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Geometrically Connected Components of Lubin-Tate Deformation Spaces with Level Structures

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Dedicated to Jean-Pierre Serre on the occasion of his 80th birthday

Abstract: We determine the geometrically connected components of the generic fibre of the deformation space $\mathcal{M}_m$ which parameterizes deformations of a one-dimensional formal $\mathfrak{o}$-module equipped with Drinfeld level-m-structures. It is shown that the geometrically connected components are defined over a Lubin-Tate extension of the base field, and the action of the covering group $GL_n(\mathfrak{o}/\varpi^m))$ on the components is given by the determinant. This furnishes a description of the action of this group on the étale cohomology of the spaces $\mathcal{M}_m^{rig}$ in degree zero.

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1. Introduction

Let F be a local non-Archimedean field, and denote by $\mathfrak{o}$ its ring of integers. Let $\mathbb{X}$ be a one-dimensional formal $\mathfrak{o}$-module of F-height n over the algebraic closure

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$\mathbb{F}$ of the residue field of $\mathfrak{o}$. Generalizing work of Lubin and Tate, cf. [LT], V. G. Drinfeld showed in [D] that the functor of deformations of $\mathbb{X}$ is representable by an affine formal scheme $\mathcal{M}_0 = \mathrm{Spf}(R)$, where $R \simeq \hat{\mathfrak{o}}^{nr}[[u_1, \ldots, u_{n-1}]]$. Here, $\hat{\mathfrak{o}}^{nr}$ is the completion of the maximal unramified extension of $\mathfrak{o}$. Moreover, Drinfeld introduced the notion of a level-m-structure, and proved that the functor of deformations of $\mathbb{X}$ which are equipped with a level-m-structure is representable by a formal scheme $\mathcal{M}_m = \mathrm{Spf}(R_m)$, where R_m is a regular local ring which is a finite flat R-module, and étale over R after inverting a uniformizer ϖ of $\mathfrak{o}$. In this paper we are interested in the geometrically connected components of the rigid-analytic space

$$M_m = \mathcal{M}_m^{rig}$$

associated to $\mathcal{M}_m$, cf. [dJ2], sec. 7. The étale cohomology groups of the spaces M_m have been investigated in the last two decades by various authors because of their significance for the local Langlands correspondence. Let us cite only [Ca], [Bo], and [HT]. According to conjectures by Carayol and Drinfeld, the inductive limit

$$H^{n-1} = \varinjlim_{m} H^{n-1}(M_m \times_{\hat{F}^{nr}} \mathbb{C}_\varpi, \overline{\mathbb{Q}_l})$$

realizes simultaneously the Jacquet-Langlands and the Langlands correspondence (cf. [Ca] for a more precise statement). Here $\hat{F}^{nr}$ is the field of fractions of $\hat{\mathfrak{o}}^{nr}$, and $\mathbb{C}_\varpi$ is the completion of an algebraic closure of $\hat{F}^{nr}$.

Whereas the spaces M_m are defined purely locally, the analysis of the inductive limit above is carried out in [Bo] and [HT] by embedding the local situation into a global one. This is done because it is very hard to understand the action of the inertia group on H^{n-1} (however, there are results for $m = 1$ or $n = 2$, cf. [Y] resp. [W]). By studying the geometry of the spaces M_m purely locally, it is possible to understand the action of the pro-covering group $GL_n(\mathfrak{o})$ and the action of $\mathrm{Aut}(\mathbb{X})$, thereby proving the assertion concerning the Jacquet-Langlands correspondence (cf. [St], where the results are unconditionally proved for the Euler-Poincaré characteristic of the cohomology). Of course, one would like to understand explicitly the cohomology in all degrees, and the most basic task is hence to determine the representation on $H^0(M_m \times_{\hat{F}^{nr}} \mathbb{C}_\varpi, \overline{\mathbb{Q}_l})$. This means to identify the group action on the set of geometrically connected components of M_m.

We are going to explain how we do this. For $m \geq 0$ put $K_m = Frac(R_m)$, $K = K_0$, and fix a separable closure K^s of K containing all K_m. Denote by $G_K = Gal(K^s/K)$ the absolute Galois group of K. Let X^{univ} be the universal deformation of $\mathbb{X}$ over R, and let

$$T_{\varpi^m} = X^{univ}[\varpi^m](K^s)$$

be the group of K^s-valued points of the ϖ^m-torsion subgroup of X^{univ}. T_{ϖ^m} is free of rank n over $\mathfrak{o}/(\varpi^m)$. Then we show, exactly as Raynaud in [R], that the induced action of G_K on $\Lambda^n(T_{\varpi^m})$ factorizes through the canonical isomorphism

$$Gal(\hat{F}^{nr}_m/\hat{F}^{nr}) \longrightarrow (\mathfrak{o}/(\varpi^m))^\times ,$$

where $\hat{F}^{nr}_m \subset K^s$ is obtained from $\hat{F}^{nr}$ by adjoining all ϖ^m-torsion points of a fixed formal $\mathfrak{o}$-module LT of height one over $\hat{\mathfrak{o}}^{nr}$. As $Gal(K^s/K_m)$ acts trivially on T_{ϖ^m}, and hence on $\Lambda^n(T_{\varpi^m})$, this implies that $\hat{F}^{nr}_m$ is contained in K_m. Then one shows that R_m/ϖ_m is reduced, where ϖ_m is a uniformizer of $\hat{F}^{nr}_m$, and using a result of de Jong, cf. [dJ2], 7.3.5, one obtains that M_m is geometrically connected over $\hat{F}^{nr}_m$.

When thinking about this problem of geometrically connected components we were inspired by de Jong's paper [dJ1]. De Jong also uses the crucial fact that the action on the determinant of the Tate module is given by the cyclotomic character (he considers only the case $F = \mathbb{Q}_p$), but his further reasoning is different from ours as he is interested in a description of the étale fundamental group of the corresponding period space, cf. [dJ1], Prop. 7.4.

