Pure and Applied Mathematics Quarterly
Volume 6, Number 3
(*Special Issue: In honor of
Joseph J. Kohn, Part 1 of 2*)
715—724, 2010

On Dominating Sets for Uniform Algebra on Pseudoconvex Domains

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Abstract: In this article we study the dominating phenomenon for uniform algebra on pseudoconvex domains in several complex variables. We prove that a subset E of a bounded domain D is a dominating set for $A(D)$ if and only if $\overline{E}$ contains the Shilov boundary $S(D)$ of $A(D)$.
Keywords: dominating set, uniform algebra, peak function, Shilov boundary, pseudoconvex domain

1. Introduction

Let D be a bounded pseudoconvex domain in $\mathbb{C}^n$, $n \geq 2$, and let ∂D denote the boundary of D. Denote by $\mathcal{O}(D)$ the space of holomorphic functions on D. Let $A(D)$ be the uniform algebra on D defined by $A(D) = \mathcal{O}(D) \cap C(\overline{D})$. As usual, the norm on $A(D)$ is supremum norm $\| \cdot \|_\infty$.

Definition 1.1. *A subset E of D is called a dominating set for $A(D)$ with respect to the supremum norm if every two functions $f, g \in A(D)$ with $|f(z)| \leq |g(z)|$ for $z \in E$ implies $\| f \|_\infty \leq \| g \|_\infty$.*

Received February 4, 2008.
Mathematics Subject Classification (2000): 32A38, 32T40.
The author is partially supported by the grant NSC 94(95)-2115-M-007-011(006) from the National Science Council of the Republic of China.

In this note we shall study the dominating phenomenon for uniform algebra on pseudoconvex domains in several complex variables. For some terminology of several complex variables the reader is referred to any standard texts, for instance, Krantz[19], Range[21] and Chen and Shaw[6].

First, we recall the definition about peak function.

Definition 1.2. *Let p be a boundary point of a domain D in $\mathbb{C}^n$, $n \geq 2$. p is said to be a peak point relative to $A(D)$ if there exists a function $f \in A(D)$ such that $f(p) = 1$ and $|f(q)| < 1$ for $q \in \overline{D} \setminus \{p\}$. Such an f is called a peak function for $A(D)$ at p.*

We also recall the definition of the Shilov boundary.

Definition 1.3. *A closed subset $S(D)$ of the boundary ∂D of a bounded domain D is called the Shilov boundary of $A(D)$ if $S(D)$ is the smallest closed subset of ∂D such that every function $f \in A(D)$ assumes its maximum modulus on $S(D)$.*

When D is a bounded domain in $\mathbb{C}^n$, $A(D)$ is a uniform algebra on $\overline{D}$. Clearly, the Shilov boundary must contains all the peak points. In fact, it is known that the Shilov boundary $S(D)$ of $A(D)$ on a compact metric space is the closure of the set of all peak points. For instance, see Gamelin[15].

When $n = 1$ and D is the open unit disc in the complex plane, Danikas and Hayman[9] proved the following result. See also Hayman[17].

Theorem 1.4. *Let E be a subset of the open unit disc U in the complex plane. Then E is a dominating set for $A(U)$ if and only if $\overline{E}$ contains ∂U.*

In fact, Theorem 1.4 can be stated on a more general punctured domain in one complex variable.

Theorem 1.5. *Let $\Omega = D \setminus \cup_{j=1}^{\infty} \overline{D}_j$ be a domian in $\mathbb{C}$, where D and D_j are bounded domains with Jordan curves as their boundaries such that D_j, $j \in \mathbb{N}$, are relatively compact subdomians of D and that $\overline{D}_j \cap \overline{D}_k = \emptyset$ for $j \neq k$. Let E be a subset of Ω. Then E is a dominating set for $A(\Omega)$ if and only if $\overline{E}$ contains the boundary $\partial\Omega = \partial D \cup (\cup_{j=1}^{\infty} \partial D_j)$, that is, the Shilov boundary $S(\Omega) = \partial\Omega$.*

Proof. It suffices to show every boundary point is a peak point. For outer boundary ∂D, this can be seen easily by composing a conformal mapping from the open unit disc onto D. For any one of the inner boundary ∂D_j, one can pick a point $z_j \in D_j$ and apply inversion mapping with respect to z_j. This proves the theorem.

Then, we prove the following main result in several complex variables.

