Essays on geometric group theory
Ramanujan Mathematical Society
Lectures Notes Series

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Essays on geometric group theory

Proceedings of the instructional workshop and international conference on group theory, Indian Institute of Technology, Guwahati, December 2002

Volume editor

N. S. Narasimha Sastry
Preface

Geometric ideas have played a very major role in the development of Group theory in 20th century. Coxeter groups and Coxeter complexes, Tits theory of buildings, Bass-Serre theory of Groups acting on trees, Gromov’s notion of hyperbolic groups and combinatorial methods have all been very central in several areas of mathematics.

With the objective of introducing the basic ideas and recent developments in these topics to young researchers in various Research Institutions and Universities in India, the Indian Statistical Institute at its Bangalore Centre and the Indian Institute of Technology, Guwahati, organized “Instructional Workshop and International Conference on Geometric Group Theory” during December 2–21, 2002, at the beautiful campus of the Indian Institute of Technology, Guwahati.

In the Workshop part of this programme (December 2–14, 2002), five short courses of lectures were given. The topics were:

- Coxeter Groups (10 lectures) by R.P. Shukla and Maneesh Thakur
- Buildings and Tits Systems (10 lectures) by N.S. Narasimha Sastry
- Bass-Serre Theory (10 lectures) by A. Raghuram and B. Sury
- Hyperbolic Groups (5 lectures) by A. Naolekar
- Combinatorial Group Theory (5 lectures) by I.B.S. Passi and Roman Mikhailov

In the Conference part of this programme (December 16–21, 2002), experts in this area of mathematics were invited to give research/expository lectures in the topic of their expertise. The articles in this volume are mostly the expanded versions of the lectures delivered by the authors in this Conference. It is hoped that this volume will be of some assistance to mathematicians who would like to familiarize themselves with these topics.

The Organizing Committee for this programme consisted of Professors I.B.S. Passi (Chairman), N.S. Narasimha Sastry, B. Sury, B.K. Sarma and Meenaxi Bhattacharjee.

Apart from the Indian Statistical Institute and the Indian Institute of Technology, Guwahati, the programme also received financial support from the National Board for Higher Mathematics, India; The British Council, India; Harish-Chandra Research Institute, Allahabad; Oil and Natural Gas Commission and Oil India Limited.

I wish to thank all the speakers in this programme and the authors of the articles, without whose enthusiastic support, this programme would not have achieved its objective.

As a member of the organizing committee, I also thank all the sponsors for their financial support; the Indian Institute of Technology, Guwahati, for the excellent facilities and enthusiastic and cheerful support extended during the Conference; and the Ramanujan Mathematical Society for bringing out this volume.
I also would like to record my appreciation of the enthusiasm, meticulous attention to detail and the caring of Professor Meenaxi Bhattacharjee, her colleagues and students at IIT, Guwahati, which contributed largely towards making this event a very pleasant and fruitful experience. I am confident that all the participants at this event share this sentiment.

Finally, I would like to thank Ms. Asha Lata for her help in compiling this volume.

Date: 5 December 2008
Indian Statistical Institute
Bangalore Centre

N. S. Narasimha Sastry
Editor
Talks given in the Conference

Professor Peter Abramenko  Some applications of Bass-Serre theory to the $GE_2$ question
Professor Roger M. Bryant  Metanilpotent varieties of groups
Professor Marc Burger  Tree automorphisms, products of trees and lattices
Professor Alan Camina  Block designs
Dr. Indira Chatterji  Twisted idempotent conjectures
Dr. Ahmad Erfanian  On some groups with the zero presentation ranks
Mr. Shripad M. Garge  On conjugacy of Weyl groups
Professor Narain Datt Gupta  A group with the slowest sub-exponential growth
Professor C.K. Gupta  Recent results on automorphism groups
Professor Alessandra Iozzi  Trees and hyperbolic spaces
Dr. D.L. Johnson  Word growth of groups
Professor S.S. Khare  Group actions on smooth manifolds with specific fixed point sets
Dr. Linus Kramer  Symmetric spaces, asymptotic cones and buildings
Professor Ravi Kulkarni  Idea of a branched covering
Mr. Roman Mikhailov  On localization in groups and group varieties
Professor Shahar Mozes  Tree automorphisms, products of trees and lattices
Professor Himadri Mukherjee  On the isomorphism problem and related problems
Professor I.B.S. Passi  Centralizers and homology
Dr. Laszlo Pyber  Boundedly generated groups
Professor M.S. Ragunathan  The word metric on lattices in higher rank semi-simple Lie groups
Dr. A. Raghuram  Depths and conductors of representations of $p$-adic groups
Dr. Bertrand Remy  Non-linearities of Kac-Moody groups
Professor Mark Ronan  Constructing buildings and twin buildings
Dr. Parameswaran Sankaran: On automorphisms and quasi-isometries of finitely generated groups