We conclude with a remark on an earlier approach to the problem of geometrically connected components of these spaces. Let $\hat{\mathfrak{o}}^{nr}_m$ be the ring of integers of the Lubin-Tate extension $\hat{F}^{nr}_m$. From the inclusion $\hat{\mathfrak{o}}^{nr}_m \subset R_m$ we get a morphism of formal schemes

$$\mathcal{M}_m \longrightarrow \mathcal{M}^{(1)}_m := \mathrm{Spf}(\mathfrak{o}_{\hat{F}^{nr}_m}) .$$

$\mathcal{M}^{(1)}_m$ can be intrinsically defined as the deformation space with level-m-structures of the reduction $LT_{\mathbb{F}}$ of LT. Our original aim was to define *a priori* a functorial map like this, and then to deduce that M_m is geometrically connected over $(\mathcal{M}^{(1)}_m)^{rig} = \mathrm{Sp}(\hat{F}^{nr}_m)$. A functorial map as above can be thought of as associating to a deformation of $\mathbb{X}$ which is equipped with a level-m-structure:

$$(X, (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n \xrightarrow{\phi} X[\varpi^m])$$

its *determinant*:

$$(\Lambda^n(X), \Lambda^n(\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n \xrightarrow{\Lambda^n\phi} \Lambda^n(X)[\varpi^m])\,.$$

In an unpublished manuscript J. Lubin treats the problem of defining the determinant of a one-dimensional formal module together with a determinant map, cf. [L], and we, in our earlier approach, worked along the same lines. First one has to define $\Lambda^n(X)$, what one may do using, for example, Zink's theory of displays (resp., in the equal characteristic case, the 'module des coordonnées', cf. [Bo]). Then, and more difficult it seems, one has to compare the Tate modules. Using Falting's results in [F], cf. p. 278, one may possibly do this. The approach of this paper, however, seems to be much more elementary.

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Notation. In this paper, F will be a non-Archimedean local field, with ring of integers $\mathfrak{o}$, and ϖ will be a uniformizer of F. The number of elements of the residue field will be denoted by q, and the residue field itself by $\mathbb{F}_q$. We denote by $\mathbb{F}$ an algebraic closure of $\mathbb{F}_q$. $\hat{F}^{nr}$ is the completion of the maximal unramified extension of F, and $\hat{\mathfrak{o}}^{nr}$ its ring of integers. $\mathbb{C}_\varpi$ denotes a the completion of an algebraic closure of $\hat{F}^{nr}$. If A is a local ring we denote by $\mathfrak{m}_A$ its maximal ideal. The residue field of a point x on a scheme will be denoted by $\kappa(x)$.

2. Preliminaries

In this section we recall without proof some facts about the formal deformation schemes $\mathcal{M}_m$, cf. [D], [St].

2.1. Let $\mathbb{X}$ be a one-dimensional formal group over $\mathbb{F}$ that is equipped with an action of $\mathfrak{o}$, i.e. we assume given a homomorphism $\mathfrak{o} \to \mathrm{End}_{\mathbb{F}}(\mathbb{X})$ such that the action of $\mathfrak{o}$ on the tangent space is given by the reduction map $\mathfrak{o} \to \mathbb{F}_q \subset \mathbb{F}$. Such an object is called a *formal* $\mathfrak{o}$*-module* over $\mathbb{F}$. Moreover, we assume that $\mathbb{X}$ is of F-height n, which means that the kernel of multiplication by ϖ is a finite group scheme of rank q^n over $\mathbb{F}$.

It is known that for each $n \in \mathbb{Z}_{>0}$ there exists a formal $\mathfrak{o}$-module of F-height n over $\mathbb{F}$, and that it is unique up to isomorphism [D], Prop. 1.6, 1.7.

Let $\mathcal{C}$ be the category of complete local noetherian $\hat{\mathfrak{o}}^{nr}$-algebras with residue field $\mathbb{F}$. A *deformation* of $\mathbb{X}$ over an object A of $\mathcal{C}$ is a pair (X, ι), consisting of a formal $\mathfrak{o}$-module X over A which is equipped with an isomorphism $\iota : \mathbb{X} \to X_{\mathbb{F}}$ of formal $\mathfrak{o}$-modules over $\mathbb{F}$, where $X_{\mathbb{F}}$ denotes the reduction of X modulo the maximal ideal $\mathfrak{m}_A$ of A. Sometimes we will omit ι from the notation.

Following Drinfeld [D], sec. 4B, we define a *structure of level* m or *level-m-structure* on a deformation X over $A \in \mathcal{C}$ ($m \geq 0$) as an $\mathfrak{o}$-module homomorphism

$$\phi : (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n \longrightarrow \mathfrak{m}_A \,,$$

such that, after having fixed a coordinate T on the formal group X, the power series $[\varpi]_X(T) \in A[[T]]$, which describes the multiplication by ϖ on X, is divisible by

$$\prod_{a \in (\varpi^{-1}\mathfrak{o}/\mathfrak{o})^n} (T - \phi(a)) \,.$$

Here, $\mathfrak{m}_A$ is given the structure of an $\mathfrak{o}$-module via X.