Theorem 1.6. *Let D be a bounded domain in $\mathbb{C}^n$, $n \geq 2$, and let E be a subset of D. Then E is a dominating set for $A(D)$ if and only if $\overline{E}$ contains the Shilov boundary $S(D)$.*

Proof. First, suppose that $\overline{E}$ contains the Shilov boundary $S(D)$, and that $f, g \in A(D)$ with $|f(z)| \leq |g(z)|$ for $z \in E$. Then, by continuity, we have $|f(z)| \leq |g(z)|$ for all $z \in S(D)$. Since $S(D)$ is the Shilov boundary, we have $\| f \|_\infty = \| f \|_{\infty, S(D)} \leq \| g \|_{\infty, S(D)} = \| g \|_\infty$.

On the other hand, if $p \in \partial D \setminus \overline{E}$ is a peak point, then there is a peak function $f(z)$ at p. Obviously, $\sup_{z \in \overline{E}} |f(z)| = m < 1$. Thus, if we let $g(z) \equiv m$, it is easily seen that $|f(z)| \leq |g(z)|$ for $z \in E$, but $\| f \|_\infty = 1 > m = \| g \|_\infty$. This shows that E can not be a dominating set for $A(D)$. Hence, $\overline{E}$ must contains all of the peak points, and hence, by density, the Shilov boundary $S(D)$. This completes the proof of the theorem.

Thus, from the theory of peak functions we immediately obtain the following consequences of Theorem 1.6.

Theorem 1.7. *The assertion of Theorem 1.6 holds with $S(D) = \partial D$ if D belongs to one of the following classes:*

(1) *D is a strictly convex bounded domain with C^1 boundary;*
(2) *D is a smooth bounded pseudoconvex domain such that the peak points are dense in the boundary.*

Proof. First, by strict convexity of D we mean that the tangent hyperplane $\mathbb{H}_p$ of D at the boundary point p intersects the closure of D at exactly one point, i.e., $\mathbb{H}_p \cap \overline{D} = \{p\}$. Thus, following directly from its geometric properties, every boundary point is a peak point. This proves (1). (2) is obvious. This completes the proof of the theorem.

Some remarks are in order.

Remarks. (i) It is possible that some boundary points of domains in class (1) are of infinite type. Also, class (1) contains smooth bounded convex domains with real analytic boundary.

(ii) Class (2) contains many important and interesting subclasses that have been treated independently before. We mention some of them here.

(a) D is a smooth bounded strongly pseudoconvex domain. Hence, every boundary point is a peak point. See Rossi[22] and Epe[12].

(b) D is a smooth bounded pseudoconvex domain of finite type in $\mathbb{C}^2$. Hence, every boundary point is a peak point. See Fornaess and McNeal[13]. This class also contains smooth bounded pseudoconvex domains in $\mathbb{C}^2$ with real analytic boundary. See Bedford and Fornaess[5].

(c) D is a smooth bounded pseudoconvex domain of finite type in $\mathbb{C}^n$, $n \geq 2$. Obviously, classes (a) and (b) are special cases of this class. The notion of finite type was introduced by D'Angelo[8]. Later, Catlin[4] showed that smooth bounded pseudoconvex domain of finite type satisfies condition (P). Then, Sibony showed in [23] that condition (P) is equivalent to the notion called B-regularity. Thus, again from the work of Sibony[23] we see that plurisubharmonic barrier exists at every boundary point of domains of finite type. Here, by a plurisubharmonic barrier at a boundary point p we mean that there is a function $u(z) \in C(\overline{D})$ such that u is plurisubharmonic in D and that $u(p) = 0$ and $u(q) < 0$ for $q \in \overline{D} \setminus \{p\}$. Therefore, every boundary point p of domains of finite type is in the closure of strongly pseudoconvex boundary points(see Basener[2]). Finally, by using Kohn's global regularity result for $\overline{\partial}$(see Kohn[18]), Hakim and Sibony[16] and Pflug[20] showed that every strongly pseudoconvex boundary point of D is a peak point. It follows that peak points are dense in the boundary of any smooth bounded pseudoconvex domain of finite type in $\mathbb{C}^n$.

(d) As noted in (c), (2) can be applied to any smooth bounded pseudoconvex domains as long as the strongly pseudoconvex points are dense in the boundary. Thus, a complex variety may exist in the boundary. In particular, it can be applied to the famous worm domain constructed by Diederich and Fornaess[11] on which the global regularity of the $\overline{\partial}$-Neumann problem fails. For instance, see Barrett[1] and Christ[7].

In general, the Shilov boundary $S(D)$ might be strictly smaller than the topological boundary of the domain as shown by the following examples in several complex variables.

