domain which is flat over $\mathbb{Z}$ and is unramified over all primes dividing N'. In later applications we always take R to be $\mathcal{O}_{H_D}$ or its local completion with $(D, N') = 1$. By the choice of R we know the base change $Y_k(\Gamma(N'))_R$ is again regular. Denote the generic point of $\operatorname{Spec} R$ by η. Let $\underline{x} \in X(N')(R)$ be such that the corresponding elliptic curve E over $\operatorname{Spec} R$ has CM by $\mathcal{O}_D$. Fix an embedding $\tau = \tau_E : \operatorname{End}_R(E) \to \mathbb{C}$. Let $\sqrt{D}$ be the element in $\operatorname{End}_R(E)$ with $\tau_E(\sqrt{D}) = \sqrt{D}$ in $\mathcal{O}_D$. Let Γ_D be the graph of the multiplication by $\sqrt{D}$ and write

$$Z(E) = \Gamma_D - (E \times \{0\}) - |D|(\{0\} \times E). \tag{2.1}$$

The $(k-1)$-tuple product $Z(E)^{k-1}$ defines a cycle of codimension $k-1$ in E^{2k-2}. Define

$$W_k(\underline{x}) := W_k(E) = \sum_{\sigma \in G_{2k-2}} \operatorname{sgn}(\sigma)\sigma^*(Z(E)^{k-1}) \tag{2.2}$$

and let

$$S_k(\underline{x}) := S_k(E) = c \cdot W_k(E)$$

where the positive constant c is such that the self-intersection of $S_k(E_\eta)$ in E_η^{2k-2} is $(-1)^{k-1}$. After base changing to R we have the following natural morphisms

$$Y_k(N')_R \to \mathcal{E}(N')_R^{2k-2} \to X(N')_R.$$

Through the natural morphism $E^{2k-2} \to \mathcal{E}(N')_R^{2k-2}$, the cycle $S_k(\underline{x})$ can be viewed as living in $\mathcal{E}(N')_R^{2k-2}$, and as living in $Y_k(N')_R$ as well (because CM elliptic curves have potentially good reductions). The codimension of $S_k(\underline{x})$ in $Y_k(N')_R$ becomes k. The class of $S_k(\underline{x})$ in $Ch^k(Y_k(N')_R)$ is denoted by $s_k(\underline{x})$, and is called a CM Chow cycle of weight $2k$, full level N' and discriminant D over R. The space of CM Chow cycles of weight $2k$ and level N' (for all discriminants D) over R, denoted by $CM_k(X(N')_R) \otimes \mathbb{R}$, is the $\mathbb{R}$-subspace generated by $s_k(\underline{x})$ for all CM points $\underline{x}$ in $X(N')(R)$.

Now we take R to be an integral domain flat over $\mathbb{Z}$ and unramified over primes dividing N. Let $\underline{x} \in X_0(R)$ be a Heegner point of discriminant D rational over R. We want to define the associated Heegner cycle $s_k(\underline{x})$ of weight $2k$ over $X_0(N)_R$. The Kuga-Sato variety over $X_0(N)_R$ does not have a nice regular model. So we choose N' with $N|N'$ to be any number which is a product of two coprime integers greater than or equal to 3 and such that R is also unramified over primes dividing N'. Let $\pi : X(N')_R \to X_0(N)_R$ be the natural projection, then $\pi^*(\underline{x}) = u(\underline{x}) \sum_i \underline{x}_i$ for points $\underline{x}_i$ in $X(N')(R_i)$ with $\pi(\underline{x}_i) = \underline{x}$, where $u(\underline{x}) = |\operatorname{Aut}(\underline{x})|/2$ and R_i is certain ring extension of R. Note that each elliptic curve E_i represented by $\underline{x}_i$ has CM by $\mathcal{O}_D$ (independent of the choice of i). Applying the construction described above on each x_i we obtain CM Chow cycles $s_k(\underline{x}_i) \in s_k(X(N')_{R_i}) \otimes \mathbb{R}$. Let

$$s_k(\underline{x}) = \frac{1}{u(\underline{x})\sqrt{\deg \pi}} s_k(\pi^*(\underline{x})) = \frac{1}{\sqrt{\deg \pi}} \sum_i s_k(\underline{x}_i), \tag{2.3}$$

then $s_k(\underline{x})$ is called a Heegner cycle of weight $2k$ and level $\Gamma_0(N)$ over R. We also write $Heeg_k(X_0(N)_R) \otimes \mathbb{R}$ for the space generated by all $s_k(\underline{x})$ with $\underline{x}$ a Heegner divisor rational over R of $X_0(N)$. The construction of $Heeg_k(X_0(N)_R) \otimes \mathbb{R}$ is Galois equivariant, and does not depend on the choice of the auxiliary number N'.