Define the following set-valued functor $\mathcal{M}_m$ on the category $\mathcal{C}$. For an object A of $\mathcal{C}$ put

$$\mathcal{M}_m(A) = \{(X, \iota, \phi) \,|\, (X, \iota) \text{ is a def. over } A, \, \phi \text{ is a level-}m\text{-structure on } X\}/ \simeq ,$$

where $(X, \iota, \phi) \simeq (X', \iota', \phi')$ if and only if there is an isomorphism $(X, \iota) \to (X', \iota')$ of formal $\mathfrak{o}$-modules over A, which is compatible with the level structures. For $0 \leq m' \leq m$ one gets by restricting a level-m-structure to

$$(\varpi^{-m'}\mathfrak{o}/\mathfrak{o})^n \subset (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n$$

a level-m'-structure and hence a natural transformation

$$\mathcal{M}_m \longrightarrow \mathcal{M}_{m'} .$$

Put $\mathfrak{o}_B = \mathrm{End}_{\mathfrak{o}}(\mathbb{X})$. $\mathfrak{o}_B$ is the ring of integers in a central division algebra over F with Hasse invariant $\frac{1}{n}$ ([D], Prop. 1.7). There is a natural action of $GL_n(\mathfrak{o}/\varpi^m) \times \mathfrak{o}_B^\times$ from the right on the functor $\mathcal{M}_m$ given by

$$[X, \iota, \phi] \cdot (g, b) = [X, \iota \circ b, \phi \circ g]$$

where $(g, b) \in GL_n(\mathfrak{o}/\varpi^m) \times \mathfrak{o}_B^\times$ and $[X, \iota, \phi]$ denotes the equivalence class of (X, ι, ϕ).

Theorem 2.2. *(i) The functor $\mathcal{M}_m$ is representable by a regular local ring R_m of dimension n. Hence there is a universal formal $\mathfrak{o}$-module X^{univ} over $R := R_0$ which defines on the maximal ideal $\mathfrak{m}_{R_m}$ of R_m the structure of an $\mathfrak{o}$-module. There is a universal level-m-structure*

$$\phi_m^{univ} : (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n \longrightarrow \mathfrak{m}_{R_m}$$

such that, if $a_1, ..., a_n$ is a basis of $(\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n$ over $\mathfrak{o}/(\varpi^m)$, then

$$\phi_m^{univ}(a_1), \ldots, \phi_m^{univ}(a_n)$$

is a regular system of parameters for R_m.

(ii) The ring homomorphism $R_m \to R$ which corresponds to the natural transformation $\mathcal{M}_m \to \mathcal{M}_0$ makes R_m a finite and flat R-algebra. Moreover, $R_m[\frac{1}{\varpi}]$ is étale and galois over $R[\frac{1}{\varpi}]$ with Galois group isomorphic to $GL_n(\mathfrak{o}/\varpi^m)$.

(iii) R is (non-canonically) isomorphic to $\hat{\mathfrak{o}}^{nr}[[u_1, \ldots, u_{n-1}]]$.

Proof. (i) This result is [D], Prop. 4.3.

(ii) That R_m is finite and flat over R is again [D], Prop. 4.3. For the second statement we refer to [St], Thm. 2.1.2.

(iii) This is [D], Prop. 4.2. □

Remark. The fact that $\hat{\mathfrak{o}}^{nr}[[u_1, \ldots, u_{n-1}]]$ represents $\mathcal{M}_0$ is due to Lubin and Tate (for $F = \mathbb{Q}_p$), cf. [LT]. For this reason $\mathcal{M}_0$, the deformation space without level structures, is sometines called the *Lubin-Tate moduli space*, cf. [HG], [Ch].

By the preceding theorem, R_m is a domain, and we put $K_m = Frac(R_m)$. This is a Galois extension of $K := K_0$ with Galois group canonically isomorphic to $GL_n(\mathfrak{o}/\varpi^m)$. The maps $\mathcal{M}_m \to \mathcal{M}_{m'}$ induce injections $K_{m'} \hookrightarrow K_m$. Put

$$K_\infty = \cup_{m\geq 0} K_m \,,$$

and fix a separable closure K^s of K containing K_∞.

2.3. We conclude this section by recalling that one may choose the parameters $\varpi = u_0, u_1, \ldots, u_{n-1}$ of R and the coordinate on X^{univ} such that multiplication by ϖ on X^{univ} is given by a power series $[\varpi]_{X^{univ}}(T) \in R[[T]]$ with the property that

$$[\varpi]_{X^{univ}}(T) \equiv u_i T^{q^i} \text{ mod } (u_0, \ldots, u_{i-1})\,,\ \deg(q^i+1)\,,$$

cf. [HG], Prop. 5.7. In particular, if $x \in \mathrm{Spec}(R)$ is a point where ϖ vanishes but u_1 is invertible in the residue field $\kappa(x)$ of x, the multiplication of ϖ on the formal group $X^{univ}\hat{\otimes}\kappa(x)$ has as kernel a group scheme of order q. Therefore the connected component of the associated ϖ-divisible group $(X^{univ}[\varpi^\infty]) \otimes \kappa(x)$ over $\kappa(x)$ is a formal $\mathfrak{o}$-module of height one.

3. The Galois action on the determinant of the Tate module

3.1. Denote by T the Tate module of the ϖ-divisible group $X^{univ}[\varpi^\infty] \otimes K$:

$$T = \varprojlim_m X^{univ}[\varpi^m](K^s)\,.$$

The universal Drinfeld level-structures furnish an isomorphism of $\mathfrak{o}$-modules

$$\mathfrak{o}^n \longrightarrow T\,,$$

so that $\Lambda^n_{\mathfrak{o}}(T)$ is free of rank one over $\mathfrak{o}$.

