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Lie Theory and Representation Theory

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Preface

During the period from July 13 to July 31, 2009, East China Normal University hosted the second workshop and summer school on Lie Theory and Representation Theory. This volume contains the lecture notes of three courses in that summer school, together with the lecture notes of one course given in the first summer school which was held in 2006.

This volume consists of articles by Shun-Jen Cheng and Weiqiang Wang, Rolf Farnsteiner, Daniel K. Nakano, and Toshiyuki Tanisaki. These articles focus on different areas in Lie theory and representation theory. The article jointly by Cheng and Wang introduces some recent developments of representations of Lie superalgebras, explaining how Lie superalgebras of types $\mathfrak{gl}$ and $\mathfrak{osp}$ provide a natural framework for generalized Schur and Howe dualities, and how a super duality gives a conceptual solution to the irreducible character problem for these Lie superalgebras in terms of the classical Kazhdan-Lusztig polynomials.

Farnsteiner’s article discusses combinatorial and geometric aspects of representation theory of finite group schemes, and focuses on the “classical” theory of co-commutative Hopf algebras, the defining algebras of affine algebraic group schemes.

Nakano’s article gives a survey of recent developments in the representation theory and cohomology theory of reductive algebraic groups, their Frobenius kernels and their associated finite groups of Lie type.

Tanisaki’s article presents an overview of the theory of $D$-modules and its application to representations of Lie algebras.

This volume is well suited for graduate students in the fields of Lie theory and representation theory and related topics, and also for researchers who wish to learn about some current core areas in Lie theory and representation theory and their applications.

At last, we sincerely express our thanks to the Department of Mathematics, the International Exchange Division and the Graduate School of East China Normal University for their financial support to the summer schools and workshops in 2006 and 2009. We are grateful to National Natural Science Foundation of China.
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In Shanghai
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