Theorem 1.8. *Let* $\Omega = D\backslash K$*, where* D *is one of the domains stated in Theorem 1.7, and* K *is a compact subset of* D*. Let* E *be a subset of* Ω*. Then* E *is a dominating set for* $A(\Omega)$ *if and only if* $\overline{E}$ *contains the outer boundary of* Ω*, i.e., the Shilov boundary* $S(D) = \partial D$*.*

Proof. The assertion follows immediately from Hartogs extension theorem and Theorem 1.7.

Note that the domain Ω in Theorem 1.8 is not pseudoconvex. For related results the reader is referred to Bremermann[3].

Another interesting and important example is the Hartogs triangle defined by $\Omega = \{(z,w) \in \mathbb{C}^2 \mid \ |z| < |w| < 1\}$. Hartogs triangle is a domain of holomorphy with nontrivial Nebenhülle.

Theorem 1.9. *Let* Ω *be the Hartogs triangle, and let* E *be a subset of* Ω*. Then* E *is a dominating set for* $A(\Omega)$ *if and only if* $\overline{E}$ *contains the torus* $T = \{|z| = 1\} \times \{|w| = 1\}$*, i.e., the torus* T *is the Shilov boundary* $S(\Omega)$*.*

Proof. The key of the proof is to see that for any $f \in A(\Omega)$, $\| f \|_{\infty,\Omega} = \| f \|_{\infty,T}$. Let $w = ze^{i\theta} \neq 0$, $|z| < 1$, for some θ, be a boundary point. Then the complex disc

$$\triangle = \{(\lambda z, \lambda z e^{i\theta}) \mid \ \lambda \in \mathbb{C}, \ |\lambda| < 1/|z|\}$$

lies in the boundary $\partial\Omega$. Since $f \in A(\Omega)$, $f|_{\partial\Omega} \in A(\triangle)$. The assertion now follows from maximum modulus principle. This shows that $\| f \|_{\infty,\Omega} = \| f \|_{\infty,T}$.

On the other hand, it is also easy to see that the torus T is exactly the set formed by all peak points for $A(\Omega)$. This proves the theorem.

2. Convex domains

In this section we shall discuss briefly the problem on smooth bounded convex domain D. For $p \in \partial D$, denote by $\mathbb{H}_p$ the hyperplane tangent to D at p, and let $T_p = \mathbb{H}_p \cap \overline{D}$. Note that T_p is a compact convex subset of the boundary ∂D. Denote also by T_p^o the interior of T_p. We shall call p a strictly convex point if $T_p = \{p\}$. Clearly, a strictly convex point is a peak point.

Due to geometric simplicity of strictly convex points of a convex domain, it arises a natural question. Namely, is the set of all strictly convex points dense in the Shilov boundary $S(D)$ on any smooth bounded convex domain D? We exhibit in the following examples that, in general, this is not true.

Example 2.1. Let D be a bounded convex domain in the complex plane with C^2 boundary. It is well known that $S(D) = \partial D$. Suppose now that the boundary ∂D of D conatins a line segment L. Clearly, every boundary point $p \in L$ is not strictly convex. Hence, the set of all strictly convex points is not dense in the Shilov boundary $S(D)$ on this convex domain D.

Next example demonstrates a similar phenomenon in several complex variables.

Example 2.2. Let D be a smooth bounded convex domain in $\mathbb{C}^n$, $n \geq 2$, and let $p = (1, \cdots, 1, 0)$ be a boundary point of D. Suppose that, in some open neighborhood U of p, $D \cap U$ is defined by

$$D \cap U = \{z \in U \mid \ \rho(z) = \sum_{i=1}^{n-1} |z_i| - \frac{1}{2}(z_n + \overline{z}_n) - (n-1) < 0\}.$$

We may assume that $z_1 \cdots z_{n-1} \neq 0$ for $z \in \partial D \cap U$. Since the defining function ρ is independent of $t = \mathrm{Im} z_n$, it is not hard to see that $\dim_{\mathbb{R}} T_z^o = n$ for $z \in \partial D \cap U$.