3.2. We recall some facts from Lubin-Tate theory. Fix a formal $\mathfrak{o}$-module LT of height one over $\hat{\mathfrak{o}}^{nr}$. As the universal deformation ring of height one formal $\mathfrak{o}$-modules is just $\hat{\mathfrak{o}}^{nr}$, all such formal $\mathfrak{o}$-modules are isomorphic. Denote by $\hat{F}^{nr,s} \subset K^s$ the algebraic closure of $\hat{F}^{nr}$ in K^s. It is a separable closure of $\hat{F}^{nr}$. Let $\mathfrak{m}$ be the maximal ideal of the ring of integers in the completion of $\hat{F}^{nr,s}$. $\mathfrak{m}$ is equipped via LT with an $\mathfrak{o}$-module structure, and the torsion points of LT in $\mathfrak{m}$ are known to lie in $\hat{F}^{nr,s}$. Let $\hat{F}^{nr}_m \subset \hat{F}^{nr,s}$ be the subfield generated over $\hat{F}^{nr}$ by the ϖ^m-torsion points of LT in $\mathfrak{m}$. As all $\mathfrak{o}$-modules of height one over $\hat{\mathfrak{o}}^{nr}$ are isomorphic over $\hat{\mathfrak{o}}^{nr}$, this field is independent of the choice of LT. There is a canonical isomorphism

$$\chi_m : Gal(\hat{F}^{nr}_m/\hat{F}^{nr}) \longrightarrow (\mathfrak{o}/(\varpi^m))^\times ,$$

such that for any ϖ^m-torsion point α of LT and $\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})$ one has

$$\sigma(\alpha) = [\chi_m(\sigma)]_{LT}(\alpha) .$$

The field $\hat{F}^{nr}_\infty = \bigcup_m \hat{F}^{nr}_m$ is the maximal abelian extension of $\hat{F}^{nr}$. The characters χ_m induce an isomorphism

$$\chi : Gal(\hat{F}^{nr}_\infty/\hat{F}^{nr}) \longrightarrow \mathfrak{o}^\times ,$$

Finally, let $\tilde{\chi} : G_K \to \mathfrak{o}^\times$ be the composition of $G_K \to Gal(\hat{F}^{nr}_\infty/\hat{F}^{nr})$ with χ.

The following theorem is essentially Raynaud's theorem on the action of the Galois group on the determinant of the Tate module of a p-divisible group, cf. [R], Thm. 4.2.1. We prove it here again for the sake of completeness and because we need it in the more general context of formal $\mathfrak{o}$-modules.

Theorem 3.3. *The natural action of $G_K = Gal(K^s/K)$ on $\Lambda^n_{\mathfrak{o}}(T)$ is given by the character $\tilde{\chi}$, i.e. for all $\sigma \in G_K$ and $\lambda \in \Lambda^n_{\mathfrak{o}}(T)$ one has:*

$$\sigma(\lambda) = \tilde{\chi}(\sigma)\lambda .$$

Proof. We assume $n \geq 2$, because for $n = 1$ everything is trivial. We follow the reasoning of the proof of [R], Th. 4.2.1. Let $x \in \mathrm{Spec}(R)$ be a prime ideal containing ϖ but not containing u_1, and denote by $\kappa(x)$ the residue field at x. We already noticed in 2.3 that the connected component of the ϖ-divisible group

$X^{univ}[\varpi^\infty] \otimes \kappa(x)$ is a formal $\mathfrak{o}$-module of height one, and the étale part is hence of height $n-1$. Let R_x^{sh} be the strict henselization of R at x. Its residue field $\kappa(x)^s$ is a separable closure of $\kappa(x)$. By [D], Prop. 1.7, all formal $\mathfrak{o}$-modules of finite height over a separably closed field are isomorphic, so that the formal module associated to the connected component

$$(X^{univ}[\varpi^\infty] \otimes \kappa(x)^s)^\circ$$

is isomorphic to $LT\hat{\otimes}_{\hat{\mathfrak{o}}^{nr}}\kappa(x)^s$. Therefore, the formal module associated to

$$(X^{univ}[\varpi^\infty] \otimes R_x^{sh})^\circ$$

is isomorphic to a deformation of $LT\hat{\otimes}_{\hat{\mathfrak{o}}^{nr}}\kappa(x)^s$, hence it is isomorphic to

$$LT\hat{\otimes}_{\hat{\mathfrak{o}}^{nr}}R_x^{sh}\,,$$

because there is up to isomorphism only one deformation, cf. [Ha], Thm. 22.4.16. Let $R_x^h \subset R_x^{sh}$ be the henselization of R at x, put $K_x^h = Frac(R_x^h)$, and fix a separable closure $K_x^{h,s}$ of K_x^h together with an embedding of K^s into this separable closure. We get an embedding of Galois groups $Gal(K_x^{h,s}/K_x^h) \hookrightarrow G_K$, and it follows from what we have said above that the restriction of the G_K-action on

$$\Pi := \Lambda^n_{\mathfrak{o}}(T) \otimes \tilde{\chi}^{-1}$$

to $Gal(K_x^{h,s}/K_x^h)$ factors through the absolute Galois group of $\kappa(x)$. That means, the representation of G_K on Π is unramified at x. As the ϖ-divisible group $X^{univ}[\varpi^\infty]$ is étale at every point of $\mathrm{Spec}(R)$ where ϖ is invertible, Π is unramified at all points of $U = \mathrm{Spec}(R) - V((\varpi, u_1))$, whose complement is of codimension 2. Hence the representation of G_K on Π extends to the étale fundamental group $\pi_1(U, \mathrm{Spec}(K^s))$ of U. But by the Zariski-Nagata purity theorem, cf. [SGA2], exp. X, Thm. 3.4, [SGA1], exp. X, Cor. 3.3, $\pi_1(U, \mathrm{Spec}(K^s)) = \pi_1(\mathrm{Spec}(R), \mathrm{Spec}(K^s))$, and the latter group is trivial, as R is strictly henselian. This means that G_K acts trivially on Π and therefore by $\tilde{\chi}$ on $\Lambda^n_{\mathfrak{o}}(T)$. □

Corollary 3.4. *(i) The action of G_K on $\Lambda^n_{\mathfrak{o}}(X^{univ}[\varpi^m](K^s))$ is given by the character $\tilde{\chi}_m$, which is the composition of $\tilde{\chi}$ with the canonical map $\mathfrak{o}^\times \to (\mathfrak{o}/\varpi^m)^\times$.*

(ii) The ring $R_m = \mathcal{O}(\mathcal{M}_m)$ contains the ring of integers $\hat{\mathfrak{o}}_m^{nr}$ of the Lubin-Tate extension $\hat{F}_m^{nr}$.