The Hecke operators T_m induce a natural action on $Heeg_k(X_0(N)_R) \otimes \mathbb{R}$. If $T_m(\underline{x}) = \sum_i \underline{x}_i$, then

$$T_m(s_k(\underline{x})) = m^{k-1} \sum_i s_k(\underline{x}_i) \tag{2.4}$$

and

$$T_m T_n = \sum_{d|(m,n)} d^{k-1} T_{\frac{mn}{d^2}}. \tag{2.5}$$

Details of the above construction can be found in [9] over a field or in [11] in the general setting.

Now back to the situation of Section 1. We define

$$s_{D,r} = \sum_{x \in P_{D,r}} s_k(x) \in Heeg_k(X_0(N)_{K_D}) \otimes \mathbb{R}$$

and

$$s^*_{D,r} = \sum_{x \in P_{D,r}} s_k(x) + \sum_{x \in P_{D,r}} \overline{s_k(x)} \in Heeg_k(X_0(N)_{\mathbb{Q}}) \otimes \mathbb{R},$$

where the bar denotes the complex conjugate. Let D be a fundamental discriminant, then [8, p. 542, (3)]

$$T_m(P_{D,r}) = \sum_{m=dd'} \left(\frac{D}{d'}\right) P_{Dd^2,rd},$$

which implies that

$$T_m s^*_{D,r} = m^{k-1} \sum_{m=dd'} \left(\frac{D}{d'}\right) s^*_{Dd^2,rd}. \tag{2.6}$$

In particular, the space $Heeg_k(X_0(N)_{\mathbb{Q}}) \otimes \mathbb{R}$ is stable under all Hecke operators T_m for $(m, N) = 1$.

Similarly we define an integral version

$$\underline{s}_{D,r} = \sum_{x \in \underline{P}_{D,r}} s_k(\underline{x}) \in Heeg_k(X_0(N)_{\mathcal{O}_D}) \otimes \mathbb{R}$$

and

$$\underline{s}^*_{D,r} = \sum_{\underline{x} \in \underline{P}_{D,r}} s_k(\underline{x}) + \sum_{\underline{x} \in \underline{P}_{D,r}} \overline{s_k(\underline{x})} \in Heeg_k(X_0(N)_{\mathbb{Z}}) \otimes \mathbb{R}.$$

The construction of CM Chow cycles depends on the choice of the embedding $\tau : \mathrm{End}_R(E) \to \mathbb{C}$. If τ changes one has the following result [11, Prop. 2.4.1] or [9, p. 106].

Lemma 2.1. *If τ changes to its complex conjugate then $s_k(E_{\overline{\tau}}) = (-1)^{k-1} s_k(E_\tau)$.*

In particular, the complex conjugate has the following effect on CM cycles

$$\overline{s_k(x)} = (-1)^{k-1} s_k(\overline{x}) \in Heeg_k(X_0(N)_{H_D}) \otimes \mathbb{R}. \tag{2.7}$$

Therefore

$$\begin{aligned} s^*_{D,r} &= \sum_{x \in P_{D,r}} s_k(x) + (-1)^{k-1} \sum_{x \in P_{D,r}} s_k(\overline{x}) \\ &= \sum_{x \in P_{D,r}} s_k(x) + (-1)^{k-1} \sum_{y \in P_{D,-r}} s_k(y). \end{aligned} \tag{2.8}$$

On the other hand, if $f \in S_{2k}(\Gamma_0(N))$ is a normalized newform, then the f-isotypical component of $s_{D,r}$ has the following effect under the complex conjugate.

Lemma 2.2. *The complex conjugate of the f-isotypical component $(s_{D,r})_f$ of $s_{D,r}$ is given by*

$$(\overline{s_{D,r}})_f = (-\varepsilon_f) \cdot (s_{D,r})_f, \tag{2.9}$$

where $\varepsilon_f = \pm 1$ is the sign in the functional equation of $L(f,s)$.

Proof. See the proof of [9, Prop. 6.2]. □

By Lemma 2.2 if we assume further that $f \in S^-_{2k}(\Gamma_0(N))$, then $\varepsilon_f = -1$ and thus

$$(s^*_{D,r})_f = 2(s_{D,r})_f \in Heeg_k(X_0(N)_{K_D}) \otimes \mathbb{R}. \tag{2.10}$$

3. Height pairing between Heegner cycles

In this section, we always let $D_1, D_2 < 0$ denote coprime (but not necessarily fundamental) discriminants with $D_i \equiv r_i^2 \pmod{4N}$, and let ϱ be such that $\varrho \equiv r_1 r_2 \pmod{4N}$. Our main goal is to compute the height pairing $\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle$ for $(m,N)=1$. The height pairing is defined in [11] through the arithmetic intersection theory developed by Gillet-Soulé [5]. Roughly speaking, let

$$\widehat{s}^*_{D_i,r_i} = (\underline{s}^*_{D_i,r_i}, g_{D_i,r_i})$$

be the arithmetic cycle associated to $s^*_{D_i,r_i}$ with g_{D_i,r_i} a Green current at infinity, then

$$\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle = (-1)^k \left(\widehat{s}^*_{D_1,r_1} \cdot T_m \widehat{s}^*_{D_2,r_2}\right),$$

where the product on the right hand side means the arithmetic intersection. This height pairing has the following decomposition into local height pairings and thus into the local intersections

$$\begin{aligned} \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle &= \sum_{p<\infty} \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_p + \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_\infty \\ &= (-1)^k \left(\sum_{p<\infty} \left(\underline{s}^*_{D_1,r_1} \cdot T_m \underline{s}^*_{D_2,r_2}\right)_p \log p \right) + \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_\infty. \end{aligned}$$

