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Erratum to "Conifold transitions and Mori theory†"

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The first statement of Corollary 2 of [1] is incorrect. This error unfortunately propagates through the paper and invalidates the proofs of the main results, Theorem 1 and Corollary 1.

We recall that in [1], we study a symplectic manifold Y obtained from a symplectic manifold X by performing a conifold transition [5] in a single nullhomologous Lagrangian 3-sphere L. Corollary 2 of [1] asserted that $c_1^2(Y) = 0$ whenever $c_1^2(X) = 0$, where c_1^2 denotes the square of the first Chern class. This is incorrect. Although there is a canonical injection $\phi : H^2(X) \to H^2(Y)$ which maps $c_1(X)$ to $c_1(Y)$, as discussed in [1, Lemma 2], the map ϕ is not induced by a map of spaces from Y to X, and need not be a ring homomorphism. In fact, there is a nullhomologous Lagrangian 3-sphere in $E \times \mathbb{P}^1$ for any Kähler surface E, after suitably scaling the Kähler form on E. The analysis of the conifold transition of $\mathbb{P}^2 \times \mathbb{P}^1$ in [4, Proposition 4.2] shows that the square of the first Chern class of the conifold transition of $\mathbb{C}^2 \times \mathbb{P}^1$ is non-zero; indeed [4, Equation 4] shows that the locally finite cycles $\mathbb{C}^2 \times \{p\}$ and $\mathbb{C}^2 \times \{q\}$ have proper transforms in the conifold transition which meet cleanly along the exceptional $\mathbb{P}^1$ of the surgery.

[1, Theorem 1] asserted that when $X = E \times \mathbb{P}^1$, with E an Enriques surface, then Y is not Kähler. The proof first argued that if Y was Kähler it would be projective, and then derived a contradiction from Mori's classification of extremal rays and the resulting constraints on the intersection form on $H^*(Y;\mathbb{Z})$. With a more careful computation of $c_1^2(Y)$, the argument breaks down, since there are extremal rays compatible with the given topology. The main theorem of Jiang [3] applies Wall's classification of diffeomorphism types of smooth six-manifolds [6] to prove that the manifold Y of [1, Theorem 1] is in fact diffeomorphic to a projective 3-fold. The proof of [1, Corollary 1] relied essentially on the non-Kähler conclusion of [1, Theorem 1], hence is also invalidated.

We would like to emphasise that, so far as we know, the following two questions remain open:

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† A. Corti and I. Smith, Math. Res. Lett., **12**:767–778, (2005).

1) Is there a conifold transition of a Kähler 3-fold which is not Kähler?
2) (Donaldson's question from [2]) Is every Lagrangian sphere in an algebraic variety the vanishing cycle of an algebraic degeneration?

Finally, we note that the basic idea that Mori's classification of extremal rays should provide new constraints for a symplectic manifold to be projective, or even Kähler, as in [1, Section 6], remains sound in principle, but no longer has a valid "proof of concept" example.

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