Proof. (i) This assertion follows immediately from the G_K-equivariant isomorphism

$$\Lambda^n_{\mathfrak{o}}(X^{univ}[\varpi^m](K^s)) \simeq \Lambda^n_{\mathfrak{o}}(T)/\varpi^m\Lambda^n_{\mathfrak{o}}(T)\,.$$

(ii) The subgroup $Gal(K^s/K_m) \subset G_K$ acts trivially on $X^{univ}[\varpi^m](K^s)$, and hence trivially on $\Lambda^n(X^{univ}[\varpi^m](K^s))$. By (i) the action on the latter module is given by $\tilde{\chi}_m$. Therefore, $Gal(K^s/K_m)$ acts trivially on $\hat{F}_m^{nr}$, and hence $\hat{F}_m^{nr}$ is contained in K_m. Because R_m is integrally closed, it contains $\hat{\mathfrak{o}}_m^{nr}$. □

By the second assertion, we will view from now on R_m as an $\hat{\mathfrak{o}}_m^{nr}$-algebra, and $R_m[\frac{1}{\varpi}]$ as an $\hat{F}_m^{nr}$-algebra.

4. Geometrically connected components and the group action on π_0

4.1. Fix an integer $m \geq 1$. In this section we show first that $R_m[\frac{1}{\varpi}]$ is geometrically integral over $\hat{F}_m^{nr}$. Then we determine the structure of $\pi_0(M_m \times_{\hat{F}^{nr}} \mathbb{C}_\varpi)$ together with the action of $GL_n(\mathfrak{o}) \times \mathfrak{o}_B^\times \times G_{\hat{F}^{nr}}$.

Denote as in Cor. 3.4 by $\hat{\mathfrak{o}}_m^{nr}$ the ring of integers in $\hat{F}_m^{nr}$, and let ϖ_m be a uniformizer in $\hat{\mathfrak{o}}_m^{nr}$. Furthermore, we fix a set $\mathcal{R} \subset (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n$ of representatives of the orbits of the action of $(\mathfrak{o}/\varpi^m)^\times$ on

$$(\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n - (\varpi^{-(m-1)}\mathfrak{o}/\mathfrak{o})^n\,.$$

Finally, we abbreviate the universal level-m-structure ϕ_m^{univ}, cf. 2.2, by ϕ_m.

Proposition 4.2. *(i) For $\alpha \in \mathcal{R}$ the element $\phi_m(\alpha) \in \mathfrak{m}_{R_m}$ of the regular local ring R_m is irreducible. Moreover, the elements $\phi_m(\alpha)$ et $\phi_m(\beta)$ are not associated if $\alpha \neq \beta$, $\alpha, \beta \in \mathcal{R}$.*
(ii) Up to a unit in R_m, the element ϖ_m is equal to the product $\prod_{\alpha\in\mathcal{R}} \phi_m(\alpha)$.
(iii) The ring $R_m/\varpi_m R_m$ is reduced.
(iv) The ring $R_m[\frac{1}{\varpi}]$ is geometrically integral over $\hat{F}_m^{nr}$.

Proof. (i) An element $\alpha \in \mathcal{R}$ can be completed to a basis $\alpha = \alpha_1, \ldots, \alpha_n$ of $(\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n$. By 2.2 we know that $\phi_m(\alpha_1), \ldots, \phi_m(\alpha_n)$ is a regular set of parameters of R_m. Hence, the element $\phi_m(\alpha) = \phi_m(\alpha_1)$ is in particular irreducible. Let α, β be two different elements of $\mathcal{R}$. For a point $x \in \mathrm{Spec}(R_m)$ consider the induced homomorphism

$$\phi_{m,x} : (\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n \xrightarrow{\phi_m} X^{univ}[\varpi^m](R_m) \longrightarrow X^{univ}[\varpi^m](\kappa(x)) .$$

If h is the height of the connected part of the ϖ-divisible group $X^{univ}[\varpi^\infty] \otimes \kappa(x)$, the kernel of $\phi_{m,x}$ is a direct summand of $(\varpi^{-m}\mathfrak{o}/\mathfrak{o})^n$ of rank h over $\mathfrak{o}/(\varpi^m)$. And conversely, if the rank of the kernel of $\phi_{m,x}$ is equal to h, then the height of the connected component of $X^{univ}[\varpi^\infty] \otimes \kappa(x)$ is h. Hence, if α and β are two different elements of $\mathcal{R}$, the kernel of $\phi_{m,x}$ is at least of rank two if it contains α and β, and in this case the height of the connected component of $X^{univ}[\varpi^\infty] \otimes \kappa(x)$ is at least two. Now suppose that $\phi_m(\alpha)$ and $\phi_m(\beta)$ are associated prime elements. Let $x_0 \in \mathrm{Spec}(R_0)$ correspond to the prime ideal ϖR_0. Then, as we already stated in 2.3, the height of the connected component of $X^{univ}[\varpi^\infty] \otimes \kappa(x_0)$ is one. As R_m is finite and flat over R_0, there is a prime ideal x_m of R_m lying over x_0. Then the kernel of ϕ_{m,x_m} is a direct summand of rank one, hence generated by one element, γ say. Let $g \in GL_n(\mathfrak{o}/\varpi^m)$ be an element with $g(\alpha) = \gamma$. Then α lies in the kernel of ϕ_{m,y_m}, where $y_m = g(x_m)$, so $\phi_m(\alpha)$ is in the prime ideal corresponding to y_m. But if $\phi(\alpha)$ and $\phi(\beta)$ are associated, i.e. $\phi(\beta) = u\phi(\alpha)$ with a unit $u \in R_m$, we also have $\phi_{m,y_m}(\beta) = 0$, and so the kernel of ϕ_{m,y_m} is at least of rank two, which cannot be, because $\phi_{m,y_m} = \phi_{m,x_m} \circ g$.