We begin with the computation of the height pairing at non-archimedean places. Let W be a complete local ring with a prime element π and algebraically closed residue field of characteristic p with $(p,N)=1$. Let $\underline{x}_1 \in \underline{P}_{D_1,r_1}(W)$ and $\underline{x}_2 \in \underline{P}_{D_2,\varepsilon r_2}(W)$ for $\varepsilon = \pm 1$. Suppose $(\underline{x}_1 \cdot \underline{x}_2)_W > 0$, and let $z = (E_1 \to E_1') = (E_2 \to E_2')$ be the common reduction of $\underline{x}_1$ and $\underline{x}_2$ mod (π). Let $B(p) = \mathrm{End}(E_i) \otimes \mathbb{Q}$, then $B(p)$ is the definite quaternion algebra ramified at p. Write $R = \mathrm{End}_{W/\pi W}(z)$, then R is the right order of the left ideal class (which uniquely determines z) for the Eichler order of level N in $B(p)$. We also define $S_{[D_1,2n,D_2]}$ to be the Clifford order associated to the quadratic form $[D_1, 2n, D_2]$

$$S_{[D_1,2n,D_2]} = \mathbb{Z} + \mathbb{Z}\frac{r_1+e_1}{2} + \mathbb{Z}\frac{r_2+e_2}{2} + \mathbb{Z}\frac{r_1r_2 + r_1e_2 + r_2e_1 + e_1e_2}{4},$$

with $e_i^2 = D_i$ for $i = 1, 2$ and $e_1e_2 + e_2e_1 = 2n$. As pointed out in [8, pp. 549-551], the intersections over W of $\underline{P}_{D_1,r_1}$ and $\underline{P}_{D_2,\varepsilon r_2}$ are in one-to-one correspondence with the embeddings of $S_{[D_1,2n,D_2]}$ into right orders R of the distinct left ideal classes for the Eichler order of level N in $B(p)$. More precisely, if $\phi : S_{[D_1,2n,D_2]} \to R$ is an embedding that corresponds to the intersection between $\underline{x}_1 \in \underline{P}_{D_1,r_1}(W)$ and $\underline{x}_2 \in \underline{P}_{D_2,\varepsilon r_2}(W)$ at z, then $\mathrm{End}(z) \cong R$, $\phi(e_1) = \sqrt{D_1}$ and $\phi(e_2) = \varepsilon\sqrt{D_2}$, where $\sqrt{D_i}$ means the image of the endomorphism $\sqrt{D_i} \in \mathrm{End}(\underline{x}_i)$ in R under the reduction mod π.

Proposition 3.1. *Suppose $\underline{x}_i \in \underline{P}_{D_i,r_i}(W)$ for $i = 1, 2$, and suppose they intersect at z which corresponds to an embedding $\phi : S_{[D_1,2n,D_2]} \to R = \mathrm{End}(z)$, then*

$$(3.1)\qquad \left(s_k(\underline{x}_1) \cdot \varepsilon^{k-1} s_k(\underline{x}_2)\right)_W = (-1)^{k-1} P_{k-1}\left(\frac{n}{\sqrt{D_1D_2}}\right) \cdot (\underline{x}_1 \cdot \underline{x}_2)_W .$$

Proof. By [11, (3.3.1)] we have

$$(3.2)\qquad \left(s_k(\underline{x}_1) \cdot \varepsilon^{k-1} s_k(\underline{x}_2)\right)_W = \varepsilon^{k-1}\left(s_k(\underline{x}_1)_0 \cdot s_k(\underline{x}_2)_0\right) \cdot (\underline{x}_1 \cdot \underline{x}_2)_W ,$$

where $s_k(\underline{x}_1)_0$ and $s_k(\underline{x}_2)_0$ denote reductions of $s_k(\underline{x}_1)$ and $s_k(\underline{x}_2)$ mod (π) respectively, and $(s_k(\underline{x}_1)_0 \cdot s_k(\underline{x}_2)_0)$ denotes their intersection number inside the abelian variety $E_1^{2k-2} = E_2^{2k-2}$. To compute this intersection number we use the pairing on the l-adic cohomology (see [11, p. 129])

$$(3.3)\qquad H^{2k-2}(E_i^{2k-2}, \mathbb{Q}_l(k-1)) \times H^{2k-2}(E_i^{2k-2}, \mathbb{Q}_l(k-1)) \overset{(\cdot,\cdot)}{\to} \mathbb{Q}_l,$$

which is induced from the pairing on $H^1(E_i, \mathbb{Q}_l)$. Now we choose $l \neq p$ such that both $\sqrt{|D_1|}$ and $\sqrt{|D_2|}$ are in $\mathbb{Q}_l$, but $\sqrt{-1}$ is not in $\mathbb{Q}_l$ (such an l always exists by the Chebotarev density theorem). Let $F = \mathbb{Q}_l(\sqrt{-1})$, after fixing an isomorphism $H^1(E_1, \mathbb{Q}_l) \otimes F \cong H^1(E_2, \mathbb{Q}_l) \otimes F$ we denote both spaces by H. We choose a basis $\{X, Y\}$ of H such that

$$(3.4)\qquad [\phi(e_1)]X = -\sqrt{D_1}X, \quad [\phi(e_1)]Y = \sqrt{D_1}Y, \quad \text{and} \quad (X, Y) = \sqrt{-1} \in F.$$

