Nonlinear Analysis in Geometry and Applied Mathematics

Lydia Bieri
Piotr Chruściel
Shing-Tung Yau
Contents

Preface vii

Future-complete null hypersurfaces, interior gluings, and the Trautman-Bondi mass
LYDIA BIERI AND PIOTR T. CHRUŚCIEL 1

Quasi-local mass at the null infinity of the Vaidya spacetime
PO-NING CHEN, MU-TAO WANG, AND SHING-TUNG YAU 33

A note on the Bartnik mass
JUSTIN CORVINO 49

Linear stability of rotating black holes: outline of the proof
FELIX FINSTER AND JOEL SMOLLER 77

Lower semicontinuity of Huisken’s isoperimetric mass
DAN A. LEE 91

Mean curvature deficit and a quasi-local mass
CHRISTOS MANTOULIDIS AND PENGZI MIAO 99

BMS symmetries, soft theorems and the membrane paradigm
PRAHAR MITRA AND ROBERT PENNA 109

A summary of some new results on the formation of shocks in the presence of vorticity
JARED SPECK 133
Preface

Nonlinear partial differential equations (PDEs) on the one hand describe phenomena in science, technology, and related fields; and on the other hand provide an excellent excuse for fascinating mathematical research. Nonlinear PDEs comprise various subfields concerned with different phenomena. Within this larger frame we find mathematical general relativity, with Einstein equations at its core. The latter are a system of non-linear (in fact quasi-linear) PDEs that can be brought into hyperbolic form. They describe our universe by relating its geometric structures to matter and energy content. Hereby, gravitation acts through curvature. Solutions of the Einstein equations are Lorentzian metrics, and the universe or its parts are modeled as manifolds equipped with these metrics; we call these spacetimes. Whenever other fields are present, they obey their own equations and we obtain a coupled system. Examples include the Einstein–Maxwell and the Einstein–Euler systems.

The main goal of mathematical general relativity is to understand the global structure of solutions of Einstein equations. This is often done by solving the Cauchy problem for the Einstein equations—with initial data corresponding to situations of physical interest—and studying the resulting spacetimes. Mathematical general relativity has thus solved challenging mathematical problems and given answers to long-standing questions in physics. Along the way, it has led to many intriguing new questions in mathematics, pushing further research in related areas.

During the 2015–2016 year at the Harvard Center of Mathematical Sciences and Applications (CMSA), several researchers working in mathematical general relativity presented lectures on modern topics of research in the field of “Non-linear Equations.” This volume presents articles—by those researchers and their co-authors—drawn from those CMSA lectures.

Specific topics include the Cauchy problem for the Einstein equations in cosmological and non-cosmological settings; investigation of stability as well as singularities (black holes) of classes of spacetimes; initial data engineering; gravitational radiation; and asymptotics of spacetimes, quasi-local energies, and their limits.

The content of this volume reflects some of the activities at the Harvard CMSA during the 2015–2016 program, and provides insights into active
areas of research in mathematical general relativity that can benefit scholars working in PDEs, geometric analysis, and general relativity.

We thank the Harvard CMSA for its hospitality, the National Science Foundation and the Evergrande Group for their financial support, and the authors who contributed to this book for their efforts.

The Editors:
Lydia Bieri
Piotr T. Chruściel
Shing-Tung Yau