(ii) Let $\alpha \in \mathcal{R}$. Then $\mathfrak{p} = \phi(\alpha)R_m$ is a prime ideal, and because the kernel of $\phi_{m,\mathfrak{p}}$ contains α, the connected component of $X^{univ}[\varpi^\infty] \otimes \kappa(\mathfrak{p})$ is at least of height one. Hence $\mathfrak{p}$ contains ϖ, and therefore ϖ_m too. Because the elements $\phi_m(\alpha)$, $\alpha \in \mathcal{R}$, are pairwise not associated, and because R_m is a UFD, we see that

$$\varpi_m = f \cdot \prod_{\alpha \in \mathcal{R}} \phi(\alpha) ,$$

with some element $f \in R_m$. Now let us consider the unramified extension F' of F of degree n inside $\hat{F}^{nr}$. Denote by $\mathfrak{o}'$ the ring of integers of F' and fix a formal $\mathfrak{o}'$-module X of F'-height one. Let $\iota : \mathbb{X} \to X \times_{\mathfrak{o}'} \mathbb{F}$ be an isomorphism, such that the pair $(X \times_{\mathfrak{o}'} \hat{\mathfrak{o}}^{nr}, \iota)$ is a deformation of $\mathbb{X}$ (as a formal $\mathfrak{o}$-module of height n). This pair corresponds to a point x in $\mathrm{Spec}(R_0)$. We lift this point to a point y in $\mathrm{Spec}(R_m)$. The residue field $\kappa(y)$ at y is then an extension of F' generated by the ϖ^m-torsion points of X. Denote by v the valuation on this extension which is normalized by $v(\varpi) = 1$. Then the valuation of a ϖ^m-torsion point of X, which is not annihilated by ϖ^{m-1}, is equal to $\frac{1}{(q^n-1)q^{n(m-1)}}$. Mapping ϖ_m into $\kappa(y)$ and using the equation above we calculate

$$\frac{1}{(q-1)q^{m-1}} = v(\varpi_m) \geq \sum_{\alpha\in\mathcal{R}} v(\phi_{m,y}(\alpha)) = \sum_{\alpha\in\mathcal{R}} \frac{1}{(q^n-1)q^{n(m-1)}}$$

$$= \frac{(q^n-1)q^{n(m-1)}}{(q-1)q^{m-1}} \frac{1}{(q^n-1)q^{n(m-1)}} = \frac{1}{(q-1)q^{m-1}} .$$

And this shows that f is necessarily a unit in R_m.

(iii) This is an immediate consequence of (i) and (ii).

(iv) By [EGA], Cor. 18.9.8, it suffices to show that the fibres of

$$\mathrm{Spec}(R_m) \longrightarrow \mathrm{Spec}(\hat{\mathfrak{o}}_m^{nr})$$

are geometrically reduced. By (iii), this is the case for the fibre over the closed point. Let us now consider the generic fibre. Let E be a field containing $\hat{F}_m^{nr}$. Because $R_m[\frac{1}{\varpi}]$ is étale over $R_0[\frac{1}{\varpi}]$, the ring extension

$$R_0[\frac{1}{\varpi}] \otimes_{\hat{F}^{nr}} E \to R_m[\frac{1}{\varpi}] \otimes_{\hat{F}^{nr}} E = \prod_{\sigma\in Gal(\hat{F}_m^{nr}/\hat{F}^{nr})} R_m[\frac{1}{\varpi}] \otimes_{\hat{F}_m^{nr}} E$$

is étale too. Because $R_0[\frac{1}{\varpi}] \otimes_{\hat{F}^{nr}} E$ maps injectively into $E[[u_1, \ldots, u_{n-1}]]$, this ring is reduced. By general results on étale extensions,

$$R_m[\frac{1}{\varpi}] \otimes_{\hat{F}_m^{nr}} E ,$$

which is étale over $R_0[\frac{1}{\varpi}] \otimes_{\hat{F}^{nr}} E$, is reduced too. Therefore $R_m[\frac{1}{\varpi}] \otimes_{\hat{F}_m^{nr}} E$ is reduced. □

Theorem 4.3. *(i) Let E be a finite separable extension of $\hat{F}^{nr}$, which contains the Lubin-Tate extension $\hat{F}_m^{nr}$ of $\hat{F}^{nr}$. Then the rigid-analytic space*

$$M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(E) = \mathcal{M}_m^{rig} \times_{\hat{F}^{nr}} \mathrm{Sp}(E)$$

over E has $(q-1)q^{m-1}$ connected components. These are the fibres of the morphism

$$M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(E) \longrightarrow \mathrm{Sp}(\hat{F}^{nr}_m) \times_{\hat{F}^{nr}} \mathrm{Sp}(E) = \mathrm{Sp}(\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} E)\,.$$

Proof. For the construction of the rigid-analytic space associated to a formal scheme we refer to [dJ2], sec. 7. The first assertion clearly follows from the second. Because

$$M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(E) = \coprod_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})} M_m \times_{\hat{F}^{nr}_m} \mathrm{Sp}(E)\,,$$

we only need to show that $M_m \otimes_{\hat{F}^{nr}_m} \mathrm{Sp}(E)$ is connected. By [dJ2], 7.2.4 (g), we have

$$M_m \otimes_{\hat{F}^{nr}_m} \mathrm{Sp}(E) = \mathrm{Spf}\left(R_m \hat{\otimes}_{\hat{\mathfrak{o}}^{nr}_m} \mathfrak{o}_E\right)^{rig}\,,$$

where $\mathfrak{o}_E$ is the ring of integers in E. By [dJ2], 7.3.5, this space is connected if the ring

$$R_m \hat{\otimes}_{\hat{\mathfrak{o}}^{nr}_m} \mathfrak{o}_E = R_m \otimes_{\hat{\mathfrak{o}}^{nr}_m} \mathfrak{o}_E$$