In other words, the action of $[\phi(e_1)]$ has the following matrix with respect to the basis $\{X, Y\}$

$$(3.5)\qquad [\phi(e_1)] = \begin{bmatrix} -\sqrt{D_1} & 0 \\ 0 & \sqrt{D_1} \end{bmatrix}.$$

Similarly, let $\{X', Y'\}$ be a basis of $H = H^1(E_2, \mathbb{Q}_l)$ such that

$$(3.6)\quad [\phi(e_2)]X' = -\varepsilon\sqrt{D_2}X', \quad [\phi(e_2)]Y' = \varepsilon\sqrt{D_2}Y', \quad \text{and} \quad (X', Y') = \sqrt{-1} \in F.$$

Under the above setting we have the following result.

Lemma 3.2. *Suppose* $\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$, *then*

$$s_k(\underline{x}_1)_0 \cdot s_k(\underline{x}_2)_0 = (-1)^{k-1} P_{k-1}\left(ad+bc\right), \tag{3.7}$$

where $P_{k-1}(t)$ is the standard Legendre polynomial of degree $k-1$.

Proof. As $ad - bc = 1$, this is [11, Prop. 3.3.3]. □

We next determine the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Suppose the matrix of $[\phi(e_2)]$ with respect to the basis $\{X, Y\}$ is given by

$$[\phi(e_2)] = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}.$$

Since

$$[\phi(e_1e_2 + e_2e_1)] = [2n] \tag{3.8}$$

we obtain

$$\begin{bmatrix} -\alpha\sqrt{D_1} & 0 \\ 0 & \delta\sqrt{D_1} \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix},$$

which implies that

$$\alpha = \frac{-n}{\sqrt{D_1}}, \quad \delta = \frac{n}{\sqrt{D_1}} \tag{3.9}$$

By (3.6)

$$[\phi(e_2)]X' = [\phi(e_2)](aX + bY) = (a\alpha + b\gamma)X + (a\beta + b\delta)Y = -\varepsilon\sqrt{D_2}X'. \tag{3.10}$$

Thus

$$a(\alpha + \varepsilon\sqrt{D_2}) + b\gamma = 0 \tag{3.11}$$

and

$$a = -\frac{\gamma}{\alpha + \varepsilon\sqrt{D_2}} b \tag{3.12}$$

Similarly, by $[e_2]Y' = \sqrt{D_2}Y'$ we obtain

$$c = -\frac{\gamma}{\alpha - \varepsilon\sqrt{D_2}} d \tag{3.13}$$

Putting (3.9), (3.12) and (3.13) together,

$$\begin{aligned} ad + bc =& \frac{ad+bc}{ad-bc} = \frac{-\frac{\gamma}{\alpha+\varepsilon\sqrt{D_2}} - \frac{\gamma}{\alpha-\varepsilon\sqrt{D_2}}}{-\frac{\gamma}{\alpha+\varepsilon\sqrt{D_2}} + \frac{\gamma}{\alpha-\varepsilon\sqrt{D_2}}} \\ =& \frac{\frac{-2n}{\sqrt{D_1}}}{-2\varepsilon\sqrt{D_2}} = \frac{\varepsilon n}{\sqrt{D_1 D_2}}. \end{aligned} \tag{3.14}$$

Now (3.1) follows from (3.2), (3.7), (3.14) and $P_{k-1}(\varepsilon t) = \varepsilon^{k-1} P_{k-1}(t)$. □

Proposition 3.3. *Let* $D_i = r_i^2 - 4n_iN < 0$ *(*$i = 0, 1$*) be coprime fundamental discriminants and write* $\Delta = D_1D_2$, $\varrho = r_1r_2$. *Then the finite part of the height pairing between* $s^*_{D_1,r_1}$ *and* $T_m s^*_{D_2,r_2}$ *is given by*

$$\sum_{p<\infty} \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_p = 2m^{k-1} \sum_{\substack{|n|<m\sqrt{\Delta} \\ n\equiv m\varrho(2N)}} \sigma'_\chi \left(\left(\frac{n+m\sqrt{\Delta}}{2} \right) \mathfrak{n}^{-1} \right) P_{k-1}\left(\frac{n}{m\sqrt{\Delta}} \right),$$

where $\mathfrak{n} = \left(N, \frac{r_1r_2+\sqrt{\Delta}}{2}\right)$ *is a primitive ideal of norm* N *in* $\mathbb{Q}(\sqrt{\Delta})$*, and* $\sigma'_\chi(\mathfrak{a})$ *for each ideal* $\mathfrak{a}$ *of* $\mathbb{Q}(\sqrt{\Delta})$ *is defined in (3.17).*