Professor Marcus du Sautoy: Counting subgroups in nilpotent groups and points on elliptic curves

Professor Dan Segal: Subgroups of finite index in profinite groups

Professor J.B. Srivastava: Invariants of certain classical and reductive groups and their applications in computer vision

Dr. Geetha Venkataraman: Finite group enumerations: a brief introduction

Professor D.N. Verma: A natural representation model for the symmetric groups, and the ensuring potential for a new unified representation theory for Coxeter Groups

Dr. Christopher Voll: Zeta functions of groups and enumeration in Bruhat-Tits buildings of $Sl_n(Q_p)$

Professor John S. Wilson: Aspects of growth of groups
Conference Participants

Professor Peter Abramenko
Dr. Meenaxi Bhattacharjee
Professor Roger M. Bryant
Professor Marc Burger
Professor Alan Camina
Dr. Indira Chatterji
Professor B.P. Chetiya
Dr. Ahmad Erfanian
Professor C.K. Gupta
Professor Narain Datt Gupta
Professor Alessandra Iozzi
Dr. D.L. Johnson
Professor S.S. Khare
Dr. Linus Kramer
Professor Ravi Kulkarni
Professor Ramji Lal
Professor Shahar Mozes
Professor H.K. Mukherjee
Professor C. Musili
Dr. A. Naolekar
Professor I.B.S. Passi

Professor Dipendra Prasad
Dr. Laszlo Pyber
Dr. K.N. Raghavan
Professor M.S. Raghunathan
Dr. A. Raghuram
Dr. Bertrand Remy
Professor Mark Ronan
Dr. Paraeswaran Sankaran
Dr. Bhaba Kumar Sarma
Professor N.S.N. Sastry
Professor Marcus du Sautoy
Professor Dan Segal
Professor B.L. Sharma
Dr. Ravindra Prasad Shukla
Professor J.B. Srivastava
Dr. B. Sury
Dr. Maneesh Thakur
Professor D.N. Verma
Professor Christopher Voll
Professor John S. Wilson
Workshop Participants

Md. S K Safique Ahmad
Ms. Sasmita Barik
Mr. Dhruba Jyoti Bora
Ms. Ardeline Mary Buhphang
Mr. Anandaram Burhagohain
Mr. Moon Kumar Chetry
Ms. Jonali Choudhury
Mr. Biren Das
Dr. Partha Pratim Dey
Dr. Sanjay Kumar Dutta
Dr. Sanghita Dutta
Mr. Shripad M Garge
Mr. Sahzamal Hoque
Mr. Rafiqul Islam
Mr. Rupesh Kumar Jha
Mr. Bidyut Kalita
Mr. Roman Mikhailov
Mr. Milan Nath
Mr. Tulasi P Nepal
Mr. Binod Kumar Sahoo
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Mr. Ritumoni Sarma
Mr. Brajesh Kumar Sharma
Mr. Angom Tiken Singh
Mr. Anupam Kumar Singh
Mr. Ningthoujam Jiban Singh
Mr. Thokchom Vikramjit
Ms. Hiramala Talukdar
Ms. Madhusmita Tripathy
Dr. Geetha Venkataraman
Dr. Christopher Voll
Mr. Akhilesh Chandra Yadav
Mr. Satya Narayan Yadav
## Contents

Preface iii

Talks given in the Conference v

Conference Participants vii

Workshop Participants viii

Linear Representations and Arithmeticity of Lattices in Products of Trees
Marc Burger, Shahar Mozes and Robert J. Zimmer 1–25

Some Aspects of Profinite Group Theory Dan Segal 27–60

Ultrapowers of Lie Groups, a Question of Gromov, and the Margulis Conjecture Linus Kramer and Katrin Tent 61–77

Commensurators of Some Non-uniform Tree Lattices and Moufang Twin Trees Peter Abramenko and Bertrand Rémy 79–104

Kac-Moody Groups as Discrete Groups Bertrand Rémy 105–124

Construction and Automorphisms of Buildings Mark A. Ronan 125–143

Zeta Functions of Nilpotent Groups – Singular Pfaffians Christopher Voll 145–159