(this equality holds because $E/\hat{F}^{nr}_m$ is finite) is integrally closed. This ring is contained in $R_m \otimes_{\hat{\mathfrak{o}}^{nr}_m} E = R_m[\frac{1}{\varpi}] \otimes_{\hat{F}^{nr}_m} E$ which is integral, by the preceding proposition 4.2. By [Bou], V, §1.7, Cor. to Prop. 19, $R_m \otimes_{\hat{\mathfrak{o}}^{nr}_m} \mathfrak{o}_E$ is integrally closed. $\square$.

Let $\mathbb{C}_\varpi$ be a completion of an algebraic closure $\bar{F}^{nr}$ of $\hat{F}^{nr}$. Denote by $G_{\hat{F}^{nr}} = Gal(\bar{F}^{nr}/\hat{F}^{nr})$ the absolute Galois group of $\hat{F}^{nr}$. By continuity it acts on $\mathbb{C}_\varpi$. In the following we use the isomorphism $\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} \mathbb{C}_\varpi \simeq \prod_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})} \mathbb{C}_\varpi$ given by

$$\lambda \otimes \mu \mapsto (\sigma^{-1}(\lambda)\mu)_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})}\,.$$

This isomorphism is used to identify the connected components of $\mathrm{Sp}(\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} \mathbb{C}_\varpi)$ with the connected components of $\coprod_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})} \mathrm{Sp}(\mathbb{C}_\varpi)$ which we identify with its indexing set $Gal(\hat{F}^{nr}_m/\hat{F}^{nr})$. The latter group gets identified via the character χ_m with $(\mathfrak{o}/\varpi^m)^\times$. Note that if we let $\tau \in G_{\hat{F}^{nr}}$ act on the second factor of $\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} \mathbb{C}_\varpi$, we have, via the isomorphism above,

$$((\sigma^{-1}(\lambda)\tau(\mu))_\sigma = \tau((\sigma \circ \tau)^{-1}(\lambda)\mu)_\sigma) = \tau((\sigma^{-1}(\lambda)\mu)_{\sigma\circ\tau^{-1}})\,.$$

Therefore, on the indexing set $Gal(\hat{F}^{nr}_m/\hat{F}^{nr})$, τ acts by multiplication by $(\tau|_{\hat{F}^{nr}_m})^{-1}$, and consequently on $(\mathfrak{o}/\varpi^m)^\times$ by $\chi_m(\tau|_{\hat{F}^{nr}_m})^{-1}$.

Theorem 4.4. *(i) The morphism $M_m \to \mathrm{Sp}(\hat{F}^{nr}_m)$ induces a bijection*

$$\pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)) \xrightarrow{\simeq} \pi_0(\mathrm{Sp}(\hat{F}^{nr}_m \times_{\hat{F}^{nr}} \mathbb{C}_\varpi))\,,$$

and the set on the right is identified with $(\mathfrak{o}/\varpi^m)^\times$, as explained above. The resulting bijection

$$\pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)) \xrightarrow{\simeq} (\mathfrak{o}/\varpi^m)^\times$$

is $GL_n(\mathfrak{o})\times\mathfrak{o}_B^\times\times G_{\hat{F}^{nr}}$-equivariant if we let $GL_n(\mathfrak{o})\times\mathfrak{o}_B^\times\times G_{\hat{F}^{nr}}$ act on $(\mathfrak{o}/\varpi^m)^\times$ by

$$(g, b, \tau) \mapsto \det(g)Nrd(b)^{-1}\chi(\tau|_{\hat{F}^{nr}_\infty})^{-1} \mod (1 + \varpi^m\mathfrak{o})\,.$$

Here $Nrd : \mathfrak{o}_B^\times \to \mathfrak{o}^\times$ denotes the reduced norm.

(ii) In particular, the zero'th l-adic étale cohomology group decomposes as follows:

$$H^0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi), \overline{\mathbb{Q}_l}) \simeq \bigoplus_\omega (\omega \circ \det) \otimes (\omega \circ Nrd)^{-1} \otimes (\omega \circ rec_{\hat{F}^{nr}})\,,$$

where $\omega : (\mathfrak{o}/\varpi^m)^\times \to \overline{\mathbb{Q}_l}^\times$ runs through all $\overline{\mathbb{Q}_l}$-valued characters of $(\mathfrak{o}/\varpi^m)^\times$, and $rec_{\hat{F}^{nr}}$ is the reciprocity map from local class field theory (normalized such that an arithmetic Frobenius is mapped to a uniformizer).