Proof. For any pair of (not necessarily fundamental) discriminants D_1 and D_2, the local height pairing at a prime p is given by

$$\begin{aligned}
&\langle s^*_{D_1,r_1}, s^*_{D_2,r_2}\rangle_p \\
=&(-1)^k (s^*_{D_1,r_1} \cdot s^*_{D_2,r_2})_p \log p \\
=&2(-1)^k \left(\sum_{\underline{x}\in P_{D_1,r_1}} s_k(\underline{x}) \right) \cdot \left(\sum_{\underline{y}\in P_{D_2,r_2}} s_k(\underline{y}) + (-1)^{k-1} \sum_{\underline{y}\in P_{D_2,-r_2}} s_k(\underline{y}) \right) \log p \\
=&\frac{-1}{2^t} \sum_{\substack{n\equiv\varrho(2N) \\ n^2<\Delta}} \sum_{R_i} \left|\{S_{[D_1,2n,D_2]} \to R_i \bmod R_i^\times / \pm 1\}\right| P_{k-1}\left(\frac{n}{\sqrt{\Delta}}\right) \frac{\mathrm{ord}_p(M)+1}{2} \log p \\
=&2 \sum_{\substack{n\equiv\varrho(2N) \\ n^2<\Delta}} P_{k-1}\left(\frac{n}{\sqrt{\Delta}}\right) \cdot \ell'(M,0)
\end{aligned}$$

where $M = \frac{D_1D_2-4n^2}{4N}$, $\ell(M,s) = \sum_{d|M} \epsilon(d)d^s$, and we have used formula (3.1) together with the following two formulas from [8, p. 551]

$$\begin{aligned}
&(P_{D_1,r_1} \cdot (P_{D_1,r_1} + P_{D_2,r_2}))_p \\
=&\frac{1}{2^{t+1}} \sum_{\substack{n\equiv r_1r_2(2N) \\ n^2<D_1D_2}} \sum_{R_i} \left|\{S_{[D_1,2n,D_2]} \to R_i \bmod R_i^\times / \pm 1\}\right| \frac{\mathrm{ord}_p(M)+1}{2}
\end{aligned}$$

and

$$\frac{-1}{2^{t+1}} \sum_{R_i} \left|\{S_{[D_1,2n,D_2]} \to R_i \bmod R_i^\times / \pm 1\}\right| \frac{\mathrm{ord}_p(M)+1}{2} \log p = \ell'(M,0).$$

By (2.6) we obtain

(3.15)

$$\begin{aligned}
&\sum_p \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_p \\
=&m^{k-1}\sum_{m_i=d_i d_i'}\left(\frac{D_1}{d_1'}\right)\left(\frac{D_2}{d_2'}\right)\left(\sum_p \langle s^*_{D_1 d_1^2, r_1 d_1}, s^*_{D_2 d_2^2, r_2 d_2}\rangle_p\right)\\
=&2m^{k-1}\sum_{m_i=d_i d_i'}\left(\frac{D_1}{d_1'}\right)\left(\frac{D_2}{d_2'}\right)\left(\sum_{\substack{x\equiv \varrho d_1 d_2 (2N)\\ |x|<d_1 d_2\sqrt{\Delta}}} P_{k-1}\left(\frac{x}{d_1 d_2\sqrt{\Delta}}\right)\ell'\left(\frac{\Delta d_1^2 d_2^2 - x^2}{4N},0\right)\right)\\
=&2m^{k-1}\sum_{\substack{n\equiv m\varrho (2N)\\ |n|<\sqrt{\Delta}m}} P_{k-1}\left(\frac{n}{m\sqrt{\Delta}}\right)\sum_{d_i'|(n,m_i)}\left(\frac{D_1}{d_1'}\right)\left(\frac{D_2}{d_2'}\right)\ell'\left(\frac{\Delta m^2-n^2}{4N(d_1'd_2')^2},0\right),
\end{aligned}$$

where in the last line we set $n = d_1'd_2'x$ and interchanged the order of summation. By [8, p. 553, (3)] the inner sum is given by

$$(3.16)\qquad \sum_{d_i'|(n,m_i)}\left(\frac{D_1}{d_1'}\right)\left(\frac{D_2}{d_2'}\right)\ell'\left(\frac{\Delta m^2-n^2}{4N(d_1'd_2')^2},0\right) = \sigma'_\chi\left(\left(\frac{n+m\sqrt{\Delta}}{2}\right)\mathfrak{n}^{-1}\right).$$

Now, (3.15) and (3.16) complete the proof. □

We next consider the local height pairing at the archimedean place. Let

$$Q_{k-1}(t) = \int_0^\infty (t+\sqrt{t^2-1}\cosh u)^{-k}du,$$

and

$$G_k(z,z') = \sum_{\gamma\in\Gamma_0(N)} -2Q_{k-1}\left(1+\frac{|z-\gamma z'|^2}{2y(z)y(\gamma z')}\right),$$

where $y(z)$ denotes the imaginary part of the complex variable z.