A direct calculation of the complex Hessian gives

$$\frac{\partial^2 \rho}{\partial z_i \partial \overline{z}_i}(z) = \frac{1}{4|z_i|} > 0, \ 1 \leq i \leq n-1; \quad \frac{\partial^2 \rho}{\partial z_n \partial \overline{z}_n}(z) = 0,$$
$$\frac{\partial^2 \rho}{\partial z_i \partial \overline{z}_j}(z) = 0, \ i \neq j, \ 1 \leq i, j \leq n,$$

for $z \in \partial D \cap U$. Now, choose a basis of tangential type (1,0) vector fields

$$L_i = \frac{\partial \rho}{\partial z_n} \frac{\partial}{\partial z_i} - \frac{\partial \rho}{\partial z_i} \frac{\partial}{\partial z_n} = -\frac{1}{2} \frac{\partial}{\partial z_i} - \frac{1}{2} \frac{\overline{z}_i}{|z_i|} \frac{\partial}{\partial z_n},$$

for $z \in \partial D \cap U$ and $1 \leq i \leq n-1$. This shows that the Levi form at any $z \in \partial D \cap U$ is positive definite. It follows that all of the boundary points $z \in \partial D \cap U$ are strongly pseudoconvex, and hence, peak points. This shows that the set of all strictly convex points is not dense in the Shilov boundary $S(D)$.

However, under suitable hypothesis we show in the next theorem that the strictly convex points indeed are dense in the Shilov boundary.

Theorem 2.3. *Let D be a smooth bounded convex domain in $\mathbb{C}^n$, $n \geq 2$. Suppose that, for $p \in \partial D$, either p is a strictly convex point or $dim_{\mathbb{R}} T_p^o \geq n+1$. Let F be the set of all strictly convex points of the boundary, and let E be a subset of D. Then E is a dominating set for $A(D)$ if and only if $\overline{E}$ contains the Shilov boundary $S(D) = \overline{F}$.*

Proof. Clearly, $\overline{F}$ is contained in $S(D)$. Conversely, if $p \in \partial D \setminus \overline{F}$, then p is not a strictly convex point. Observe that p is an interior point of $\partial D \setminus \overline{F}$. Now, if we apply complex structure to the space T_p^o, we see from hypothesis that any point $q \in T_p^o$ lies in a complex disc sitting in the boundary. It indicates that p can not be a strongly pseudoconvex point. Since peak points are all in the closure of strongly pseudoconvex points, thus $\overline{F}$ contains all of the peak points. For instance, see Debiard and Gaveau[10] and Basener[2]. Hence, $S(D) = \overline{F}$. This proves the theorem.

Examples 2.1 and 2.2 show that the hypothesis on dimension of T_p^o in Theorem 2.3 is optimal. Convex domains that satisfy the hypothesis of Theorem 2.3 can be constructed easily. For instance, let B_n be the unit open ball in $\mathbb{C}^n$, and let $D_\alpha = B_n \cap \{z \in \mathbb{C}^n \mid \; y_n = \mathrm{Im} z_n < \alpha\}$ for some $0 < \alpha < 1$. Then let D be obtained from D_α by rounding the edge of D_α. It is not hard to see that D is a smooth bounded convex domain that is Levi-flat on a real $(2n-1)$-dimensional closed ball V sitting in the hyperplane $\{z \in \mathbb{C}^n \mid \; y_n = \alpha\}$. We may also assume that D is strictly convex outside V.

Next, we consider the problem on convex Reinhardt domains, and show that the hypothesis on dimension of T_p^o can be reduced by one on such domains. Recall that a domain D is called Reinhardt if $z = (z_1, \cdots, z_n) \in D$ implies $(z_1 e^{i\theta_1}, \cdots, z_n e^{i\theta_n}) \in D$ for all real θ_j, $1 \leq j \leq n$.

Theorem 2.4. *Let D be a smooth bounded convex Reinhardt domain in $\mathbb{C}^n$, $n \geq 2$. Let F be the set of all strictly convex points of the boundary, and let E be a subset of D. Suppose that, if $p \in \partial D \setminus F$, $dim_{\mathbb{R}}(T_p^o) \geq n$. Then E is a dominating set for $A(D)$ if and only if $\overline{E}$ contains the Shilov boundary $S(D) = \overline{F}$.*

Proof. Let $p \in \partial D \setminus \overline{F}$, then p is not a strictly convex point. Hence, $\dim_{\mathbb{R}}(T_p^o) \geq n$. If $\dim_{\mathbb{R}}(T_p^o) \geq n+1$, then we argue as in the proof of Theorem 2.3. Thus, we may assume that $\dim_{\mathbb{R}}(T_p^o) = n$.