Proof. (i) As R_m is normal, by [dJ2], 7.3.5., the rigid space $M_m = \mathrm{Spf}(R_m)^{rig}$ is connected. By [Co], Cor. 3.2.3, there is a finite separable extension E of $\hat{F}^{nr}$ such that the canonical map

$$M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi) \longrightarrow M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(E)$$

induces a bijection of the connected components:

$$\pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)) \xrightarrow{\simeq} \pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(E))\,.$$

By theorem 4.3, we can take here $E = \hat{F}^{nr}_m$, and get:

$$\begin{aligned}\pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)) &= \pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\hat{F}^{nr}_m)) = \pi_0(\mathrm{Sp}(\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} \hat{F}^{nr}_m))\\ &= \pi_0(\mathrm{Sp}(\textstyle\prod_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})} \hat{F}^{nr}_m))\\ &= Gal(\hat{F}^{nr}_m/\hat{F}^{nr}) \xrightarrow{\chi} (\mathfrak{o}/\varpi^m)^\times\,.\end{aligned}$$

Here we used the isomorphism $\hat{F}^{nr}_m \otimes_{\hat{F}^{nr}} \hat{F}^{nr}_m \simeq \prod_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})} \hat{F}^{nr}_m$ given by

$$\lambda \otimes \mu \mapsto (\sigma^{-1}(\lambda)\mu)_{\sigma \in Gal(\hat{F}^{nr}_m/\hat{F}^{nr})}\,.$$

The action of the Galois group G_K on R_m factors through $GL_n(\mathfrak{o})$, so let $g \in GL_n(\mathfrak{o})$ come from some $\rho \in G_K$. Then, for a ϖ^m-torsion point a of LT in $\hat{F}^{nr}_m$ we have

$$\rho(a) = [\tilde{\chi}(\rho)]_{LT}(a)\,.$$

But $\tilde{\chi}(\rho)$ is the element by which ρ acts on the Tate module $\Lambda^n(T)$, by theorem 3.3. On the other hand, g acts as $\det(g)$ on $\Lambda^n(T)$. Therefore, g acts on the torsion point $a \in \hat{\mathfrak{o}}^{nr}_m \subset R_m$ by $[\det(g)]_{LT}(a)$. Via the above identification of $\pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi))$ with $(\mathfrak{o}/\varpi^m)^\times$, we get that $GL_n(\mathfrak{o})$ acts on $(\mathfrak{o}/\varpi^m)^\times$ via the determinant ($\mod (1+\varpi^m\mathfrak{o})$). The action of $G_{\hat{F}^{nr}}$ we already computed above. Finally, we consider the action of $\mathfrak{o}_B^\times$. Let $\mathfrak{o}' \subset \mathfrak{o}_B$ be the maximal unramified extension of $\mathfrak{o}$ in $\mathfrak{o}_B$. Because the reduced norm on $\mathfrak{o}_B^\times$, when restricted to $(\mathfrak{o}')^\times$, maps $(\mathfrak{o}')^\times$ surjectively onto $\mathfrak{o}^\times$, it suffices to calculate the action of $(\mathfrak{o}')^\times \subset \mathfrak{o}_B^\times$. Let X be a formal $\mathfrak{o}'$-module of height one over $\mathfrak{o}'$. Then $X \hat{\otimes}_{\mathfrak{o}'} \hat{\mathfrak{o}}^{nr}$, when equipped with an isomorphism of its special fibre with $\mathbb{X}$, corresponds to a point x in $\mathrm{Spec}(R_0)$ which is fixed by the action of $(\mathfrak{o}')^\times$. Let $y \in \mathrm{Spec}(R_m)$ be a point over x. Then $\mathfrak{o}'_m := R_m/y$ is the ring of integers in $\hat{F}^{nr}.F'_m$, where F'_m is the Lubin-Tate extension of $F' = \mathfrak{o}'[\frac{1}{\varpi}]$ generated by the ϖ^m-torsion points of X. By local class field theory, the action of an element $b \in (\mathfrak{o}')^\times$ on the subfield $\hat{F}^{nr}_m \cap F'_m \subset F'_m$ is given by the norm of $N_{F'/F}(b) = Nrd(b)$ (on torsion points of LT inside $\hat{F}^{nr}_m$). But the action of $b \in \mathfrak{o}_B^\times$ on a triple (X, ι, ϕ) is defined by $(X, \iota \circ b, \phi)$, and the latter object is equivalent to $(X, \iota, b^{-1} \circ \phi)$, because X has multiplication by $\mathfrak{o}'$. Hence the action of b (via R_m and specialisation) on the ϖ^m-torsion points of X is given by $[b^{-1}]_X$. Therefore, it acts on the ϖ^m-torsion points of LT by $[Nrd(b)^{-1}]_{LT}$. Via the above identification of the connected components of $M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)$ with $(\mathfrak{o}/\varpi^m)^\times$, b acts on the latter set by multiplication with $Nrd(b)^{-1}$. This proves that $\mathfrak{o}_B^\times$ acts on $(\mathfrak{o}/\varpi^m)^\times$ by Nrd^{-1}.

(ii) It is well known, that the local reciprocity map (normalized such that an arithmetic Frobenius is mapped to a uniformizer) fullfills

$$rec_{\hat{F}^{nr}}(\tau) = \chi(\tau)^{-1}\,.$$

From this and the first part of the theorem the assertion follows immediately. $\square$.

We can finally also pass to the limit over all m and get a natural $GL_n(\mathfrak{o}) \times \mathfrak{o}_B^\times \times G_{\hat{F}^{nr}}$-equivariant map

$$\varprojlim_m \pi_0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi)) \xrightarrow{\simeq} \mathfrak{o}^\times\,,$$

where on the right side the group $GL_n(\mathfrak{o}) \times \mathfrak{o}_B^\times \times G_{\hat{F}^{nr}}$ acts by

$$(g, b, \tau) \mapsto \det(g) Nrd(b)^{-1} rec_{\hat{F}^{nr}}(\tau)\,.$$

And hence:

$$\varinjlim_m H^0(M_m \times_{\hat{F}^{nr}} \mathrm{Sp}(\mathbb{C}_\varpi), \overline{\mathbb{Q}_l}) \simeq \bigoplus_\omega (\omega \circ \det) \otimes (\omega \circ Nrd)^{-1} \otimes (\omega \circ rec_{\hat{F}^{nr}})\,,$$

where ω runs through all continuous characters $\mathfrak{o}^\times \to \overline{\mathbb{Q}_l}^\times$ (with the discrete topology on $\overline{\mathbb{Q}_l}^\times$).

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