Lemma 3.4. *For any two coprime discriminants D_1 and D_2, we have*

$$\langle s^*_{D_1,r_1}, s^*_{D_2,r_2}\rangle_\infty = G_k(P_{D_1,r_1}, P_{D_2,r_2}) + (-1)^{k-1}G_k(P_{D_1,r_1}, P_{D_2,-r_2}),$$

where $G_k(P_1,P_2)$ is defined by extending $G_k(x,y)$ bi-linearly to divisors P_1 and P_2.

Proof. By [11, Prop. 4.1.2] for $x\in P_{D_1,r_1}(\mathbb{C})$ and $y\in P_{D_2,\epsilon r_2}(\mathbb{C})$ one has

$$\langle s_k(x), s_k(y)\rangle_\infty = \frac{1}{2}G_k(x,y).$$

Thus

$$\begin{aligned}&\langle s^*_{D_1,r_1}, s^*_{D_2,r_2}\rangle_\infty\\ =&2\left\langle \sum_{x\in P_{D_1,r_1}} s_k(x), \sum_{y\in P_{D_2,r_2}} s_k(y) + (-1)^{k-1}\sum_{z\in P_{D_2,-r_2}} s_k(z)\right\rangle_\infty\\ =&G_k(P_{D_1,r_1}, P_{D_2,r_2}) + (-1)^{k-1} G_k(P_{D_1,r_1}, P_{D_2,-r_2})\end{aligned}$$

as desired. □

Proposition 3.5. *The local height pairing* $\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_\infty$ *is given by*

$$m^{k-1}\left(G_k(P_{D_1,r_1}, T_m P_{D_2,r_2}) + (-1)^{k-1} G_k(P_{D_1,r_1}, T_m P_{D_2,-r_2})\right).$$

Proof. Evident from Lemma 3.4. □

Let $F(z) = \sum_{m=1}^{\infty} a_m e^{2\pi i m z} \in S^-_{2k}(\Gamma_0(N))$ be such that its m-th Fourier coefficients are given by

$$\begin{aligned}a_m =&(m\sqrt{\Delta})^{k-1}\sum_{\substack{|n|<m\sqrt{\Delta}\\ n\equiv m\varrho\,(2N)}} \sigma'_\chi\left(\left(\frac{n+m\sqrt{\Delta}}{2}\right)\mathfrak{n}^{-1}\right) P_{k-1}\left(\frac{n}{m\sqrt{\Delta}}\right)\\ &-(m\sqrt{\Delta})^{k-1}\sum_{\substack{n>m\sqrt{\Delta}\\ n\equiv m\varrho\,(2N)}} \sigma_{0,\chi}\left(\left(\frac{n+m\sqrt{\Delta}}{2}\right)\mathfrak{n}^{-1}\right) Q_{k-1}\left(\frac{n}{m\sqrt{\Delta}}\right)\\ &-(m\sqrt{\Delta})^{k-1}\sum_{\substack{n>m\sqrt{\Delta}\\ n\equiv -m\varrho\,(2N)}} \sigma_{0,\chi}\left(\left(\frac{n+m\sqrt{\Delta}}{2}\right)\mathfrak{n}'^{-1}\right) Q_{k-1}\left(\frac{n}{m\sqrt{\Delta}}\right)\end{aligned}$$

where χ is the genus character associated the decomposition of $\Delta = D_1D_2$, $\mathfrak{n} = \left(N, \frac{\varrho+\sqrt{\Delta}}{2}\right)$, and $\sigma_{0,\chi}(\mathfrak{a})$ and $\sigma'_\chi(\mathfrak{a})$ denote the value and derivative of

$$\sigma_{s,\chi}(\mathfrak{a}) = \sum_{\mathfrak{b}|\mathfrak{a}} \chi(\mathfrak{b}) N(\mathfrak{b})^s \tag{3.17}$$

at $s = 0$ respectively. See [8, p. 530] or Section 4 for more detail of F.

Theorem 1. *For* $m \geq 1$ *relatively prime to* N*, the Fourier coefficient* a_m *of* F *is given by*

$$a_m = \frac{\sqrt{\Delta}^{k-1}}{2}\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle.$$