Let $q \in T_p^o$. Write $q = (r_1(q)e^{i\theta_1(q)}, \cdots, r_n(q)e^{i\theta_n(q)})$. We shall denote the complex line $\mathbb{L}_i = \{z \in \mathbb{C}^n \mid \ z_j = r_j(q)e^{i\theta_j(q)},\ j \neq i\}$, and denote

$$e^{i\theta_j}T_p^o = \{(z_1, \cdots, z_{j-1}, e^{i\theta_j}z_j, z_{j+1}, \cdots, z_n) \mid \ z \in T_p^o\}.$$

We claim that $\mathbb{L}_i \cap T_p^o$ contains more than one point for some i. If not, we will have $\mathbb{L}_i \cap T_p^o = \{q\}$ for $1 \leq i \leq n$. Since $e^{i\theta_j}T_p^o$ lies in the boundary by rotational symmetry for $1 \leq j \leq n$, this shows that, in some open neighborhood of q, the boundary ∂D will have dimension $2n$ which is not possible.

Thus, we may assume that $\mathbb{L}_1 \cap T_p^o$ contains more than the point q, say, $q_1 \neq q$. Note that $q_1 \in T_p^o$, so q_1 is also a boundary point. Now, by convexity of D and $q, q_1 \in \mathbb{H}_p$, we see that the line segment $\overline{qq_1}$ lies in the boundary, and hence, in $\mathbb{L}_1 \cap T_p^o$. Then, by rotation in z_1 direction, the line segment $\overline{qq_1}$ generates an annulus in z_1 contained in the boundary. Again, by convexity it will force a whole complex disc U in z_1 centerred at the origin to sit in the boundary. Obviously, $q \in \overline{U}$. It indicates that p can not be a strongly pseudoconvex point. Then, we argue as in the proof of Theorem 2.3, and the proof of the theorem is now completed.

A related result concerning the peak points on Reinhardt domains can be found in Gamelin[14].

Again, as shown in the following example, the hypothesis $\dim_{\mathbb{R}}(T_p^o) \geq n$ stated in Theorem 2.4 is optimal on convex Reinhardt domains.

Example 2.5. Let D be a smooth bounded convex Reinhardt domain in $\mathbb{C}^n$, $n \geq 2$. Suppose that a piece of the boundary W is defined by

$$W = \{z \in \partial D \mid \ \rho(z) = \sum_{i=1}^{n} a_i|z_i| - 1 = 0,\ a_i > 0,\ 1 \leq i \leq n\},$$

where $0 < \alpha_i < |z_i| < \beta_i < 1/a_i$ for some appropriate positive real α_i, β_i, $1 \leq i \leq n$. Obviously, W is an open subset of the boundary, and points in W are not strictly convex. Note that $\dim_{\mathbb{R}}(T_p^o) = n - 1$ for $p \in W$.

Then, a direct calculation of the complex Hessian gives

$$\frac{\partial^2 \rho}{\partial z_i \partial \overline{z}_i}(z) = \frac{a_i}{4|z_i|} > 0,\ 1 \leq i \leq n; \quad \frac{\partial^2 \rho}{\partial z_i \partial \overline{z}_j}(z) = 0,\ i \neq j,\ 1 \leq i, j \leq n,$$

for z in W. This shows that the Levi form at any z in W is positive definite. It follows that all of the boundary points z in W are strongly pseudoconvex, and hence, peak points. Thus, the set of all strictly convex points is not dense in the Shilov boundary $S(D)$.

3. Product domains

For product domains we have the following result.

Theorem 3.1. *Let $D = \prod_{j=1}^{k} D_j$ be a bounded domain in $\mathbb{C}^{\sum_{j=1}^{k} n_j}$, where D_j, $j = 1, 2, \cdots, k$, is a bounded domain in $\mathbb{C}^{n_j}$. Suppose that $S(D_j)$ is the Shilov boundary of $A(D_j)$. Let E be a subset of D. Then E is a dominating set for $A(D)$ if and only if $\overline{E}$ contains the Shilov boundary $S(D) = \prod_{j=1}^{k} S(D_j)$ of D.*

Proof. First, observe that $(p_1, \cdots, p_k) \in \partial D$ is a peak point for $A(D)$ if and only if $p_j \in \partial D_j$, $j = 1, 2, \cdots, k$, is a peak point for $A(D_j)$. Hence, the set of all peak points of D is dense in $S(D) = \prod_{j=1}^{k} S(D_j)$. To see any function $f \in A(D)$ assumes its maximum modulus in $S(D)$, simply observe that the restriction of f to any coordinate z_j is in $A(D_j)$. This proves the theorem.

Acknowledgment. The author is indebted to professors Tai-Chung Lee and Alan Noell for fruitful conversation and communication during the preparation of this article. The author also thanks the referee for the comments on Example 2.1.

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