

# Convexity in discrete structures

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Lecture Notes Series

Volume 5

# Convexity in discrete structures

Proceedings of the international instructional  
workshop on convexity in discrete structures,  
Thiruvananthapuram, India, 2006; and the  
international workshop on metric and convex graph  
theory, Barcelona, Spain, 2006

*Volume editors*

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Volume Editors:  
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## Foreword

This special volume of the Ramanujan Mathematical Society Lecture Note Series is the joint proceedings of the International Instructional Workshop on Convexity in Discrete Structures held at Thiruvananthapuram, Kerala, India (March 22–April 2, 2006) and the International Workshop on Metric and Convex Graph Theory held at Barcelona, Spain (June 12–16, 2006). The volume contains 12 selected refereed articles (surveys and contributed papers) on which talks at the conference were based. They are all devoted to abstract and discrete convexity and to discrete and graphic metric spaces.

The paper by B. D. Acharya reviews the set systems which can be realized as families of neighborhoods and iterated neighborhoods of graphs and their relationships with abstract interval structures. Four papers are devoted to the investigation of various transit functions on graphs, posets, and lattices, another structure closely related to intervals, convexity, and betweenness. The paper by H. M. Mulder presents the general theory of transit functions and formulate the open questions in this area. A. Mathews and J. Mathews investigate the transit functions on posets and lattices, while M. Changat and J. Mathews characterize the graphs for which the induced path transit function is monotone. Finally, Prasanth G. N. studies a transit function closely related with the geodesic transit function on graphs. P. Duchet investigates the convexity parameters of segment spaces and presents a sharp upper bound of Radon numbers of these spaces in terms of their Helly numbers. The paper by Kannan B. and M. Changat surveys the results about the hull number and the rank of induced path and triangle-path convexities of graphs. The paper by I. M. Pelayo overview the results on extremal vertices and Krein-Milman property of various graph-convexities. K. Reji Kumar investigates the convexity property of minimal edge dominating functions of graphs. C. Hernando, M. Mora, P. Slater, and D. R. Wood present a relationship between the fault-tolerant metric dimension and the classical metric dimension of graphs and an investigation of the first concept for trees. S. Klavžar presents a survey of recent results on the structure and the constructions of cubical partial cubes and formulates some remaining open questions. Finally, M. Changat and V. Chepoi give an overview of classical and recent results in the theory of finite metric spaces, their embedding and related graph classes.

In conclusion, the proceedings present a valuable collection of original and survey papers in the growing area of convexity studies and metric graph theory. Therefore the collection will be a welcome source for researchers in these areas.

# Acknowledgement

The International Instructional Workshop on Convexity in Discrete Structures (IIWCDS) was organized by the Department of Futures studies, University of Kerala, Thiruvananthapuram from March 22 to April 2, 2006 at Kovalam, Thiruvananthapuram, India. The workshop was sponsored by the Department of Science and Technology (DST), Govt. of India and co-sponsored by University of Kerala with its University Grants Commission's (UGC) unassigned grant and Kerala State Council for Science, Technology and Environment (KSCSTE), Govt. of Kerala. We express our sincere gratitude to DST, UGC and KSCSTE. The authorities of the University of Kerala and the staff and research scholars of the Department of Futures Studies have helped wholeheartedly for the success of the workshop. Many thanks to all of them for making this a fruitful academic reality.

The International Workshop on Metric and Convex Graph Theory (IWMCGT) was organized by Ignacio M. Pelayo and Carlos Seara and hosted by the research group in Computational Geometry and Combinatorial Geometry of the Polytechnic University of Catalonia from June 12th to 16th, 2006 in Barcelona, Spain. The workshop was sponsored by research group in Computational Geometry and Combinatorial Geometry, Applied Mathematical Department II, and by the Polytechnic University of Catalonia. We express our sincere gratitude to the UPC research group in Computational Geometry and Combinatorial Geometry for hosting and sponsoring the workshop.

**Manoj Changat**  
Coordinator, IIWCDS

**Ignacio M. Pelayo & Carlos Seara**  
Organizers, IWMCGT

# Preface

Convexity is basically a branch of geometry. It has innumerable connections with the other areas of mathematics such as analysis, algebra, topology, and combinatorial optimization. One of the most attractive aspects of convexity theory is the large number of easily stated and intuitively appealing unsolved problems that it still contains. The significance of convexity theory is that it deals analytically with global properties of objects and enables one to deal with extremal problems. There is a vast amount of literature on convexity theory from different perspectives. The axiomatic approach to convexity developed in the works of Levi, Jamison, Sierksma, Van de Vel and others, caught the attention of discrete mathematicians, especially graph theorists and resulted in vast literature of convexities in discrete structures like graphs, ordered sets, lattices etc. The myriad applications of convexity in discrete optimization, computational geometry etc. makes it a very interesting and important theme for discussion. The two workshops, one in India and the other in Spain, were held to discuss the problems on convexity mainly in graphs and other discrete structures.

This volume is the joint proceedings of the International Instructional Workshop on Convexity in Discrete Structures held at Thiruvananthapuram, Kerala, India during March 22–April 2, 2006 and the International Workshop on Metric and Convex Graph Theory held at Barcelona, Spain during June 12–16, 2006. The articles in this volume consist of the summary of lectures delivered and discussed in both workshops. There are 12 refereed articles consisting of surveys and contributed papers. Article 5 and article 7 are contributions from the Barcelona workshop and the rest are from the Thiruvananthapuram workshop.

With deep sense of gratitude, we register our indebtedness to the Ramanujan Mathematical Society (RMS), India for bringing out this joint proceedings. We are also extremely thankful to all the referees who devoted their valuable time for critically analyzing the articles and all the authors who contributed articles to this volume.

March 2007

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