Proof. By [8, p. 556, Th. 2] (the factor $\left(m\sqrt{\Delta}\right)^{k-1}$ is missed on the second line there)

$$\begin{aligned} a_m =& \sqrt{\Delta}^{k-1} \sum_{\substack{|n|<m\sqrt{\Delta} \\ n\equiv m\varrho(2N)}} m^{k-1} P_{k-1}\left(\frac{n}{m\sqrt{\Delta}}\right) \sigma'_\chi\left(\left(\frac{n+m\sqrt{\Delta}}{2}\right)\mathfrak{n}^{-1}\right) \\ & + \sqrt{\Delta}^{k-1} m^{k-1}\left(G_k(P_{D_1,r_1}, T_m P_{D_2,r_2}) + (-1)^{k-1} G_k(P_{D_1,r_1}, T_m P_{D_2,-r_2})\right)/2 \\ =& \frac{\sqrt{\Delta}^{k-1}}{2}\left(\sum_{p<\infty} \langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_p\right) + \frac{\sqrt{\Delta}^{k-1}}{2}\left(\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}\rangle_\infty\right) \\ =& \frac{\sqrt{\Delta}^{k-1}}{2}\langle s^*_{D_1,r_1}, T_m s^*_{D_2,r_2}, \rangle \end{aligned}$$

where in the second equation we have used Proposition 3.5. □

4. Consequences

We retain the notation of Section 3, and let $D_i = r_i^2 - 4n_iN < 0$ $(i = 1, 2)$ be two coprime fundamental discriminants with $(D, 2N) = 1$. In this section we also make the following assumption.

Assumption 4.1. *The height pairing is positive definite on $Heeg_k(X_0(N)_{\mathbb{Q}}) \otimes \mathbb{R}$.*

This positive definiteness is a special case of the general conjectures of Gillet-Soulé [6] and Bloch-Beilinson [1] [2] on positive definiteness of the height pairing, also see Conjectures 1.1.1 and 1.3.1 in [11].

By Assumption 4.1 and Theorem 1, we have the following spectral decomposition of $s^*_{D_i,r_i}$ (see [11, Prop. 5.1.2] for the precise argument)

$$s^*_{D_i,r_i} = \sum_j (s^*_{D_i,r_i})_{f_j},$$

where $\{f_j\}$ is an orthogonal eigenbasis for $S_{2k}(\Gamma_0(N))$ with $f_1 = f$ and $(s^*_{D_i,r_i})_{f_j}$ is the f_j-isotypical component of $s^*_{D_i,r_i}$, that is for for m with $(m, N) = 1$

$$T_m((s^*_{D_i,r_i})_{f_j}) = a_m(f_j)(s^*_{D_i,r_i})_{f_j}.$$

Moreover, the f_j can be chosen to have real Fourier coefficients.

Now define

$$G(z) = \sum_j \langle (s^*_{D_1,r_1})_{f_j}, (s^*_{D_2,r_2})_{f_j}\rangle f_j \in S_{2k}(\Gamma_0(N)).$$

Lemma 4.2. *Suppose $(m, N) = 1$, then*

$$a_m = \frac{\sqrt{\Delta}^{k-1}}{2} a_m(G).$$

Proof. By the spectral decomposition of $s^*_{D_i,r_i}$ and $\langle (s^*_{D_1,r_1})_{f_j}, (s^*_{D_2,r_2})_{f_{j'}}\rangle = 0$ for $j \neq j'$ we get

$$\begin{aligned}a_m(G) =&\sum_j \langle (s^*_{D_1,r_1})_{f_j}, a_m(f_j)(s^*_{D_2,r_2})_{f_j}\rangle\\ =&\langle \sum_j \langle (s^*_{D_1,r_1})_{f_j}, \sum_j a_m(f_j)(s^*_{D_2,r_2})_{f_j}\rangle\\ =&\langle (s^*_{D_1,r_1}), T_m(s^*_{D_2,r_2})\rangle.\end{aligned}$$

The lemma now follows from Theorem 1. □

Theorem 2. *Let $f \in S^-_{2k}(\Gamma_0(N))$ be a normalized newform. Then the subspace of $Heeg_k(X_0(N)_{\mathbb{Q}}) \otimes \mathbb{R}$ generated by $(s^*_{D,r})_f$ (for all fundamental discriminants D with $(D, 2N) = 1$) has dimension 1 if $L'(f,k) \neq 0$ and 0 if $L'(f,k) = 0$. Moreover,*

$$|D|^{\frac{k-1}{2}}(s^*_{D,r})_f = c\left(\frac{r^2-D}{4N}, r\right)\cdot s^*_f,$$

where $s^*_f \in (Heeg_k(X_0(N)_{\mathbb{Q}}) \otimes \mathbb{R})_f$ is independent of D and r, and such that

$$\langle s^*_f, s^*_f\rangle = \frac{(2k-2)!N^{k-1}}{2^{2k-1}\pi^k(k-1)!(\phi_f,\phi_f)}\cdot L'(f,k).$$

Proof. First let $D_i = r_i^2 - 4n_iN < 0$ be two coprime fundamental discriminants. By Lemma 4.2 it follows that $F - \frac{1}{2}\sqrt{\Delta}^{k-1}G$ is an old form in $S_{2k}(\Gamma_0(N))$, so $(G,f) = 2\sqrt{\Delta}^{1-k}(F,f)$. But

$$(G,f) = (f,f)\langle (s^*_{D_1,r_1})_f, (s^*_{D_2,r_2})_f\rangle,$$

thus

$$\langle (s^*_{D_1,r_1})_f, (s^*_{D_2,r_2})_f\rangle = 2\sqrt{\Delta}^{1-k}\frac{(F,f)}{(f,f)}. \tag{4.1}$$

The inner product (F,f) is given by [8, p. 536]

$$(F,f) = \frac{i^{k-1}\Gamma(k-\frac{1}{2})}{2^{k+1}\pi^{k+1/2}}L'(f,k)r_{k,N,\Delta,\varrho,D_1}(f), \tag{4.2}$$

where $r_{k,N,\Delta,\varrho,D_1}(f)$ is the cycle integral whose precise definition is not important here and can be found in [8, p. 518]. Theorem A of [8] gives the following formula regarding the value of this cycle integral

$$r_{k,N,\Delta,\varrho,D_1}(f) = \left(\frac{i}{2N}\right)^{1-k}\frac{c(n_1,r_1)c(n_2,r_2)(f,f)}{(\phi_f,\phi_f)}, \tag{4.3}$$

where $c(n,r)$ are Fourier coefficients of the Jacobi form ϕ_f that corresponds to f as in (1.1).

Combining (4.1), (4.2) and (4.3) we conclude

$$\begin{aligned}\langle (s^*_{D_1,r_1})_f, (s^*_{D_2,r_2})_f\rangle &= \frac{\Gamma(k-\frac{1}{2})N^{k-1}}{2\pi^{k+\frac{1}{2}}\Delta^{\frac{k-1}{2}}}\cdot\frac{c(n_1,r_1)c(n_2,r_2)}{(\phi_f,\phi_f)}\cdot L'(f,k)\\ &= \frac{(2k-2)!N^{k-1}}{2^{2k-1}\pi^k(k-1)!\Delta^{\frac{k-1}{2}}}\cdot\frac{c(n_1,r_1)c(n_2,r_2)}{(\phi_f,\phi_f)}\cdot L'(f,k).\end{aligned} \tag{4.4}$$

On the other hand, by [11, Cor. 0.3.2] and (2.10)

$$\begin{aligned}\langle (s^*_{D_i,r_i})_f, (s^*_{D_i,r_i})_f\rangle =&\frac{1}{2}\langle (s^*_{D_i,r_i})_f, (s^*_{D_i,r_i})_f\rangle_{K_{D_i}}\\ =&\frac{(2k-2)!\sqrt{|D_i|}L'(f,k)}{2^{4k-2}\pi^{2k}(f,f)}L(f,D_i,k)\end{aligned}$$

and by [8, p. 527, Cor. 1]

$$\frac{c(n_i,r_i)^2}{(\phi_f,\phi_f)} = \frac{(k-1)!|D_i|^{k-1/2}}{2^{2k-1}\pi^k N^{k-1}}\cdot\frac{L(f,D_i,k)}{(f,f)},$$

so

$$\langle (s^*_{D_i,r_i})_f, (s^*_{D_i,r_i})_f\rangle = \frac{(2k-2)!N^{k-1}}{2^{2k-1}\pi^k(k-1)!|D_i|^{k-1}}\cdot\frac{c(n_i,r_i)^2}{(\phi_f,\phi_f)}\cdot L'(f,k). \tag{4.5}$$

Comparing (4.4) and (4.5) we see that

$$c(n_2,r_2)|D_1|^{\frac{k-1}{2}}(s^*_{D_1,r_1})_f - c(n_1,r_1)|D_2|^{\frac{k-1}{2}}(s^*_{D_2,r_2})_f$$

has height 0, so Assumption 4.1 implies that

$$c(n_2,r_2)|D_1|^{\frac{k-1}{2}}(s^*_{D_1,r_1})_f = c(n_1,r_1)|D_2|^{\frac{k-1}{2}}(s^*_{D_2,r_2})_f. \tag{4.6}$$

Next suppose D_1 and D_2 are not necessarily coprime. By [10, Lem. 3.2] we can find a fundamental discriminant $D' = r'^2 - 4n'N$ coprime to D_1, D_2 and $2N$ with $c(n',r')\neq 0$. Applying (4.6) to the pairs (D_i,D') for $i=1,2$, we get

$$|D_1|^{\frac{k-1}{2}}(s^*_{D_1,r_1})_f = \frac{c(n_1,r_1)}{c(n',r')}|D'|^{\frac{k-1}{2}}(s^*_{D',r'})_f$$

and

$$|D_2|^{\frac{k-1}{2}}(s^*_{D_2,r_2})_f = \frac{c(n_2,r_2)}{c(n',r')}|D'|^{\frac{k-1}{2}}(s^*_{D',r'})_f.$$

The proof of the theorem is now complete by letting $s^*_f = \frac{\sqrt{|D'|}^{k-1}}{c(n',r')}(s^*_{D',r'})_f$. □

Note that the dimension 1 assertion for the cohomology classes of $(s^*_{D,r})_f$ (under the l-adic Abel-Jacobi map) is an immediate consequence of the main theorem of Nekovář [9], while our result holds at the level of Chow class space. It would be interesting to obtain an unconditional proof of Theorem 2 following Borcherds' approach [3].

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